

## TIME-DEPENDENT STOCHASTIC FAILURE OF FIBRE NETWORK

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Fibre network is commonly found in both industrial materials (paper and nonwoven) and also biological tissues. When subjected to time-dependent mechanical stresses, it is known to exhibit stochastic failure. Stochastic failure is most evident in creep and fatigue loading conditions when the fibre networks are used as structural members. In this case, the coefficient of variation of service lifetime often reaches “100%” or even more. Stochastic failure is also exhibited as web breaks when paper web is running on paper machine. In the latter case, the failure is rare, once in every, for example, 600 km of paper production, depending on the machine speed and break frequency, and it often takes place without any deterministic cause.

In this paper we attempt to formulate this time-dependent, stochastic failure in a macroscopic level by going through a structural hierarchy of fibre network. The objective is to determine cumulative distribution function of lifetime for a macroscopic fibre network under a uniaxial but general loading history.

Random fibre network, such as paper, has no obvious structural hierarchy. However, it does show some correlation structures in the scales of segment length (bond-to-bond distance) and fibre length. In a typical dense fibre network, significant auto and cross correlations exist in elastic properties, fibre density, and local deformation up to the length scale of about 10 times of the typical segment length (tentatively called critical length) [1]. In other words, within this length scale, one needs to resolve micromechanics in the fracturing process accurately, whereas beyond this scale, one may treat the entire structure as an independent (uncorrelated) random structure. This feature is very attractive to making multi-scale transformation of the distribution function by using the well-investigated, chain of fibre bundle model (for example, [2-3]).

We have constructed a chain of bundle model, where the chain direction is taken in the loading direction. A typical bundle length is the length when the weakest-link scaling starts (e.g., 15 mm); and the width of the bundle element is set to be the critical length so that each element is treated as statistically independent. The deviation from a typical fibre bundle model is that each element of the bundle receives different stresses due to the disordered structures. By using the renormalisation procedure [4] we have shown that we can recover the typical fibre bundle model consisting of Coleman’s fibre [5] with a minor reinterpretation of one of the parameters. We have also performed Monte-Carlo simulation to examine the validity range of the above conjecture.

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