Students, agency and mathematical subjectivity

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This paper reports on a study of 18 Swedish year 1 students' (7–8 years) work with mathematics textbooks analysed according to the concept of agency. The empirical data consisted of video material, students' representations and mathematics textbooks. The result showed that some exercises enable agency, and some do not. Also, students' opportunities for agency are affected by the notion that, according to the students, mathematical symbols are the resource that should be used to be considered successful in mathematics. A conclusion from this is that the textbook needs to be used consciously, offering different learning situations based on both opportunities for agency and multimodal aspects to provide all students learning situations that benefit both learning and the opportunity to discover themselves as mathematical individuals.

Introduction

In mathematics education, the textbook is a widespread learning resource (Mullis, Martin, Foy, & Arora, 2012). According to TIMSS (Trends in International Math-ematics and Science Study) and other comparative international studies, the use of mathematics textbooks is higher in the Baltic and Nordic countries than in other parts of the world (Grevholm, 2017). Regardless of how much the teacher plans and stages the teaching, the students' mathematics teaching often involves individual work with mathematics textbooks. Students are thus assumed to be able to work individually with their mathematics textbook.

This study built on multimodal social semiotics (Kress, 2010), where communication in different resources or *modes* (Kress, 2010) such as images, mathematical symbols and writing is studied. From this perspective, the student's work with the mathematics textbook is understood as the student's *meaning making* (e.g., Kress, 2010) working with the mathematics textbook. First this study sheds light on what the textbooks are designed to offer students and, second, the students' meaning-making working with the mathematics textbook. The purpose of what the textbook is designed to offer is understood as the learning resource's meaning potential, a relational concept (van Leeuwen, 2005).

To deepen the understanding of students' meaning making the concept of *agency* was used. Agency refers to an individual's possibility to take active participation (Bezemer & Kress, 2016). In this study, the concept is used based on the student's opportunity to participate when working with mathematics textbooks. Students' opportunities to take agency is then discussed concerning the concept of mathematical subjectivity (Palmer, 2009), referring to that an individual *becomes* mathematical in different situations rather than *is* mathematical, and that the concept concerns "how to understand oneself and to be

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understood by others in relation to mathematics" (my translation, Palmer, 2009, p. 17). This means that mathematical subjectivity is connected to meaning making and actions.

Thus, the aim of the study is to discuss how students' meaning making when working with mathematics textbooks can be understood from the concept of agency. The analysis has been guided by the following research questions: Which meaning potentials are designed into the exercises? How do the students make meaning when working with the textbooks? What opportunities for students' agency is possible when working with the mathematics textbook?

Literature review

Research on agency linked to students' mathematics learning is based on different theoretical perspectives. Louie (2019) describes learning as socially constructed by studying mathematics teaching of five American elementary school teachers through discourse analysis. She used the term *agency discourse* and highlighted agency to achieve equality. Louie concluded that agency can be the path to all students' right to discover a rich mathematics education but must be adapted to different student groups, focusing on vulnerable groups. Louie emphasized the importance of supporting teachers in this development work without blaming the teaching staff. Discourses are also concentrated on by Norén (2011). She starts from Foucault's power perspective and studied linguistic discourses for multilingual students in year eight and national mathematics tests. One conclusion that was drawn was that an essential factor in solving text problems consisted of the possibility of being able to switch between different linguistic discourses and students' ability to take agency in the test situation.

The agency concept is used from cognitive as well as social perspectives on learning. In a method development study based on social semiotics and Halliday's functional linguistics, Morgan (2016) used the term *human agency* in a discourse analysis of how mathematics teaching enables agency linked to mathematics learning. Alshwaikh and Morgan (2013) used the term *learner agency* in a multimodal discourse analysis of Palestinian mathematics textbooks, focusing on the type of activity the student is expected to engage in and the choices available. Björklund Boistrup (2010) used agency from a design-oriented perspective, similar to this study. She studied assessment documents in mathematics teaching communication through a case study in year 4 (7–8 years), where the opportunities for students' agency and learning were in focus. The results showed that the students' agency increased if the teachers showed interest in what the students showed knowledge about, instead of assuming an assessing role where the students' performance was valued in praise or dissatisfaction. She also concluded that restrictions on which modes are offered to students might limit students' opportunities for agency.

Conceptual framework: Meaning potential and agency

The concepts used in this study builds on a social semiotic approach referring to the individual's meaning making in a social and cultural context where social processes create conditions for learning and the individual make meaning of information (Kress, 2010).

A meaning potential (van Leeuwen, 2005) is understood as an offer or as opportunities and limitations. The concept of meaning potential originates from the concept of affordance developed in social semiotics to a relational discovery, which implies that a meaning potential only exists when individuals and resources meet. Although there may be more given meaning potentials linked to an individual's encounter with a resource, different individuals can also focus on different meaning potentials (van Leeuwen, 2005). The mathematics textbook, is designed to offer a specific meaning potential, the textbook author's purpose with the exercise, or the *designed meaning*. In this study, a comparison between (1) the mathematics textbook's designed meanings and (2) the student's meaning making in her/his work was made. In the student's encounter with this potential meaning, space for agency can arise.

The concept of agency is understood as the individual's active participation and the ability to act independently (Bezemer & Kress, 2016). The individual's temporary involvement in different environments, for example, the student's work with mathematics textbook, and the individual's response to this can be understood as the individual's capacity to make choices and be linked to activity and passivity (Björklund Boistrup, 2010). Kress (2010) writes that meaning involves a creator, which connects to active participation or agency. In this study, the concept of agency is directed towards the student's opportunity for agency in her/his individual work with the mathematics textbook.

Methodology

Video transcripts from 18 Year 1 (aged 7–8) students were collected chosen out of a convenience sample. In order to getting to know the students one week was spent in the class before the data collection. Here, my 12 years of experience as a compulsory teacher, came into use. I sat with one child at a time in a room next to the classroom which made it possible to focus on the students' meaning making in detail. This approach gave me the opportunity to ask the child questions while working with the textbook.

The video material consists of 450 minutes of film, approximately 25 minutes per child, with a range from 19 to 44 minutes. A tablet was used for documentation., Tablets were used in the students' everyday teaching and therefore did not mean any special focus from the students. The tablet was placed obliquely above me and the child. This allowed both the child, the textbook, and me to be seen in the video. The child started working on the exercise on her/his own. After some time, I asked questions of an investigative nature, such as "Can you tell me how you went along on this side?", "How did you know how to work with this exercise?", or "I saw that you did something with this image here, can you show me?" If the child had difficulties getting started with the exercise, I provided support in the form of questions such as "Can you use the images to solve the exercise?".

The choice of textbook series was guided by the textbooks used in the class, which is well known in Sweden. It consists of two tracks: Favoritmatematik (Favourite Mathematics) 1A and 1B and Mera favoritmatematik (More Favourite Mathematics) 1A (Ristola, Tapaninaho, & Tirronen, 2012a, 2012b; Haapaniemi, Mörsky, Tikkanen, Vehmas & Voima, 2013). Mera favoritmatematik is considered a more challenging textbook. The exercises were chosen based on the results of a quantitative study (Norberg, 2021). They should address subtraction as an arithmetic operation, the design of the exercises should be commonly used, show breadth according to how the different modes were used, and the mathematical content should not be new for the students. The exercises (see Figures 1–7) were colour-copied and handed out to the child, one at a time.

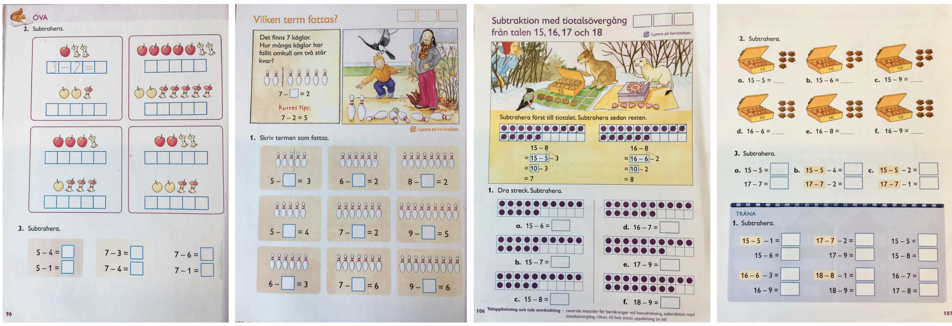

Figure 1 (from left to right): Apples and cores (Ristola, Tapaninaho & Tirronen, 2012a, p. 96).

Figure 2: Bowling pins (Haapaniemi, Mörsky, Tikkanen, Vehmas & Voima, 2013, p. 150).

Figure 3: The hare and the ermine (Ristola, Tapaninaho & Tirronen, 2012b, p. 106).

Figure 4: Gingerbread cookies (Ristola, Tapaninaho & Tirronen, 2012b, p. 107).

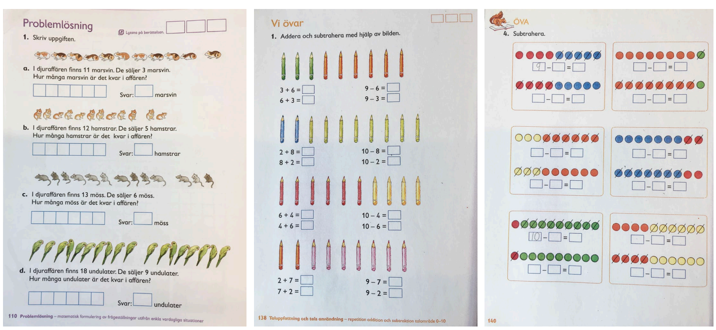

Figure 5 (from left to right): Petstore (Ristola, Tapaninaho & Tirronen, 2012b, p. 110).

Figure 6: Pencils (Ristola, Tapaninaho & Tirronen, 2012a, p. 138).

Figure 7: Dots (Ristola, Tapaninaho & Tirronen, 2012a, p. 140).

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Framework for analysis

An analysis in three steps was conducted in order to understand the students' meaning making. First, a textbook analysis of the exercises was made to capture the designed meanings (the purpose). Second, the video material and the students' representations (their answers) were analysed. Third, the designed meanings and the students' meaning making of the exercises were analysed according to the concept of agency.

The textbook exercises were analysed in the textbook analysis, referring to the research question 1: *Which meaning potentials are designed into the exercises?* The exercises' mathematical content and the mode or modes that carry information for solving the exercises were studied using the teacher's guide to answer how subtraction is addressed. Carpenter and Moser's (1982) categorisation was used to document subtraction types: *joining, separating, part-part-whole, comparison, equalizing-add on and equalizing-take away.* Examples in Figure 1; for instance, the exercise named 2 consists of a separating situation, the apples have been eaten. The exercise named 3 involves subtraction without a specific subtraction type; the information does not contain a specific subtraction type but subtraction in general.

In the *video material with the students' representations*, research question 2: *How do the students make meaning when working with the textbooks?* guided the coding of the data. To analyse the video transcripts, they were first transcribed based on different modes and using three headings: speech, image, and body language. In the image column, the students' use of images and cases in which they drew an image to support their calculations were documented. The students' representations (on the copied papers) were used as support for this analysis. In the next step of the analysis, I noted whether the child solved the exercises as designed, according to the subtraction content. After that, the transcripts were coded by reading through the material several times and highlighted using a colour code. Then condensed meanings were summarised in a matrix, from which various categories about how the student's made meaning when working with the textbook emerged.

To further deepen the understanding of students' meaning making when working with mathematics textbooks, *an analysis was also made based on the concept of agency* and the question: What opportunities for students' agency is possible when working with the mathematics textbook? Interest was directed towards the student's space to act independently and to choose different ways to make meaning when working with the mathematics textbook. The textbook exercises and students' meaning making were analysed based on the concept of agency or student's independent participation (Bezemer & Kress, 2016) and what opportunity for agency is allowed to the student when working with the mathematics textbook. For the textbook exercises, I searched for opportunities for agency concerning an imaginary work with the exercise based on the sentence offered. Next, I searched for whether the student's meaning meant opportunities to act independently and choosing how to make meaning when working with the exercise.

An example is given here of how the analysis of the *textbook exercises* was carried out on the basis of the exercise "Dots" (see Figure 7) to concretise. The subtraction type is separating, as dots are crossed out. The exercise's designed meaning was that the information needed to solve the tasks is found in the images and answers are made with mathematical symbols. With the help of the concept of agency, this could be understood as meaning that the exercises did not offer opportunities for agency for the students in that the design of the exercise does not allow different working methods.

An example of the analysis of *students' meaning making* is the students' work with the exercise "Gingerbread cookies" (see Figure 4). The students worked in different ways with that exercise. Some of the students based solely on the mathematical symbols in their work, some used images and mathematical symbols, and a couple of students drew images which they used to support the calculation. This example shows that the design of the exercise provided an opportunity for students' agency.

Results: Students' possibilities to take agency when working with mathematics textbooks

The data shows examples of exercises that enable agency and exercises that do not when students are working with them. The data also show that students' opportunities for agency are affected by the notion that mathematical symbols is the mode that should be used if the student should be considered successful in mathematics.

When exercises enable student's agency

The analysis showed that there are exercises where the design of the mathematics book provides opportunities for agency and the opposite. Exercises designed to enable agency are designed so that the student can choose working methods, and this can be done by selecting the order in which the modes are used or deciding which modes to use.

An example of an exercise that enables student's agency is *"The hare and the ermine"* (see Figure 3). The subtraction type in this exercise is separating and bridging ten. The student can choose between starting their meaning making through the image mode or the mathematical symbols mode. The students who began their meaning making from the images started by crossing out the number of dots to be subtracted and then counted the remainder and finally wrote mathematical symbols on the line. The students who began their meaning making by starting from the mathematical symbols made a calculation using the mathematical symbols and then crossed out dots or did not use the image at all. In the former way of working, the image can be a support for the calculation. In contrast, in the latter, the image is used as an additional representation for the student's subtraction calculation.

Another example where the exercise enables student's agency is seen in the exercise "Pens" (see Figure 6). The subtraction type here is part-part-whole, pencils of different colours are subtracted. Here the student can choose between using the image in the calculation or basing her/his meaning solely on the mathematical symbols. The data showed that some of the students used only the mathematical symbols, while others also used the images.

In exercises that enable agency, the students can thus, to a greater extent, decide for themselves how they want to work with the exercise, agency is possible. Here, the students' own choices can form conditions for which learning situation they want to face by choosing the rules of procedure or the mode they wish to use in their meaning making.

When exercises limit student's agency

The analysis of data shows the occurrence of exercises whose design limits students' ability to take agency. Such exercises are designed so that a specific way of working with the book is required to solve the exercise based on its designed meaning. It may be necessary that the work takes place in a particular order or is based on a specific mode and that the work is then done with a specific mode or certain modes.

An example of an exercise where *the work needs to be done in a particular order* is in the exercise "Apples and cores" (see Figure 1). The designed meaning is that the student first uses the information in the solved example and then solves the tasks in the same way. The image is to be interpreted as a sequence of events, and the calculation is written in empty boxes using mathematical symbols. The results showed that the students worked on this exercise by first using the example. Then they started from the image and interpreted it in different ways. Some of the students interpreted the image as first; for example, there were three apples, and then someone ate an apple. Others interpreted the apples as one term and the apple cores as the other, and used the image as a static image. Finally, they wrote a mathematical symbol in each box to make the calculation. So, some of the students interpreted the image as a change take from-situation and some did not.

An example of an exercise where the student's meaning making is based *on a specific mode* is in the exercise "Dots" (see Figure 7). The exercise is designed so that the student's meaning making is done based on the mode image. The designed meaning is that the student's meaning making should be based on the images, and then the answers are represented with the help of mathematical symbols in the empty boxes. When the students worked on this exercise, they started by looking at the image, counting the number of dots and crossed out dots. The subtraction calculations were then made using that information, and answers written in mathematical symbols.

In exercises that limit agency, students are, to a lesser extent, given the opportunity to decide for themselves how they want to work with the exercise. The learning situation is shaped by the fact that the work takes place in a particular order or based on a specific mode and that the work is then done with a specific mode or certain modes.

Students' opportunities for agency and expressions about the subject of mathematics

The results also showed that some students expressed a notion that mathematical symbols are "better" to use than the other modes. Choosing the mathematical symbols mode to a greater extent over other modes was described by the students as something that shows that they are successful in mathematics, demonstrated in both words and actions. Based on the concept of agency, it is made clear that the student's opportunity for agency is affected by the notion that students who are successful in mathematics do not use the images but start from the mathematical symbols. This is expressed by several students, both students who use the image and those who do not use the image to solve the tasks. In an answer to my question about whether it is possible to use the cones to count in the exercise in Figure 2 "Bowling cones", one of the students answered: "Yes, but you do not have to if you are good at math”. The space for agency that the design sometimes entails, such as in the exercise with the cones, shrinks from the prevailing notion that it is more desirable to solve the tasks without using the images. So, although it is possible to choose to work with the images, there is a resistance to this as it is not a desirable way to solve the tasks.

Discussion and concluding remarks

The results based on the agency concept showed examples of exercises that do not enable students’ agency. The exercises can be based on a designed rule of procedure that states how the different modes are to be used to solve the exercise according to the designed meaning. If, on the other hand, the design allows for different rules of procedure, students’ agency is made possible. To a greater extent, the student can then decide for herself how she wants to work with the exercise. Here, the student’s own choice could shape the conditions for which learning situation the student encounters. However, the results showed that in such exercises, there was a tendency for students to choose mathematical symbols for their meaning making over other modes. In the following, I discuss this from two different aspects.

First, meaning making starting from mathematical symbols can contribute to the student being able to miss out on a learning situation that can support the conceptual understanding of the mathematical content. For example, the image mode may show a subtraction type not captured in mathematical symbols (see for instance Figure 7 “Dots”). However, working with the mathematics textbook based on mathematical symbols can also mean that the student already possesses the knowledge required in the exercise. This makes the situation complex and, it is important not to assume that a student who works with the mathematics textbook based on the mathematical symbols automatically belongs to the latter group of students.

Secondly, students’ tendency to start from the mathematical symbols to a greater extent can contribute to students already at the age of 7–8 not understand themselves as mathematical individuals. If the student cannot meet the notion of choosing mathematical symbols over other modes, it can help shape an identity as “non-mathematical”. This can be compared with what Palmer (2009) describes as mathematical subjectivity with the meaning of *becoming* mathematical as an individual rather than *being* mathematical and that this is related to the students meaning making. The way students’ make meaning when working with textbooks and how they value their own, and other’s work, affect their understanding of themselves as mathematical or not. Mathematics teaching should be based on the assumption that all students are experiencing themselves as mathematical individuals.

If learning situations where all students at the age of 7–8 understand themselves as mathematical individuals could be created, the chances increase that students develop identities as mathematical individuals. Of course, the way to get there is complex, and the work with the mathematics textbook is only part of this identity creation. However, the mathematics textbook is widely used in mathematics teaching. It is in many ways, structured as a question-and-answer textbook, and the answers are almost always in demand in mathematical symbols mode. It is not surprising that the students develop a notion that mathematical symbols are the most important mode. Here, a question arises as to whether the students' descriptions linked to discovering themselves as mathematical individuals or not would look different if the mathematics textbooks were designed in a way that to a greater extent recognizes students' displayed knowledge through various modes. This can be compared with Björklund Boistrup's (2010) conclusion that restrictions in which modes are offered can limit students' opportunities for agency.

A question that arose in this study was which represented knowledge is recognized in mathematics textbooks for year 1. The results showed that mathematical symbols have a superior position already in year 1. If representations through modes other than mathematical symbols are recognized to a greater extent, more students would have the opportunity to discover themselves as mathematical individuals. This supports the students' further mathematics learning and the possibility that they continue to see themselves as mathematical individuals. The design of the teaching determines which agency is created for the students (Kress, 2010). With a good foundation in early mathematics teaching, it is easier to continue working towards education where more students develop identities as mathematical individuals.

Based on the reasoning given, an interpretation could be that the possibility of agency in the student's individual work with the mathematics textbook combined with mathematics textbooks that better utilize the student's represented knowledge through different modes would be desirable. Agency has been described as desirable in most studies (Björklund Boistrup, 2010; Louie, 2019; Norén, 2011). Though, a mathematics textbook that opens up to students' agency would automatically lose clarity regarding the designed meaning. Suppose the student has great opportunities to choose working methods. In that case, this may mean that parts of the exercises' design can be deselected and thus indirectly that certain mathematical content is deselected. This is because some modes are better suited than others for specific information; for example, a subtraction type cannot be displayed using only mathematical symbols. Based on this, a mathematics textbook with increasing opportunities for agency would mean a textbook that is, to a lesser extent, suitable for individual work since the teacher is needed in order to highlight the mathematical content.

A conclusion is that the work with mathematics textbooks for primary school's youngest students should be done with great consciousness. A suggested design based on this is mathematics teaching that requires representations in different modes and two different learning situations. (1) One type of learning situation offering opportunities for agency where the working method is not based on individual work, and (2) another type of learning situation where the student can work individually based on clearly designed offers to consolidate content already known to the student. These two different kinds of learning situations could benefit learning situations where all students learn and also discover themselves as mathematical individuals.

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