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Core Loss Calculation of Symmetric Trapezoidal Magnetic Flux Density Waveform

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ABSTRACT Existing empirical core loss models for symmetric trapezoidal flux waveform (T_zFW) still suffer some issues such as the inaccuracy and the complexity. These issues are mainly due to the lack of an accurate model of the relaxation loss generated during the off-time. This paper aims to understand the relaxation loss and develop an accurate model using the superposition technique. The developed model gives an accurate prediction of the on-time loss and the relaxation loss and shows the dependency of each on the duty cycle. The research shows that the core loss at low duty cycle is several times the core loss at full duty cycle. The developed model is verified with experimental results and compared to the Improved Steinmetz equation (ISE). The model error is reduced to lower than 15% compared to 50% of the ISE. Finally, an easy method using multiplication factors with the ISE model is given to simplify the developed model.

INDEX TERMS Symmetric Trapezoidal flux waveform, relaxation loss, core loss, improved Steinmetz equation.

NOMENCLATURE

D:	duty cycle
T_1, T_0, T_2:	on-time, off time and period
f_1, f_0, f_2:	on-time, off-time and switching frequencies
B:	magnetic flux density
(k_{e1}, α_{e1}, β_{e1}):	Steinmetz parameters during on-time
(k_{e2}, α_{e2}, β_{e2}):	Steinmetz parameters during off-time
P_{tr}:	total core loss of the trapezoidal flux waveform
P_r:	core loss during on-time in the frame T_1
P_{reff}:	core loss during on-time in the frame T_2
P_0:	relaxation loss

I. INTRODUCTION

Thanks to its significant impact on the volume, the efficiency and the cost of the converter, the magnetic devices are considered to be the most critical components in the design of DC/DC converters. Moreover, the sizing of the remaining converter components, such as the switching devices and the capacitors, significantly depends on the sizing of inductive components. The design of the magnetic devices involves several parameters and variables like the core shape, the magnetic material, the number of turns, the switching frequency, the flux density and the temperature. What characterizes these variables is their interdependence and their non-linear relationship with the core loss equations [1]. Core loss are also very dependent on the excitation waveform and the duty cycle [2], [3]. The diversity in the excitation waveforms, generated by wide DC/DC converters topologies, makes the calculation of the core loss more complicated. So, the challenging task is to accurately model the magnetic loss for the optimization of the magnetic devices [4]–[8].

There has been much effort spent on predicting the core loss for the triangular flux waveform. As a result of this effort, various models were developed based on the Steinmetz equation (SE), which is suitable for sinusoidal excitation only [9]. These models are mainly the modified SE (MSE) [10], the generalized SE (GSE) [11], the improved GSE (IGSE) [12] and the improved SE (ISE) [2] successively. Results show the improvement of the work performance of each model. However, the ISE model has shown higher accuracy by including the effect of the duty cycle on the core loss. Also, the results in [2] shows that the core loss can, at a duty cycle of 0.1, be 6 times higher than the one of 0.5. This point is very important for worst case designs because the required size of the transformer at duty cycle of 0.1, is much greater than the one of 0.5. Reference [2] showed also the importance of changing the Steinmetz parameters as the duty cycle changes to improve the model accuracy.

Unlike the triangular flux waveform (TFW), the core loss models developed for the trapezoidal waveform (T_zFW) still lack the accuracy.

The voltage waveform corresponding to the symmetric trapezoidal flux waveform usually includes a zero-voltage component. During the zero-voltage time (off-time), the flux B keeps constant. At constant B, the core loss is supposed to be null, however, due to the relaxation phenomenon, there is still an excess loss during this time [13]–[16]. This excess is called the relaxation loss and depends on several factors such as the zero-voltage time and the previous core loss history generated during the on-time [17]–[19].

In the literature, three main attempts were introduced to model the core loss of T_zFW [20]. In order to simplify the understanding of these limitations to the reader, the principle of core loss and its causes are explained first in the following. Then, the limitations are shown in Section III.

From a physical point of view, core loss is generated by two technical mechanisms which are the displacement of the domain walls and the rotation of the domain magnetization after the application of a magnetic field [17]–[19]. The strength and the rate of variation of the magnetic field are the main causes of these mechanisms. The relationship relating the core loss to the magnetic field is non-linear. It can be mathematically written using a power function of two variables (f and B). This model is known as the Steinmetz equation and f is the frequency which represents the rate of variation of B [9]. For equal on-time and off time, f is same as the switching frequency. However, for unequal on-time and off-time, there exist two effective frequencies: one for the on-time and one for the off-time [2]. To fully model the non-linear phenomenon as function of B and f, three parameters are required, k, α and β known as the Steinmetz parameters. The most important characteristic of these parameters is they are frequency dependent. As an example, for triangular flux waveform with unequal on-time and off-time, two sets of the Steinmetz parameters are needed for the core loss calculation.

There is an excess loss generated during the off-time for the case of trapezoidal flux waveform which is a complicated phenomenon to model. It is known in the literature as the relaxation loss which can be explain as follows. For a T_zFW and during one operating cycle (Fig. 2), the energy loss during the on-time reaches its maximum at DT. Then, the magnetic flux density keeps constant during the off-time, so that B will not contribute to any magnetic loss. At the instant DT, the core starts its relaxation process generating some excess loss which depends on the previous loss history. As the off-time increases as the energy loss increases until it is fully dissipated [21], [17]–[20]. So, depending on the length of the off-time period, the energy loss can be fully or partially dissipated. Therefore, from a power loss point of view, at a given off-time,

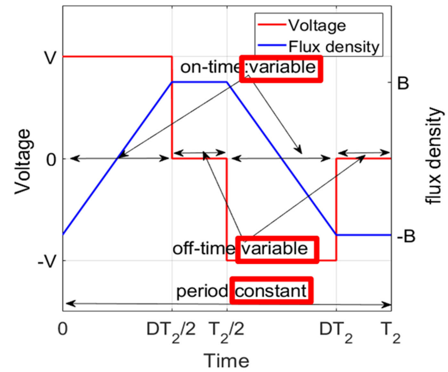


FIGURE 1. Typical waveform of the voltage and the flux density in full-bridge converter investigated in this study, operating under constant frequency.

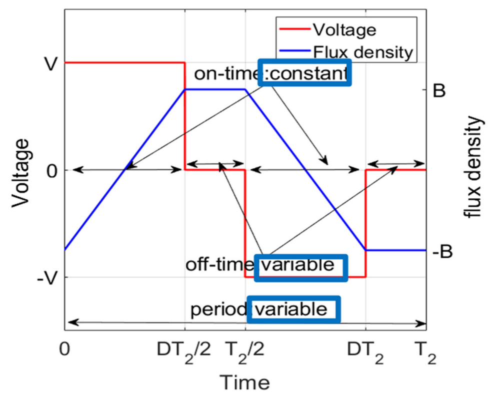


FIGURE 2. Typical waveform of the voltage and the flux density in full-bridge converter investigated in [21], operating under variable frequency.

the relaxation loss should have a maximum. The corresponding off-time to this maximum is called the critical relaxation time [21]. In power electronics field, the relaxation loss was first investigated in [17]. More effort was performed in [21] showing that the relaxation loss has maximum at a given off-time confirmed by the fundamental theory of the Richter type relaxation loss [17]–[20]. The details of the relaxation loss and their existing models are presented in Section III.

To summarize the shortcomings of the previous works, [21] has presented core loss models which are valid only for converters working under the following two conditions: variable frequency and constant on-time (Fig. 2). Therefore, it is not applicable for converters operating under constant frequency and variable on-time which is the most utilized PWM technique (Fig. 1). The model in [16] could be applied for the waveform presented in Fig. 1 but still has serious issues, explained in Section III. In addition, both models are very complex and need additional parameters which are material dependent and have to be characterized through experimental measurements.

This paper is a continuation of the previous work performed in [21]. It mainly aims to characterize the core loss of the symmetric trapezoidal flux waveform under constant frequency

(Fig. 1). The key feature of the simplified version of the developed model is it uses the Steinmetz parameters only with few correction factors.

A. RESEARCH CONTRIBUTIONS

The main contributions of this paper could be summarized as follows:

- Develop an accurate and a straightforward model for the symmetric T_ZFW. through an accurate prediction of the relaxation loss.
- Study the evolution of the on-time loss and the relaxation loss with respect to the duty cycle.
- Evaluate the developed model with experimental results.
- Review the existing core loss calculation models of the symmetric T_ZFW, and discuss their limitations.

This paper is structured as follows: Section III reviews the existing core loss models for the trapezoidal flux waveform and the measurement techniques. Section IV details and analyze the used approach of the developed model, and then by referring to the results obtained from the experiments, we will assess its accuracy .Finally, the paper's findings are outlined in the conclusion.

II. REVIEW OF THE CORE LOSS MODELS AND THE MEASUREMENT TECHNIQUES FOR TRAPEZOIDAL INDUCTION

A. CORE LOSS MODELS

Core loss of the trapezoidal flux waveform has been an area of investigation over the last several years. As confirmed in the works published in [13]–[15], the core loss model shows a loss increase in comparison with the triangular waveform. The disparity can amount to 40, 50% under same peak flux density and frequency.

The first attempt towards predicting the core loss was realized by proposing the IGSE model with the principle of the composite waveform hypothesis. Yet, this method proved to be unsatisfactory, because of two major limitations.

The first limitation is that the Steinmetz parameters, calculated at the switching frequency, may lead to a serious error when the duty cycle fall from 0.5. As analyzed in [2], the Steinmetz parameters should be determined to an effective frequency which represents the time variation of B and not to the switching frequency. Fig. 3 clearly shows the dependency of the Steinmetz parameters on the duty cycle for the case of 3C92 ferrite material. Hence, a negligible error in the determination of these parameters can lead to a huge error in the loss calculation because of the power function.

The second limitation is that the model does not consider the relaxation loss generated during the off-time which was shown in [16] and [21].

The second attempt was performed in [16]. An improved equation of the IGSE, called the Improved IGSE (1), was developed to include the effect of the relaxation loss (2) in order to improve the accuracy. Although the IIGSE managed

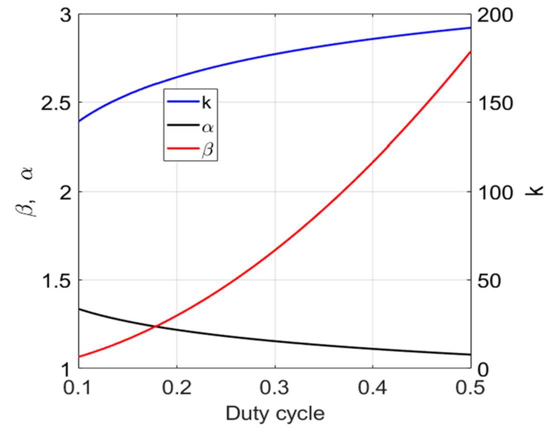


FIGURE 3. Dependency of the Steinmetz parameters on the duty cycle for 3C92 (f = 100 kHz).

to overcome the second limitation of the IGSE, it still contains serious issues. A detailed explanation of these issues is discussed in [21] and can be summarized as follows. The first limitation of the IGSE is not solved by the IIGSE. As a result, part of the generated loss during the on-time is included in the relaxation loss term. This is because the IGSE underestimates the on-time loss as shown in [2]. Therefore, the second term of eq (1) overestimates the true relaxation loss.

One other issue of the IIGSE is related to the general model, which is designed for the triangular flux waveform (TFW). It includes the relaxation loss term which is physically inaccurate because it is supposed that there is no relaxation time for this waveform. It was stated in [2] that the core loss of the TFW could be accurately calculated using the ISE without the need of any additional parameter.

$$\langle P_c \rangle = \frac{1}{T}\int_0^T k_i \left|\frac{dB}{dt}\right|^{\alpha-\beta} (\Delta B)^{\beta} dt + \sum_{l=1}^{n} P_{rl} Q_{rl} \quad (1)$$

$$\langle P_{rl} \rangle = \frac{1}{T} k_r \left|\frac{d}{dt} B\left(t_-\right)\right|^{\alpha_r} \Delta B^{\beta_r} \left(1 - e^{-t_1/\tau}\right) \quad (2)$$

$$Q_{rl} = \begin{cases} e^{-q\left(\frac{D}{1-D}\right)}, & Triangular\ waveform \\ 1, & Trapezoidal\ waveform \end{cases} \quad (3)$$

In [21], an improved model was introduced to characterize the relaxation loss of the trapezoidal flux waveform (5)–(7). The model is based on the superposition technique by summing up both the on-time loss and off-time loss (relaxation loss). The main idea of the work presented in [21] is to firstly calculate the relaxation loss (off-time loss) by subtracting the on-time loss from the total loss obtained from measurement results and then develop an empirical equation for the remaining loss (5). The on-time loss (6) or the first term can be accurately calculated using the ISE model (4). The ISE model is given by the following equation.

$$P_c = \frac{\pi}{4}\left[k_{e1}\, D\, (f_{e1})^{\alpha_{e1}} B^{\beta_{e1}} + k_{e1}\,(1-D)\,(f_{e2})^{\alpha_{e2}} B^{\beta_{e2}}\right] \quad (4)$$

TABLE 1 **Advantages and Limitations of The Reviewed and Developed Core Loss**

methods	advantages	Limitations
IGSE, ISE	•Easy to apply •Only Steinmetz parameters required	•Not valid for trapezoidal flux waveform
IIGSE[16]	•Good accuracy •Valid for trapezoidal flux waveform Fig.2	•Inaccurate off-time loss calculation •5 additional parameters are required •Impractical due to the missing parameters
Model [21]	•Good accuracy •Valid for trapezoidal flux waveform of Fig.2 •Accurate off-time loss calculation	•3 additional parameters are required
Model (20)	•Good accuracy •Valid for trapezoidal flux waveform of Fig.1 •Accurate off-time loss calculation	•3 additional parameters are required
Simplified Model (22)	•Valid for trapezoidal flux waveform of Fig.1 •No additional parameters are required •Acceptable accuracy and straightforward to apply	•Sensitive to magnetic materials •Suitable for worst case designs

where (k_{e1}, α_{e1}, β_{e1}) and (k_{e2}, α_{e2}, β_{e2}) are respectively the Steinmetz parameters for the on-time and the off-time of the triangular flux waveform.

The relaxation loss is expressed in [21] as follows:

$$P_0 = \left(x \cdot R^y + z\right)\ P_{reff} \tag{5}$$

where (x, y, z) are parameters determined by curve fitting and R is defined in (10).

The on-time loss P_{reff} is calculated by applying (4) on frame T_2. We note that the on-time parts represent the triangular flux waveform.

$$P_{reff} = D\,\frac{\pi}{4}\left(k_{e1}\ \ {f_{e1}}^{\alpha_{e1}} B^{\beta_{e1}}\right) \tag{6}$$

Finally, the total loss of the trapezoidal flux waveform is:

$$P_c = P_0 + P_{reff} \tag{7}$$

Similar to the experiments achieved in [13]–[15], the measurements in [21] are carried out by varying the off-time and by keeping the on-time constant. As a result, it cannot be used to calculate the core loss of the investigated waveform of Fig. 1 (constant frequency). On the other hand, the work in [21] has presented two interesting outcomes: first, it allows to understand the physical evolution of the relaxation loss with respect to the off-time and secondly it proposed an empirical model to calculate the relaxation loss.

To conclude this section, we summarize the advantages and the limitations of the discussed models as well as the developed models in this paper in Table 1.

B. CORE LOSS MEASUREMENT TECHNIQUES

The choice of the suitable measurement technique is an important factor for an accurate modeling of core loss. Generally, the core loss measurement methods can be classified into two main groups: the electrical methods and the calorimetric methods. The calorimetric techniques are more accurate because all the core loss can be transformed into heat and measured with a good precision. Its major flaw is the time consuming and the high cost. On the other side, the electric methods have easier set up but their accuracy can degrade due to the limitations of the measurement instruments especially at high frequency.

A fast-calorimetric measurement technique that uses a thermoelectric device to measure power loss is given in [21] and [23]–[24]. This method is based on Peltier effect where the heat flow can generate electric power that can be accurately measured with a high precision digital meter. The reader is referred to [21] and [24] for more details about the calibration of the Peltier device and the set-up of this technique.

The core loss generated in the trapezoidal flux waveform includes two types of losses. Whereas the first type is the losses generated during the on-time because of to the flux variation, the second type is the relaxation loss generated during the off-time. It is obvious that the second type cannot be measured using the electric techniques because during the off-time, the voltage is null which eventually results in a zero-power loss. Thus, the core loss of the trapezoidal flux waveform can only be measured accurately through the calorimetric techniques. The electric methods for the trapezoidal waveform can lead to a huge error because the relaxation loss can reach up to 30% of the total loss as has already been demonstrated in [21].

Its easy implementation, along with good accuracy and low testing time, the fast-calorimetric technique that uses the thermo-electric device is our best choice.

III. MODEL DEVELOPMENT AND ANALYSIS

The methodology presented in [21] is used in this work to develop an improved model for the calculation of the core loss of the trapezoidal waveform under constant frequency. Firstly, the on-time loss is calculated using the ISE. Among the many suggested models, the selection of ISE has been made in view of its accuracy and simplicity to calculate the on-time loss. Second, the relaxation or off-time loss are calculated using the model developed in [21]. Ultimately, the total loss is obtained by summing up the on and off-time losses using the superposition technique.

A. MODELING APPROACH

First of all, it is useful to determine the relationships relating the effective frequencies of the on-time and the off-time (f_1, f_0) to the switching frequency f_2.

$$f_0 = \frac{1}{T_0} = \frac{1}{T_2 - T_1} = \frac{1}{(1-D)\ T_2} = \frac{f_2}{(1-D)} \tag{8}$$

$$f_1 = \frac{1}{T_1} = \frac{1}{T_2 - T_0} = \frac{1}{D\,T_2} = \frac{f_2}{D} \tag{9}$$

$$R = \frac{f_0}{f_1} = \frac{D}{(1-D)} \tag{10}$$

The energy loss of the trapezoidal flux waveform is the sum of the energy losses during the on-time and the off-time.

$$E_{tr} = 2\,(E_1 + E_0) \tag{11}$$

The power loss is defined as the energy per unit time as follows:

$$P_{tr} = \frac{2\,(E_1 + E_0)}{T_2} = 2f_2E_1 + 2f_2 \tag{12}$$

It can be written by the following form:

$$P_{tr} = \frac{f_2}{f_1}P_r + P_0 \tag{13}$$

where P_r is the power loss in the frame T_1 and P_0 is the relaxation loss in the frame T_2. P_r can be calculated using the ISE model (4). It is written as follows:

$$P_r = \frac{\pi}{4}\left(k_1\ \ f_1{}^{\alpha_1}B^{\beta_1}\right) = \frac{\pi}{4}D^{-\alpha}\left(k_1\ \ f_2{}^{\alpha_1}B^{\beta_1}\right) \tag{14}$$

Let's denote by P_{reff} as the power loss P_r projected in the frame T_2. Using (9) and (12), it can be written as a function of P_r and the duty cycle as follows:

$$P_{reff} = \frac{f_2}{f_1}P_r = D\,P_r \tag{15}$$

Hence, we get:

$$P_{tr} = P_{reff} + P_0 \tag{16}$$

Using the model developed in [19], the relaxation loss is expressed by (5):

$$P_0 = g\,P_{reff} = g\,D\,P_r \tag{17}$$

where g is a power function given as follows:

$$g = x \cdot R^y + z \tag{18}$$

Substituting (14), (15) and (17) into (16), P_{tr} is equal to:

$$P_{tr} = \frac{\pi}{4}D^{1-\alpha_1}\,(g+1)\ \ \left(k_1\ \ f_2{}^{\alpha_1}B^{\beta_1}\right) \tag{19}$$

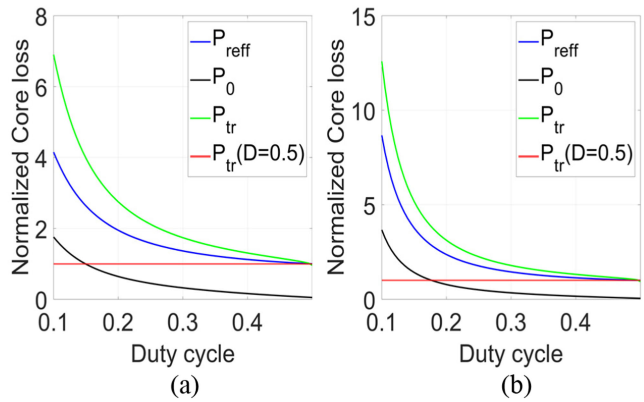


FIGURE 4. Normalized core loss ($\frac{P_{tr}}{P_{tr}(D=0.5)}$)evolution with respect to the duty cycle using model (19), f = 100 kHz, T = 100 °C, (a)-ER32/6/25-3C92, (b)-ER32/5/21-N87.

B. THEORETICAL ANALYSIS OF THE LOSS MODEL

The present section discusses the evolution of the on-time loss, the relaxation loss and the total loss with respect to the duty cycle. Fig. 4 shows the implementation of developed loss model for 3C92 and N87 ferrite materials. At full duty cycle, the trapezoidal waveform is a typical triangular waveform and its corresponding total loss are minimum. As the duty decreases, the on-time loss P_{reff} increases due to the increase of the rate of variation of B. Similarly, the relaxation loss significantly increases from zero at full duty cycle until it dominates $P_{tr}(D = 0.5)$ at a particular duty cycle. This clearly shows the large portion of the relaxation loss which should never be neglected. In this study, the relaxation loss does not show a maximum as found in [21] and this is because the on-time loss is variable and is not constant as in [21]. It should be noted, however, that the loss curves proves that: the rate of evolution of the relaxation loss and P_{reff} are quite similar. In other words, the variation of the relaxation loss can be expressed as the function of the duty cycle and P_{reff}, which in turn helps to simplify the model of the relaxation loss. Therefore, P_{tr} can be predicted using only the ISE model (4) and henceforth the three additionally parameters *(x, y, z)*, of function g (18), can be eliminated.

IV. EXPERIMENTAL VERIFICATION AND DISCUSSION

A. TEST SET-UP DESCRIPTION

The test set-up is described in Fig. 5. The tested materials are 3C92 and N87 using ER32/6/25 and ER32/5/21 cores respectively [25]–[26]. The winding is 5:2 turns for each core. To compensate the coupling factor of the transformer, the required magnetic flux density is measured across the secondary winding. For a given case (B, f). The duty cycle is swept in the range [0.1:0.05:0.5] and the input voltage is adjusted accordingly to keep the flux density constant. For cases (b) and (d) (see Fig. 9 for the test conditions), the measurements were stopped at the shown duty cycle because the magnetic cores had seen a thermal run away due to the loss increase at low duty cycle. The calorimetric measurements are achieved using

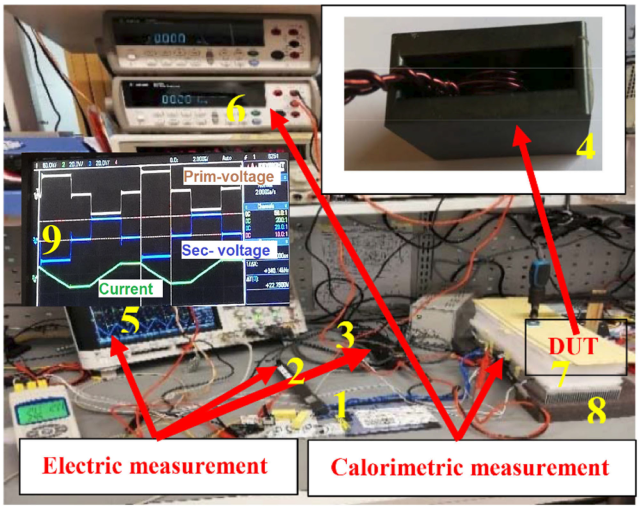


FIGURE 5. Test setup of the core loss measurement: 1-Full-bridge converter, 2- current probe, 3-voltage probe, 4-DUT (tested core), 5-oscilloscope, 6-digital meter, 7-insulation material, 8-heat sink, 9-example of the voltage and current waveforms [21].

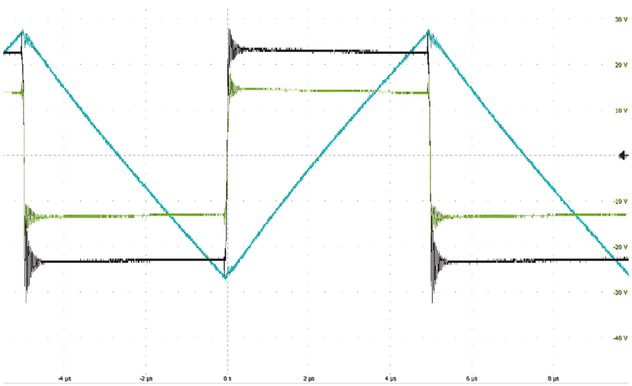


FIGURE 6. Typical waveforms of the primary voltage (green, 20V/div), the secondary voltage (black, 5V/div) and the magnetizing current (blue, 20mA/div) for D = 0.5, B = 0.1T, f = 100 kHz, 3C92, (time = 2μs/div).

TABLE 2 Approximate Loss Factor Between the Trapezoidal Flux Waveform Loss and the ISE Model

D	0.45	0.4	0.35	0.3
D_F	1.05-1.1	1.1-1.2	1.15-1.25	1.2-1.3
D	0.25	0.2	0.15	0.1
D_F	1.2-1.35	1.3-1.4	1.35-1.45	1.4-1.5

the Peltier device TEC1-12706 [22]. The Seebeck voltage of the Peltier device is measured using a high-resolution digital multi-meter (HCM8012) (Fig. 5). The trapezoidal waveform is generated using a full-bridge converter. An example of the applied waveform of the voltage and the current at the extreme duty cycle (0.5 and 0.1) is presented in Fig. 6 and Fig. 7.

B. ASSESSMENT OF MODEL (19) WITH MEASUREMENTS

To verify the accuracy of model (19), we need to consider two main effect as follows:

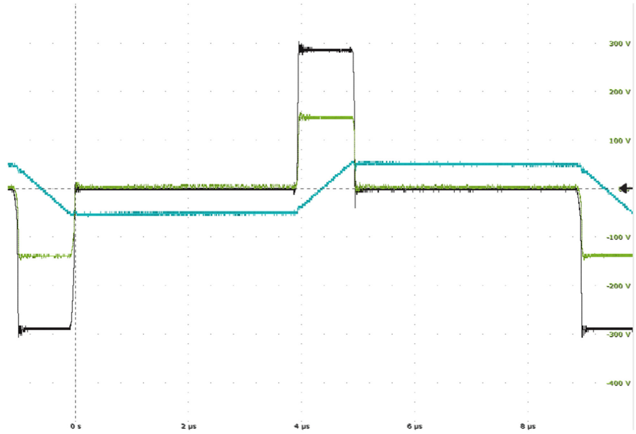


FIGURE 7. Typical waveforms of the primary voltage (green, 100V/div), the secondary voltage (blue, 20V/div) and the magnetizing current (blue, 0.1A/div) for D = 0.1, B = 0.1T, f = 100 kHz, 3C92, (time = 2μ/div).

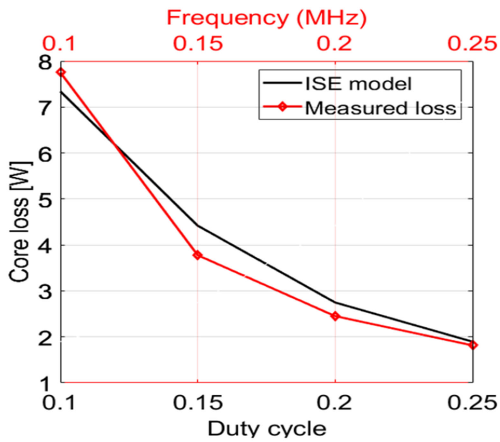


FIGURE 8. Evolution of ISE model at (f = 100 kHz, D = [0.25, 0.2, 0.15, 0.1]) and the measured loss of a square voltage waveform at the equivalent frequencies shown in the top x-axis for core (ER32/6/25) and material (3C92).

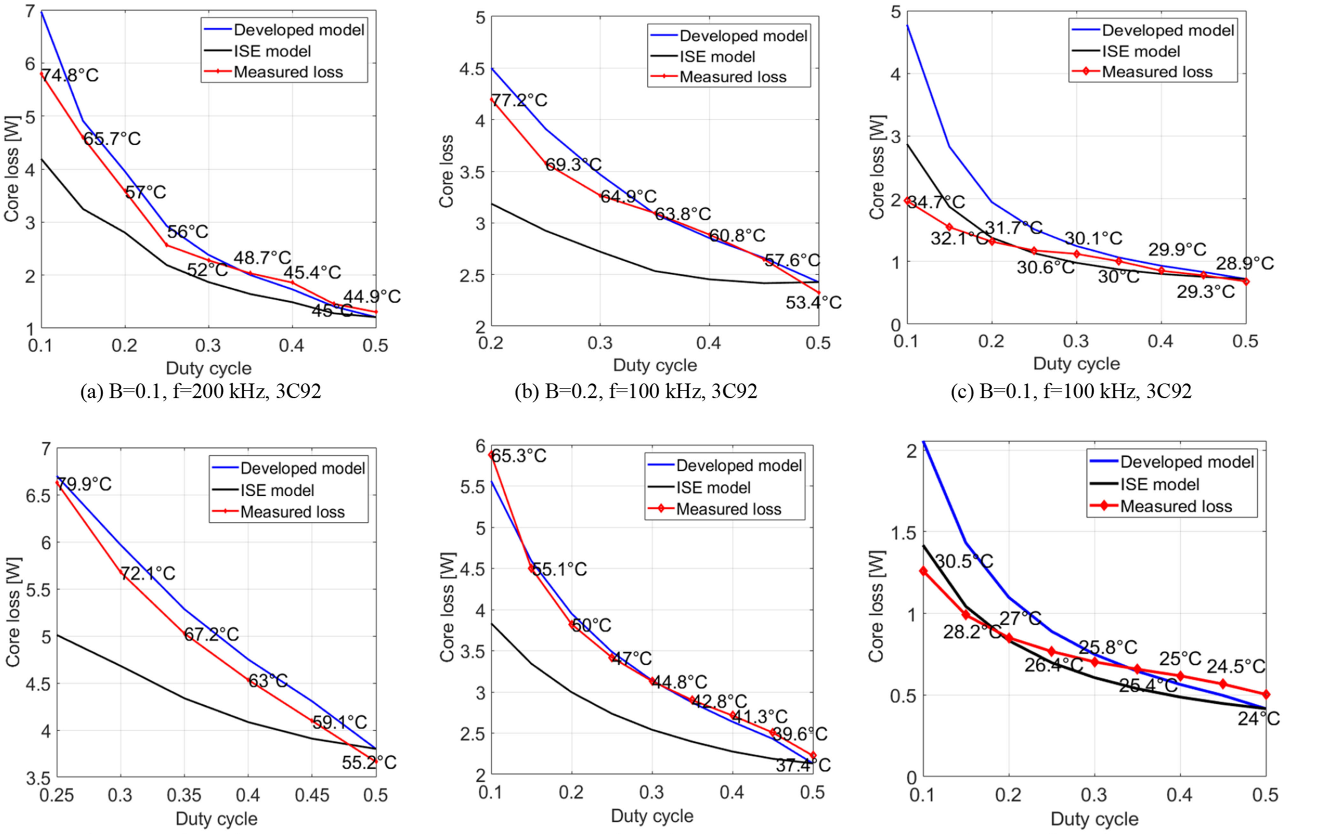
1) TEMPERATURE EFFECT

The first step is to include the temperature effect and that's because the core loss changes with the change of the core loss using the fast-calorimetric technique. The core loss dependency on the temperature can be obtained from the datasheet using curve fitting technique. The relationship is given by a normalized second order polynomial function as follow:

$$f(T) = \frac{P_c(T)}{P_c(T = 100\,^\circ C)} = a_0 T^2 + a_1 T + a_2 \tag{20}$$

2) GEOMETRY EFFECT

The core loss depends also on the core geometry which needs to be accounted. A geometry factor correction is also applied to model (19) to take into consideration the geometry effect of the core due to the non-homogenous distribution of B as stated in [24].



(a) B=0.1, f=200 kHz, 3C92 (b) B=0.2, f=100 kHz, 3C92 (c) B=0.1, f=100 kHz, 3C92

(d) B=0.15, f=200 kHz, 3C92 (e) B=0.2, f=100 kHz, N87 (f) B=0.1, f=100 kHz, N87

FIGURE 9. Core loss evolution with respect to the duty cycle using model (19), f = 100 kHz, T = 100 °C, (a)-ER32/6/25-3C92, (b)-ER32/5/21-N87.

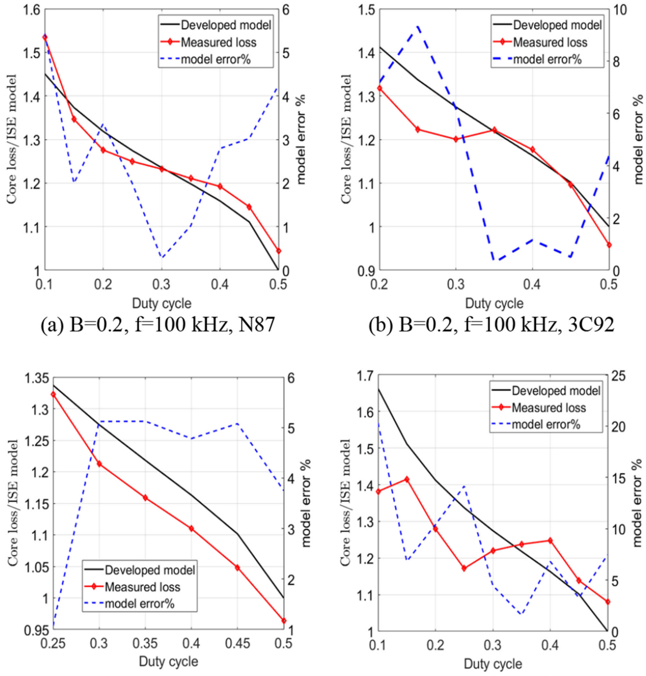
Finally, the model to be implemented and verified is the following.

$$P_{tr} = C_{ge} \cdot P_{tr} \cdot f\,(T) \tag{21}$$

Fig. 9 show the core loss evolution of the measured loss, the developed model (21) and the ISE model with respect to the duty cycle. It is worth reminding that the difference between the developed model and the ISE model is exactly equal to the relaxation loss. The first remark we notice is that a decrease in the duty cycle leads to a significant increase in the core loss. Take the following example, the core loss at 0.1 is about 3 times higher than the core loss at 0.5 for the case (N87, 0.2 T, 100 kHz). It should be mentioned that the evolution is expected to be much higher in real application. This is because, the core loss evolution shown in this figure is slowed down by the effect of the temperature increase. In other words, if the temperature remains constant all along the duty cycle range, the core loss evolution is expected to be sharper like presented in the theoretical analysis (Fig. 4).

The second observation that can be concluded from Fig. 9 is that the relaxation loss is a true physical phenomenon, which significantly increases as the duty cycle decreases. This can be synthesized from the gap-increase between the measured loss and the ISE model as the duty cycle decreases. Two main factors that leads to this increase: the increase in the effective core loss (P_{reff}) and the increase in the relaxation time (off-time). The failure of the ISE model in predicting the core loss of the trapezoidal waveform is very huge in the duty cycle range of [0.1, 0.3]. On the other hand, the developed model, in general, shows a better accuracy overall the duty cycle range with some discrepancy. This discrepancy can be seen in cases (c and f). It could be due to two possible reasons.

The first reason is related to the model error resulting from the inaccuracy of the Steinmetz parameters. The second reason is of physical origins resulting from the behavior of the relaxation loss at low temperature. To verify the possibility of the first reason, we proceeded as follows. The ISE model and the Steinmetz parameters accuracy are verified by measuring the core loss of a triangular flux waveform having same frequency as the effective frequency (*f/2D*). As an example, for the case (B = 0.1, f = 100 kHz, D = [0.25, 0.2, 0.15, 0.1], 3C92), a rectangular voltage waveform is generated for the following frequency [200, 250, 333.33. 500] kHz and the core loss are measured and compared to the ISE model. The results are in Fig. 8. The ISE model is enough accurate which explains that the error in cases (c, f) is not due to the Steinmetz parameters. Therefore, what remains is that the error could be resulted from the relaxation loss phenomenon. It could have some reversible effects on the on-time loss. According to our understating of this problem, this effect is of great dependence on the temperature. It was shown in [21] that the



(a) B=0.2, f=100 kHz, N87 (b) B=0.2, f=100 kHz, 3C92

(d) B=0.15, f=200 kHz, 3C92 (d) B=0.1, f=200 kHz, 3C92

FIGURE 10. Core loss-to-ISE model evolution and error of the developed model.

relaxation loss decreases at low temperature and vice-versa. Additionally, for low duty cycle, the off-time is much higher than the on-time, which gives enough time for the core to be cooled down and then reduces the total average loss. This issue is not our focus in this work, but it will be investigated in future works.

3) MODEL SIMPLIFICATION

Although the developed model (21) shows a good accuracy (maximum error% is lower than 15%), it includes three unknown parameters (*x*, *y* and *z*) which are not available in the material datasheet (Fig. 10). for worst case designs, it much worth to simplify the model by replacing the unknown parameters with suitable correction factors. From the experimental results, model (21) could be replaced by the following:

$$P_{tr} = D_F \cdot C_{ge} \cdot P_r \cdot f(T) \tag{22}$$

D_F and C_{ge} are the duty cycle correction factor and the core geometry factor.

D_F factors are obtained by a simple division using the experimental results and model (22). The results are given in Table 2 and shows that D_F are quite independent on the tested materials. However, we think that the verification of this assumption with measurement of more different materials is necessary in the future".

V. CONCLUSION

Constructing on the analysis of the physical explanation of the relaxation loss and the superposition technique, so far, this paper has proposed an accurate model to calculate the core loss of the symmetric trapezoidal flux waveform (21). An easier method, using a multiplication factor depending on the duty cycle, was also proposed to simplify the relaxation loss calculation. The error of the developed model (<15%) is much lower than of the existing models (>50%). In general, this work has presented not only a simplified and accurate model, but also an insightful and physical explanation of the loss change of the trapezoidal flux waveform with respect to the duty cycle. The first reason of this change is the increase in the rate of variation of B resulting from the decrease in the duty cycle, which leads to a significant increase in the on-time loss. The second reason is the relaxation loss or the off-time loss which depends on R and much greatly on the temperature. Finally, despite the important results accomplished in the present paper concerning the core loss calculation of the symmetric trapezoidal flux waveform, we think that more effort should be devoted for the thermal characterization of the relaxation loss.

REFERENCES

[1] S. Barg and K. Bertilsson, "Multiobjective pareto and GAs nonlinear optimization approach for flyback transformer," *J. Elect. Eng.*, vol. 101, pp. 995–1006, Elsevier, Sep. 25, 2019.

[2] S. Barg, M. Hanen, K. Ammous, and A. Ammous, "An improved empirical formulation for magnetic core losses estimation under non-sinusoidal induction," *IEEE Trans. Power Electron.*, vol. 32, no. 3, pp. 2146–2154, Mar. 2017.

[3] A. Van den Bossche, V. C. Valchev, and G. B. Georgiev, "Measurement and loss model of ferrites with non-sinusoidal waveforms," in *Proc. IEEE 35th Annu. Power Electron. Specialists Conf.*, (IEEE Cat. No.04CH37551), vol. 6, 2004, pp. 4814–4818.

[4] T. M. Colonel, *Transformer and Inductor Design Handbook*. New York, NY, USA: Marcel Dekker, Inc, 2004.

[5] M. K. Kazimierczuk, *High Frequency Magnetic Components*. Hoboken, NJ, USA: Wiley, 2014.

[6] V. Cekov Valchev and A. Van den Bossche, *Inductors and Transformers for Power Electronics*. Boca Raton, FL, USA: CRC Press, 2018.

[7] W. G. Hurley and W. H.Wolfle, *Transformers and Inductors for Power Electronics: Theory, Design and Applications*. Hoboken, NJ, USA: Wiley, 2013.

[8] A. I. Pressmann, K. L. Billings, and T. Morey, *Switchmode Power Supply Design*, 3rd ed. New York, NY, USA: McGraw-Hill, 2009.

[9] C. P. Steinmetz, "On the law of hysteresis," *Proc. IEEE*, vol. 72, no. 2, pp. 196–221, Feb. 1984.

[10] J. Reinert, A. Brockmeyer, and R. W. De Doncker, "Calculation of losses in ferro- and ferrimagnetic materials based on the modified Steinmetz equation," *IEEE Trans. Ind. Appl.*, vol. 37, no. 4, pp. 1055–1061, Jul./Aug. 2001.

[11] J. Li, T. Abdallah, and C. Sullivan, "Improved calculation of core loss with nonsinusoidal waveforms," in *Proc. IEEE Ind . Appl. Soc. Annu. Meet.*, Oct. 2001, pp. 2203–2210.

[12] J. Li, T. Abdallah, and C. Sullivan, "Accurate prediction of ferrite core loss with nonsinusoidal waveforms using only Steinmetz parameters," in *Proc. IEEE Workshop Comput. Power Electron*, Jun. 2002, pp. 36–41.

[13] C. R. Sullivan, J. H. Harris, and E. Herbert, "Core loss predictions for general PWM waveforms from a simplified set of measured data," in *Proc. 25th Annu. IEEE Appl. Power Electron. Conf. Expo.*, 2010, pp. 1048–1055.

[14] C. R. Sullivan and E. Herbert, "Testing core loss for rectangular waveforms: Phase II final report," [Online]. Available: http://engineering.dartmouth

[15] E. Herbert, "Magnetic core losses, the PSMA-Dartmouth studies: Pilot project, phase II project, a supplement report," Dec. 2011, Accessed: Jan. 2012. [Online]. Available: https://www.psma.com

[16] J. Mühlethaler, J. Biela, J. W. Kolar, and A. Ecklebe, "Improved core loss calculation for magnetic components employed in power electronic systems," *IEEE Trans. Power Electron.*, vol. 27, no. 2, pp. 964–973, Feb. 2012.

[17] F. Preisach, "On the magnetic after effect," *IEEE Trans. Magn.*, vol. 53, no. 3, Mar. 2017, Art. no. 0700111.

[18] S. Chikazumi, *Physics of Ferromagnetism*. London, U.K.: Oxford Univ. Press, 1997.

[19] R. M. Bozorth, *Ferromagnetism*. Hoboken, NJ, USA: Wiley, 2003.

[20] J. B. Goodenough, "Summary of losses in magnetic materials," *IEEE Trans. Magn.*, vol. 38, no. 5, pp. 3398–3408, Sep. 2002.

[21] S. Barg and K. Bertilsson, "Core loss modeling and calculation for trapezoidal magnetic flux density waveform," *IEEE Trans. Ind. Electron.*, vol. 68, no 9, pp. 7975–7984, Sep. 2021.

[22] Thermoelectric cooler TEC1-12706: HB corporation, [Online]. Available: http://www.hebeiltd.com.cn/peltier.datasheet/TEC1-12706.pdf

[23] Y. Zhang and T. M. Jahns, "Power electronics loss measurement using new heat flux sensor based on thermoelectric device with active control," *IEEE Trans. Ind. Appl.*, vol. 50, no. 6, pp. 4098–4106, Nov./Dec. 2014.

[24] S. Barg, F. Alam, and K. Bertilsson, "Modeling of the geometry effect on the core loss and verification with a measurement technique based on the seebeck effect and FEA," in *Proc. Accepted 45th Annu. Conf. IEEE Ind. Electron. Soc., IECO*N, 2019, pp. 1832–1837.

[25] Planar ER Cores and Accessories ER32/6/25, Ferroxcube [Online]. Available: https://www.ferroxcube.com/upload/media/product/file/Pr_ds/ER32_6_25.pdf

[26] Planar Cores Ferrites and Accessories ER32/5/21, TDK [Online]. Available: https://www.tdk-electronics.tdk.com/inf/80/db/fer/er_32_5_21.pdf