Homogenization of a hyperbolic-parabolic problem in a perforated domain

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We study the homogenization of the hyperbolic-parabolic system

$$\begin{aligned} \partial_{tt}^2 u_\varepsilon(x,t) + \partial_t u_\varepsilon(x,t) - \nabla \cdot \left(a\Big(\frac{x}{\varepsilon}, \frac{t}{\varepsilon}\Big) \nabla u_\varepsilon(x,t) \right) &= f_\varepsilon(x,t) \text{ in } \Omega_\varepsilon \times (0,T), \\ a\Big(\frac{x}{\varepsilon}, \frac{t}{\varepsilon}\Big) \nabla u_\varepsilon(x,t) \cdot n &= 0 \text{ on } \partial\Omega_\varepsilon - \partial\Omega, \\ u_\varepsilon(x,t) &= 0 \text{ on } \partial\Omega \times (0,T), \\ u_\varepsilon(x,0) &= g_\varepsilon(x) \text{ in } \Omega_\varepsilon, \\ \partial_t u_\varepsilon(x,0) &= h_\varepsilon(x) \text{ in } \Omega_\varepsilon \end{aligned}$$

by means of adaptions of the evolution setting of multiscale convergence and very weak multiscale convergence [1] to periodically perforated domains Ω_ε [2]. Such domains mean that periodically arranged identical holes appearing with a period of characteristic size ε have been removed from an open bounded set $\Omega \subset \mathbb{R}^N$ except in the layer closest to the boundary $\partial\Omega$ .The set $\partial\Omega_\varepsilon - \partial\Omega$ means the boundary of these holes and the coefficient $a(y,s)$ is periodic with respect to $(0,1)^N \times (0,1)$. Moreover, let Y^* be the upscaling of one period of the perforated domain Ω_ε to the size of a unit cube and $\theta = \mu(Y^*)$. Under certain assumptions on the convergence of $\{f_\varepsilon\}$, $\{g_\varepsilon\}$ and $\{h_\varepsilon\}$, $\{u_\varepsilon\}$ approaches the solution u to the homogenized system

$$\begin{aligned} \theta \partial_{tt}^2 u(x,t) + \theta \partial_t u(x,t) - \nabla \cdot (b \nabla u(x,t)) &= f(x,t) \text{ in } \Omega \times (0,T), \\ u(x,t) &= 0 \text{ on } \partial\Omega \times (0,T), \\ u(x,0) &= g(x) \text{ in } \Omega, \\ \partial_t u(x,0) &= \theta^{-1} h(x) \text{ in } \Omega \end{aligned}$$

for $\varepsilon \to 0$. The homogenized coefficient b is identified by the local problem

$$\begin{aligned} \partial_{ss}^2 u_1(x,t,y,s) - \nabla_y \cdot \big(a(y,s) \left(\nabla u(x,t) + \nabla_y u_1(x,t,y,s) \right) \big) &= 0 \text{ in } \Omega \times (0,T) \times Y^* \times (0,1), \\ a(y,s) \left(\nabla u(x,t) + \nabla_y u_1(x,t,y,s) \right) \cdot n &= 0 \text{ on } \Omega \times (0,T) \times (\partial Y^* - \partial Y) \end{aligned}$$

through

$$b \nabla u(x,t) = \int_0^1 \int_{Y^*} a(y,s) \left(\nabla u(x,t) + \nabla_y u_1(x,t,y,s) \right) dyds.$$

Keywords: Homogenization, parabolic, hyperbolic, perforated domains.

References

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