Predicting Stochastic Failure of Fibre-based Materials in Long Term and Short Term

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1 INTRODUCTION

The future of sustainable society requires materials to be lighter, durable and reliable, rather than simply strong. Currently, there is no established way to characterise these aspects of the material property. Accordingly there are no design criteria for the new material developments. The most challenging fact is that one must deal with both time-dependency and stochastic aspects of material failure. Time-dependency means that the failures are not simply determined by the stress applied, but how long or how it is applied as a function of time. The stochastic nature refers to the fact that even under the same conditions, failures happens in an unexpected manner. This can be illustrated by Fig. 1, where all samples are subjected to the same creep load, but the failure points (lifetime) show a large scatter. The coefficient of variation of service lifetime often reaches "100%" or even more. This is very different from static strength, whose coefficient of variation resides in only 4-8% range.

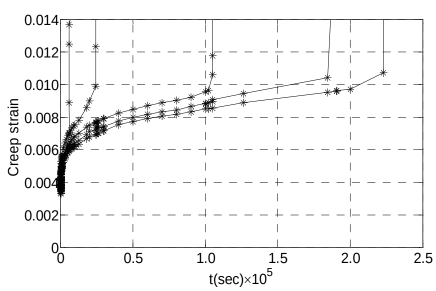

Figure 1. Typical compression creep curves and failures of linerboard [1]

The literature, however, is highly concentrated in the area of time-independent, deterministic failures, although time-dependent stochastic failure is most common in reality.

In our earlier work we proposed a new set of material parameters [1] that describe both long- and short-term failure phenomena under arbitrary loading histories. In this study, we apply the method to derive the relation between creep failure and standard compression failure for paperboard.

2 THEORETICAL BACKGROUND

In a real end-use condition, the material is subjected, not only to creep, but also to vibrations, impulse loading, and random loading. In order to deal with such complex loading conditions, we applied Coleman's formulation of time-dependent, statistical failure [2]. This model has been later extensively investigated by various authors in statistical mechanics area [3, 4]. For example, when the material is subjected to a general loading history f(t), the probability that failure occurs before time t_B (cumulative distribution function) is given by:

$$F(t_B) = 1 - exp\left\{-\left[\int_0^{t_B}\left(\frac{f(t)}{\alpha}\right)^{\rho} dt\right]^{\beta}\right\} \quad (1)$$

where α, ρ and β are the *material parameters* that characterize the probability of failure to a general loading history f(t). In other words, by determining these three parameters we can calculate failure probability to any loading history according to Eq. (1).

In the case of creep, i.e. when a constant load is applied, Eq. (1) can be rewritten as:

$$F(t_B) = 1 - exp\left\{-\left(\frac{f_0}{\alpha}\right)^{\rho\beta} t_B^{\beta}\right\} \quad (2)$$

where f_0 is creep load. Equation (2) showed that lifetime follows Weibull distribution, which was observed by our earlier study [1] in compression creep failure of liner board. An important point of this expression is that by measuring creep lifetime distributions at different creep loads, it is then possible to determine the unknown three parameters, α, ρ and β. (The method is described later.)

For a constant loading test, such as typical strength tests, Eq. (1) can be written to derive strength distribution function G(f*) as:

$$G(f^*) = 1 - exp\left\{-\frac{1}{\alpha^{\rho\beta}(\rho+1)^{\beta}\lambda^{\beta}}(f^*)^{\beta(\rho+1)}\right\} \quad (3)$$

where f* is strength and λ is loading rate. In the same way as for creep testing, this equation offers a way to determine the three unknown parameters, α, ρ and β by varying loading rate.

The three parameters involved in the above equations have special physical meaning [1], (1) α is approximately proportional to short-term strength (i.e., normal average strength), (2) ρ represents load sensitivity (the higher the ρ, the more sensitive for adding more load), but ρ also represents durability at a given strength (the higher the ρ, the higher the durability), and (3) β is a reliability factor that gives the uniformity of creep lifetime distribution (the higher the β, the more uniform lifetime distribution). From this view point, it is interesting to note that strength distribution (Eq.(3)) follows Weibull, but with the different exponent (β(ρ+1)) from that of creep life time (i.e., β). In other words, a seemingly narrow distribution of standard strength (COV~4-

8%) is governed by the high stress sensitivity (ρ dominates), whereas a large distribution of creep lifetime (COV~100-200%) is due to a diffused distribution of defects (β dominates). The theory presented by Coleman seems to explain very well the difference between the fast failure and creep failure. These results also provide the possibility to predict creep lifetime by using much faster standard compression tests, as described below.

3 MATERIALS AND METHODS

The parameters, α, ρ and β, can be determined by performing either creep tests or standard compression tests, according to Eqs. (2) and (3). For creep tests applied load is varied, whereas for strength tests loading rate is varied. Since Eqs. (2) and (3) have Weibull form, one can use Weilbull plots to least-square-fit the distribution data to determine the three parameters. In this study, however, we used *Maximum Likelyhood Estimation (MLE)* method to obtain the statistically-unbiased estimates of the three parameters. One of the advantages of MLE is that with increasing the sampling size, the estimates approaches to more accurate values (consistency), which may not always be guaranteed for the method involving nonlinear transformation (Weibull plots).

Both creep tests and standard strength tests were performed in compression for corrugated board components (liner and flute). The number of repeats for each test is about 100. The parameters from the two different data sets are then compared in order to investigate the validity of the approach.

4 RESULTS

4.1 Parameter estimate from creep tests

The result from creep tests for the three parameters (α, ρ and β) for two different liner boards and one flute is presented in Table 1.

Table 1. α, ρ and β obtained from creep tests for two different liner boards and one flute

	Liner 1	Liner 2	Flute
α=	49.48	42.66	56.30
ρ=	38.92	44.90	35.10
β=	0.5689	0.5542	0.5487

As seen in Table 1, parameter β (Weibull exponent, describing the spread of the lifetime distribution), is less than 1 for all three materials. The coefficient of variation is a unique function of β (see Eq. (4)). In Fig. 2, it can be seen that β-values less than 1 gives COV above 100%, meaning extremely large scatter of lifetime. These values are comparable with those found in the literature for glass fibres and Kevlar fibres [5-8]. The values for ρ (load sensitivity and durability factor) are very high, but also comparables to those for other materials found in the same literature.

$$COV = \left[\frac{\Gamma(1+\frac{2}{\beta})}{\Gamma^2(1+\frac{1}{\beta})} - 1\right]^{1/2} \quad (4)$$

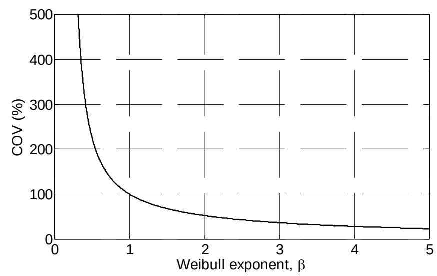

Figure 2. Coefficient of variation as a function of Weibull exponent, β

4.2 Comparison of the parameter estimates from creep and strength tests

The result from strength tests for the three parameters (α, ρ and β) for two different liner boards and one flute is presented in Table 2.

	Liner 1	Liner2	Flute
α=	39.25	34.65	39.05
ρ=	45.58	34.13	41.27
β=	0.3646	0.8657	0.4564

Table 2. α, ρ and β obtained from strength tests for two different liner boards and one flute

When comparing Table 1 and 2, it can be seen that creep and strength tests show the same range of the values of the three parameters. In this stage, it is not possible to find any consistent differences between the two methods, because of the lack of confidence intervals for the three parameters. Currently no analytical method is available for estimating the confidence interval, but we plan to use and develop a Monte Carlo method for this purpose.

4.3 Lattice model simulation of creep and strength tests

Coleman's formulation concerns, strictly speaking, one-dimensional chain where the weakest-link postulate is applied. Although the weakest-link scaling is expected to hold even in 2 and 3D cases, the formulation has not been subjected to critical tests, and thus it is still a postulate in the higher dimensions. Recently we have succeeded to demonstrate that similar scaling laws appear in the creep behaviour of a 2D system by using a central-force triangular lattice model. In this presentation we use the same approach to calculate a response to constant-rate loading, and examine the applicability of Coleman's postulate.

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