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Gliding- induced ski vibrations: approaching proper modeling

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Abstract

Phenomena of the ski and snow boards vibrations generated in gliding are known for years. In the cross country and jumping skis such vibrations are not very obvious but can play quite positive role reducing the effective gliding friction. The research into the nature of friction-induced vibrations and the factors influencing their frequencies and magnitudes is driven by the desire to control them for improving ski gliding performance. Significant amount of experimental data acquired in the field and laboratory studies is already available making it possible to formulate certain qualitative conclusions. But so far it did not bring comprehensive understanding of the phenomenon and specifically of the mechanisms controlling such vibrations. Modeling is one of the potent tools allowing to deeper understand experimentally studied phenomena and it can provide much stronger quantitative prediction capacity. Present paper discusses possible approaches to modeling of the phenomenon and first results of constructing simplified models.

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Keywords: skis; gliding efficiency; induced vibrations; nonlinear; parametric resonance; experimental; modeling

1. Introduction

The studies into the cross-country ski vibrations are inspired by the clear practical interest. There are multiple indications, that such vibrations can and indeed do have a serious impact on gliding efficiency for the cross-country and alpine skis, and the snowboards (Koptyug and Kuzmin (2011), Koptyug et al (2012, 2013), Shinoya and Sato (2013)). Reduction of the effective friction caused by the self-generation of friction-induced vibrations is also well

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known for the mechanical systems. Vibrations induced by the running surface friction of the skis and snowboards with the snow have relatively high intensity in the lower acoustic frequency range – up to few KHz (Lehtovaara (2005), Koptyug and Kuzmin (2011), Shinoya and Sato (2013)). The existence of the mechanical system instabilities leading to the intense oscillations is known for the systems with both dry and wet friction, including the case of bodies sliding on snow and ice (Buhl et al (2001), Lehtovaara (2005), Bäurle (2006)). So there exists an intriguing possibility that certain improvements into the gliding efficiency of the cross country skis can be achieved by controlling the ski vibrations generated during the gliding. Such control is not possible without a profound knowledge of the mechanisms causing these vibrations and the effects influencing their major characteristics. With this in mind an extensive research into the cross country ski vibrations is carried out both in the laboratory conditions and in the field. Present paper reports on the analysis of experimental data acquired and on the possible approaches in modeling the phenomenon.

2. Modeling of the ski vibrations- results and discussion

Modeling is one of the potent tools for the generation of new knowledge. As any of the advanced tools it has certain application rules following which allows for the efficiency and precision (Zeigler et al (2000), Sokolowski and Banks (2009)). According to the general rules, firstly the models are intrinsically limited by their purpose and the selection of facts they should take into account. Secondly, to be effective models should be based upon certain simplifications of the chosen system otherwise the excessive amount of details can cause the loss of clarity. We also are interested in a specific class, namely mathematical or computer models, allowing for the quantitative forecasts. At the moment the main purpose of the desired model is better understanding of the ski vibration phenomenon (including this possibility of making quantitative forecasts). At the initial stages of the modeling one selects for the consideration most prominent facts and major effects. At the early modeling stages it also pays to see if there are similar systems with already constructed models that can be used at least partly for our case.

One of the most interesting facts formulated basing on the large amount of data acquired with the field experiments is a significant difference in the ski vibration frequencies detected by the pressure sensors placed over the ski track. Detected vibration frequencies are different even under very similar experimental conditions: same day, same skier and same slope, similar decent rates (Koptyug and Kuzmin (2011)). One of the most probable reasons for these variations is a difference in ski loading caused by the changes of the skier posture in different descends also recorded experimentally (Koptyug and Kuzmin (2011), Koptyug et al (2012)).

Laboratory experiments with the forced excitation have confirmed the presence of the frequency offset of the acoustic resonances of loaded skis. The same time the offset dependence on the loading is quite nonlinear. For some of the most intense resonance modes the frequency offset first grows with increasing loading, but for higher loads is decreasing down to almost zero values (Koptyug et al (2012-2013)). Thus we are dealing with a nonlinear system. Experimental studies have also shown that in modern cross country skis some of these most intense modes have a very peculiar nature and corresponding resonances are parametric (Koptyug et al (2012-2013)). Most interesting, in our recent studies we were not able to detect any parametric resonances in the old all-wooden skis and in stiff alpine skis. Thus one can speculate that corresponding strong nonlinearities leading to parametric resonances are characteristic only for relatively soft skis with composite materials in the construction having significantly nonuniform cross-sectional area.

Thus we need to develop a model allowing for the nonlinearity of the studied system (skis), revealing the mechanism of the self-generation of friction- induced vibrations and describing a very special type of resonances and offset behavior of the resonance modes with loading. Also for the purposes of initial modeling one can possibly approximate the ski by an “arched beam with variable cross section and variable elasticity properties”.

Analysis of the literature has shown that models describing self-generation of the friction induced vibrations do exist. In the approach suggested by Hoffman (2007) resonance system with the losses is modeled by the ordinary differential equation with the stochastic excitation force representing the stick-slip character of the friction of the sliding body. In the free gliding of the skis and snowboards stick-slip character of the friction is quite prominent. Also a large number of small disturbances in the snow and even freshly prepared ski track will also produce stochastic excitation forces. Interestingly, that this model shows that corresponding resonances are parametric, with all the peculiarities of such described in the literature (Napoli et al (2003), Butikov (2005)) and detected in our experiments (Koptyug et al (2012-2013)). Application of this approach in our case demands representing our nonlinear system as an oversimplified resonator with effective parameters (effective mass, effective loss factor, effective elasticity constant). This may be a viable way forward, but one needs reliable ways of extracting the equivalent parameters needed for this model from experimental data or using other modeling approaches.

Extracting effective resonance parameters for the curved nonuniform beams (such as cross country ski) with precision is not a trivial task. Literature search yields two major approaches used for such beams, but only for extracting of the self-resonance frequencies. Both approaches use modeling, but one is using quasi-analytical or approximately- solution of differential equations and the other- finite element computations (e.g. Blumenfeld and Cizmas (2004)). Though both approaches are well developed, feeding one model with the results of another modeling seems little too far fetching at the initial stages of model development. Thus we have chosen much simpler approach, based on the calculation expressions derived for the uniform beams.

Extracting effective resonance parameters for the uniform beams (uniform material properties and uniform beam cross-section) is much simpler problem. Though such approach is very approximate it can still be valuable at the initial steps of the modeling and equivalent "beam parameters" of the skis can be relatively easily extracted from the experimental data. For the free-hanging cantilever beam (one end of the beam is clamped, another- left free, Fig. 1a) with constant cross section loaded by a distributed force (e.g. its own weight) the deflection of the free end x can be found as (Stokey (2010)):

$$x = \frac{Wl^3}{8EI}, \; I = \frac{1}{12}bh^3 \tag{1}$$

where $\boldsymbol{W}$ is total load, $\boldsymbol{l}$ is the total length of the beam, $\boldsymbol{E}$ is the modulus of elasticity, $\boldsymbol{I}$ is the moment of inertia, $\boldsymbol{b}$ and $\boldsymbol{h}$ are the breadth and height in the cross-section of the beam. For the same beam but additionally loaded at its free end (Fig. 1b) one can use another simple expression (Stokey (2010)):

$$x = \frac{Wl^3}{3EI} \tag{2}$$

To determine the natural frequencies for the normal modes of the uniform beam with rectangular cross-section one can use the following expressions (Finot et al (2008), Stokey (2010) Table 7.3):

$$\omega_1 = (1.875)^2 k\,,\; \omega_2 = (4.694)^2 k\,,\; \omega_3 = (7.855)^2 k\,,\; \omega_4 = (10.996)^2 k\,, \text{ where } k = \sqrt{\frac{EI}{ml^4}} \tag{3}$$

where $\boldsymbol{m}$ is the overall mass and $\boldsymbol{l}$ is the overall length of the beam, $\boldsymbol{E}$ is the modulus of elasticity and $\boldsymbol{I}$ is the moment of inertia. All of the above expressions are quasi- analytical (derived from approximate solutions of differential equations) and are widely used in the courses of advanced engineering mechanics.

Clamping the ski at its rear end and measuring the deflection of its front bending under its own weight and varying load one can extract the product $\boldsymbol{EI}$ of the elasticity modulus and the moment of inertia by solving equations (1) and (2) for known length and weight of the skis and the loading force. Using these values one can calculate self-resonance frequencies of the skis in the approximation of the uniform beam using expressions (3). To compare these values with experimental ones we have measured the forced- excitation resonance frequencies of different skis fixed at the back end or in the middle and excited at the front.

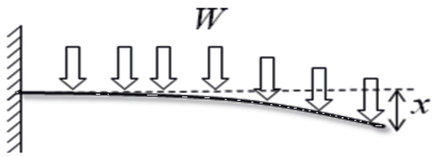


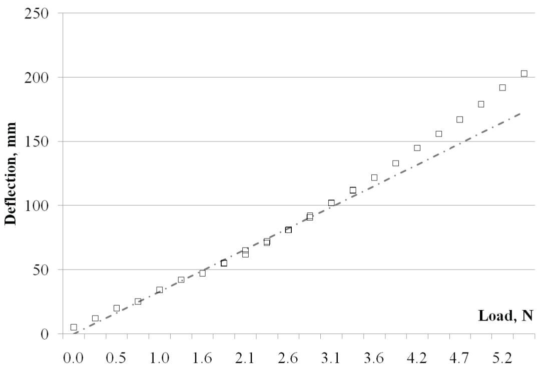
Fig. 1. (a) Cantilever beam loaded by distributed force; (b) Cantilever beam loaded at its floating end

Table 1 presents the parameters of the skis used in the comparison together with the bending constants (deflection to loading force ratio) measured for increasing loads extracted using linear fit (Bending[1] for small, Bending[2] for medium and Bending[3] for larger deflecting loads).

Table 1. Major parameters and bending constants for the skis used in the experiment.

Manufacturer	type	ski make	length	weight	Bending[1]	Bending[2]	Bending[3]
			cm	g	mm/N	mm/N	mm/N
Madshus	classic	Hypersonic 252 Cold classic	210	631	30.25	49	66
Rossignol	classic	Silver 44 Carbon	195	472	31.7	40.7	-
Atomic	skating	RS11 nomex	190	528	54.4	62.8	-
Karhu	classic	Kick Easy Wax	205	893	47.4	84.2	-
Edsbys	old classic	Ski Master Racing #748	210	1004	26.5	29	-
Fischer	alpine	R14 Powercore, carved	140	1620	12.3	13.5	-

Fig. 2 presents the dependences of the deflection of the ski front on the loading force for the modern composite ski (Fig. 2a) and older all-wooden ski (Fig 2b). Straight lines present the results of the linear fitting for the small loads (up to about 2.6 N in case Fig. 2a and about 6.5 N in case of Fig. 2b). These two graphs illustrate, that deflection of the modern composite skis is quite nonlinear even for relatively small loads, while wooden skis are linear for much larger load range. It is also clear that all-wooden ski (Fig. 2a) is much stiffer than modern lightweight composite one (Fig 2b).



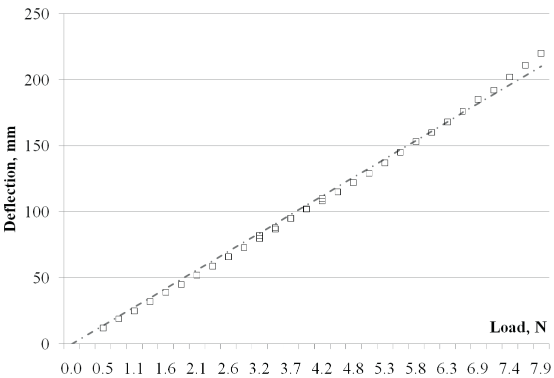


Fig. 2. (a) Deflection Vs load curve, Rossignol (see Table 1) (b) Deflection Vs load curve, Edsbys (see Table 1)

Fig. 3 illustrates the comparison of the experimental frequency response spectra with forced excitation and corresponding natural frequencies calculated using the parameters extracted from the deflection measurements. Spectra are represented by the overlaid traces from three tri-axis accelerometers placed on the upper ski surface at its front, middle and rear parts (see Koptyug et al (2012-2013) for the detail on method and measurement set-up).

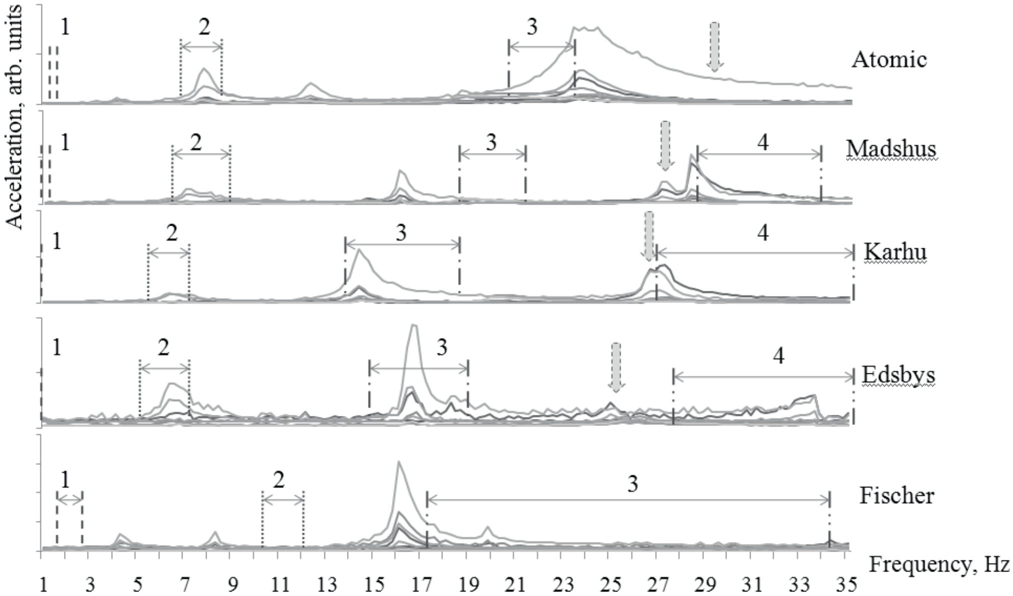


Fig. 3 Comparison of the measured and calculated frequencies for the ski vibration normal modes

The frequency values for the ski natural resonances were calculated in the approximation of the uniform cantilever beam according to the equations (3). Linear functions were fit to the nonlinear curves of the deflection Vs load dependence of the sis fixed as the cantilever beam (Fig. 1) for the small, medium and larger load values yielding up to three values for the product ***EI*** of the elasticity modulus and the moment of inertia. Corresponding natural frequencies were than calculated according to equation (2). Additional ***EI*** product value was extracted according to equation (1) from the deflection value of the free hanging skis. Calculated values for the frequencies of the natural resonances have significant spread, as indicated by the differences in the bending constants shown in Table 1. In Fig. 3 corresponding spread zones for the calculated frequencies of natural resonances are shown by dotted lines and numbered according to the natural resonance number. Vertical grayed arrows indicate the positions of the calculated first natural resonances for the cantilever beam excitation in the case when the ski is fixed at the middle.

For all studied skis the efficiency of first natural mode excitation is very low, just a hint is often present on the measured frequency curves. Despite the significant spread in the values of the calculated resonance frequencies due to the nonlinearity of the deflection Vs load relations positions of the natural resonances are correctly reproduced in the majority of cases with the exception of the Fischer alpine skis (very stiff ones, carved, with metal facings) and the Madshus (cross country skis), in the latter case the calculated values are some higher than the ones measured experimentally. Interestingly, the width of the experimentally measured resonances is also with the reasonable agreement with the spread of the calculated values.

3. Conclusions

Analysis of the models constructed for the similar engineering examples reveal that two approaches can be fruitful in the future. Solving differential equations with the stochastic excitation force seems to be most adequate for the studies of the mechanisms of the self-generation of friction induced vibrations. But this approach uses a very simple approximation of the resonance system. Finite element methods and semi-analytical approach seem to be most adequate for calculating of the natural frequencies of the skis approximated as curved beams with nonuniform cross-section and elastic properties. Unfortunately this method does not allow for the analysis of the self oscillation onset. Most probably, adequate modeling of the friction-induced ski and snowboard self-oscillations in free gliding will require combining both of these approaches.

Simplified alternative for the analysis of the ski vibration modes and natural mode frequencies is provided by the expressions derived for the uniform beams. The comparison of the calculations performed in such way for one of the configurations (cantilever beam, or clamped-free configuration) with the experimentally measured data given in present paper shows reasonable agreement. Simple way of extracting "effective beam parameters" from the experimentally measured deflection values of the clamped-free and clamped-loaded skis used in this comparison may be efficiently employed for developing more complex models.

One of the necessary future steps is the comparison of the calculated frequencies for the clamped-clamped (fixed at both ends) and hinged-hinged (laying on the supports at both ends) beam ski models with the experimentally measured frequencies for the loaded skis lying on the rig. Another step is extracting the "effective beam parameters" basing on the load dependences of the clearance between the ski and solid plane floor, which should be closer to the reality of ski gliding.

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