# Optimal distribution of power output and braking for corners in road cycling

D Sundström, M Bäckström, P Carlsson, and M Tinnsten

Keywords: pacing; corner; power output; braking; road cycling

Background: Optimal pacing strategies have previously been studied with different approaches for varying terrain as well as shifting head- and tail wind [1-3]. However, there is limited research where the pacing strategy is examined for courses with corners. Sharp corners may be a considerable obstacle in road cycling and therefore the distribution of power output and braking, for such corners, may affect performance.

Purpose: The aim of this study was to examine the optimal power output and braking power distribution for cornering in road cycling.

Methods: The optimal power output distribution was examined with a numerical model in the Matlab software. The mechanics of motion was modeled with a motion equation including power output, gravity, air resistance, rolling resistance and bearing resistance. This equation was expressed as

$$\frac{d^2x}{dt^2}\left(m+\frac{I_w}{r_w^2}\right)=\frac{P}{\frac{dx}{dt}}+C_{RR}mg+b_1\left(\frac{dx}{dt}\right)^2+b_2\frac{dx}{dt}+\frac{C_DA\rho}{2}\left(\frac{dx}{dt}\right)^2 \tag{1}$$

where $x$ was the position coordinate along the flat course, $t$ was time, $m$ was the rider and bicycle mass, $I_w$ was the moment of inertia of the wheels and $r_w$ was the outer radius of the wheels. Furthermore $P$ was the power output at the rear wheel, $C_{RR}$ was the coefficient of rolling resistance, $g$ was the acceleration of gravity, $C_DA$ was the drag area and $\rho$ was the air density. Coefficients $b_1$ and $b_2$ were derived from the study of Dahn et al. [4]. To simulate road cycling, equation (1) was solved numerically with the fourth order Runge-Kutta method. Moreover, the optimization of the power output distribution was carried out with the method of moving asymptotes [5]. To restrict the optimization, constraints were modeled for the bioenergetic limitations of the rider and for the friction of the tires. The hydraulic three vessel model of Morton [6] (Figure 1a) was used to model the bioenergetics of the rider and restrictions was set on the maximal attainable power output ($P_m$) so that it decreased linearly with the level in the $L$-vessel. The maximal attainable power output was set to be maximal when $L$ was full and decreased to a threshold value when $L$ was empty. The limitation on the maximal friction between tires and road was expressed as:

$$\sqrt{\left(\frac{d^2x}{dt^2}\right)^2+\frac{\left(\frac{dx}{dt}\right)^4}{r^2}}\leq \mu g \tag{2}$$

where $r$ was the turning radius of the corner and $\mu$ was the static friction coefficient between the tires and the road. Equation (2) incorporates both the acceleration in the direction of travel due to braking (translational component) and the normal acceleration due to cornering (rotational component).

Results: An 82.2 kg rider-bicycle system was simulated on a 500 m flat course with one 90° corner with 6.5 m minimal turning radius at 250 m. The optimization routine finished 40 iterations and the power output distribution resulting in the shortest finishing time was considered optimal. The optimized power output distribution is presented in Figure 1b. This optimized distribution gave a finishing time of 40.5 s and an average speed of 12.3 m·s$^{-1}$. Note that negative power output is interpreted as braking power.

Conclusion: The main finding of this study is that optimal power output distribution for a 500 m course with one sharp corner includes: (I) A maximal power phase and (II) a rolling phase followed by (III) a braking phase before the corner and (IV) an all-out acceleration with maximal power output immediately after the corner.

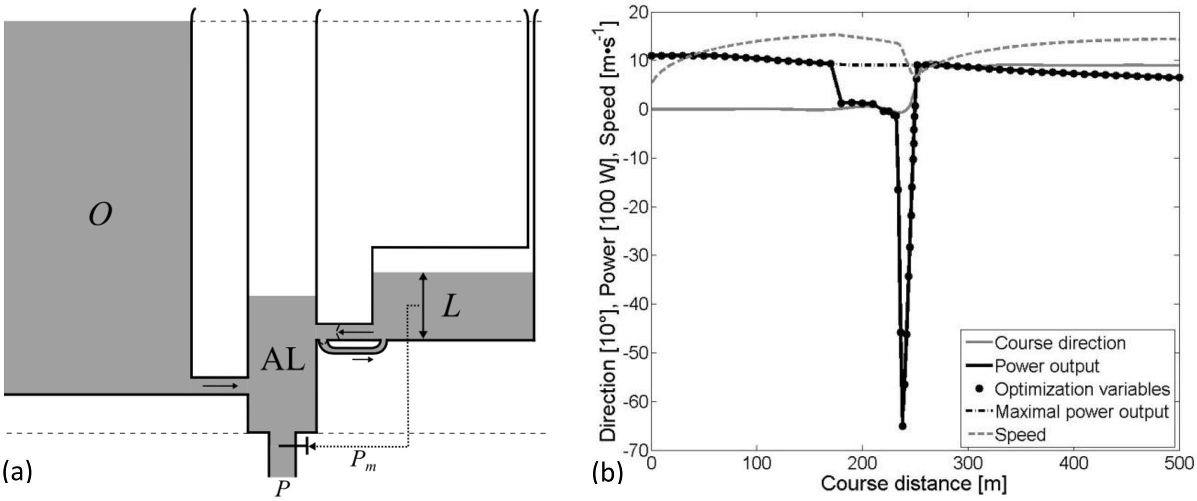


Figure 1. (a) The Margaria-Morton model works like a connected hydraulic system where $O$ is the oxidative vessel of fat and glycogen, $AL$ is the alactic vessel of ATP and phosphocreatine and $L$ is the Lactic vessel of glycogen contributing to the build-up of lactate. (b) The optimized distribution of power output.

[1] Atkinson, G, Peacock, O,Passfield, L. Variable versus constant power strategies during cycling time-trials: Prediction of time savings using an up-to-date mathematical model. *J Sport Sci* 2007;**25**:1001-9.

[2] Swain, DP. A model for optimizing cycling performance by varying power on hills and in wind. *Med Sci Sport Exer* 1997;**29**:1104-8.

[3] Sundström, D, Carlsson, P,Tinnsten, M. On optimization of pacing strategy in road cycling. *Procedia Engineering* 2013;**60**:118-23.

[4] Dahn, K, Mai, L, Poland, J,Jenkins, C. Frictional resistance in bicycle wheel bearings. *Cycl Sci* 1991;**3**:28-32.

[5] Svanberg, K. The method of moving asymptotes: a new method for structural optimization. *Int J Numer Meth Eng* 1987;**24**:359-73.

[6] Morton, RH. Modelling human power and endurance. *J. Math. Biol.* 1990;**28**:49-64.