STUDENTS' NARRATIVES FROM GRAPHICAL ARTEFACTS

Exploring the use of mathematical tools and forms of expression in students’ graphicacy

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Department of Science Education and Mathematics
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Cover picture: A bubble graph of items success rate from PISA 2003 for the Nordic countries including the OECD average

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NEVER GIVE IN (Feat. Mr. Lynx)
Written by Mark Mohr and Andrew Carwright
Arranged by Solomon Jabby, Mark Mohr, Bobby Cressey, Rob Alo and Reggie Grisham


I can't let them stop you from what you gotta be doing you know? Especially for the Most High/Emotion/Chorus: Oh, never give up/Don't give in and don't you ever give up(Repeat)/I won't go down quietly into the night/I won't go down quietly without a fight/I won't give up/No won't give in/I won't go down quietly into the night/This love is far too precious to requite/It's far too precious got to fight/I won't give up/No won't give in/I won't go down quietly into the night/You are reluctant to continue/And your resistance won't let you go through/In this instant you gotta change what you do and have faith/For you can take it just so far in length/You won't make it, not on your own strength/Don't bother fake it in your moment of angst just have faith/(CHORUS)/I say we never give up/Never gonna give it up (Repeat)/Sing out/Oh never give up/Youth never give up/With Jesus in your life--He's more than enough/Youth never give up/Please never give up/You got to keep it up/Oh never give up/Youth never give up/Sweet Jesus in your life and He's more than enough/Oh never give up/Please never give up/So no matter what they say or do/Don't you let them ever discourage you/From the things that you've got to do/I know Jah will be there for you/I know He cares for you/So you just keep pressing through/The mark of the prize that is in Him and not you/(CHORUS)/I said we're never giving up though they fight this/I got to stand up and then be a witness/I'm gonna preach the Word of God like John the Baptist/Even if they wanna kill me and leave me headless/I still preach it everywhere that I go/So all the people in the World they can know that Jesus alone yes Him love them so/We a real warrior/We nah stopping now! (CHORUS).
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ABSTRACT

The research concerns presented in this dissertation addresses aspects of students’ interaction with commonly occurring graphical artefacts in teaching and learning environments. In particular, focus is on how the students make sense of these artefacts in relation to subject specific tools and forms of expression.

The dissertation comprises of four studies guided by a semiotics cultural-historical perspective to cognition. The first study which is largely quantitative, analyses the percentage scores from students’ responses to selected items from OECD PISA surveys for items containing graphical elements. The second and the third studies in keeping with a more sociocultural perspective to learning as point of departure, examine the students’ collaborative interaction around tasks containing graphical elements. The fourth study explores the nature of students’ solutions from the Swedish national test in mathematics based on a tools and forms of expression sensitive empirically derived construct of Identification contra Critical-analytical approach to graphicy.

The main outcomes of these studies can be summarised as follows: first there is justification for re-examining the predominant characterisation of students’ interaction with graphical artefacts. Secondly, while it is not uncommon for students to take a more visual-perceptive and intuitive approach to graphicy, results from task items interactions indicate that a Critical-analytical approach seems to be more reliable and capable of yielding desirable outcomes. The outcomes of these studies call for vigilance on the type of tasks used in relation
to graphicacy and how these can be used to foster students’ Critical-analytical disposition.

**Keywords:** Critical-analytical approach, Graph comprehension, Graphicacy, Graphical artefacts, Statistical literacy, Mathematics literacy, Semiotics
SAMMANFATTNING

De forskningsresultat som presenteras i denna avhandling behandlar aspekter av elevers interaktion med vanligt förekommande grafiska artefakter i lärande miljöer. En särskilt fokus riktas på hur eleverna tar del av kärnan [innehållet] i dessa artefakter i relation till ämnesspecifika redskap och uttrycksformer.


De viktigaste resultaten från dessa studier kan sammanfattas på följande sätt: För det första finns det belägg för att ifrågasätta den vanliga förekommande forskningsbaserade karakteriseringen av elevernas interaktion med grafiska artefakter. Det andra är att det inte är ovanligt att eleverna har ett mer visuellt och intuitivt tillvägagångssätt när de handskas med grafiska artefakter. Samtidigt visar resultatet att interaktioner som präglas av ett kritisk-analytiskt tillvägagångssätt verkar vara mer tillförlitligt och ha tendenser att ge korrekt resultat. En slutsats som kan dras av resultaten av dessa studier är att det är viktigt att vara medveten om vilka typer av uppgifter som används i undervisningen i samband med grafiska artefakter och hur dessa uppgifter kan användas för att främja elevers kritiska-analytiska förhållningssätt.
FOREWORD

As I write the foreword to this dissertation I am reminded of myself as a lad listening to luo benga hits. I noticed that often when the music was reaching its climax, [thum oguro], the lead musician apportioned accolades and acknowledgements to the accompanying musicians and it dawned on me that this almost always signalled the end of the music. While there might be a few parallels between writing a PhD dissertation and benga music, I find it necessary to acknowledge the fact that the process attaining its climax in the submission and defence of this dissertation would not have been possible without the direct and indirect intervention of several persons, my accompanying musicians. To all of you, named and unnamed, you’ve meant a lot to me and I hope that we will all find satisfaction in the ‘rhythm’ of the dissertation.

I would like to express my gratitude to my main supervisor Prof. KG Karlsson and assistant supervisor Prof. Astrid Pettersson, I deeply appreciate your being there for me. You have been a source of encouragement, inspiration and support throughout a process that, at times, felt quite hopeless. KG, your ‘hawk-eyed’ approach to my texts was especially appreciated. That it was fine with you to have some supervision meetings in Stockholm highlights your quality as an ‘understanding man’. Thanks also for getting me started with PISA database. Astrid, your insightful comments on my texts and involvement in connecting me with specialised persons illustrates in many ways, your concern for the quality of this dissertation. Thanks for connecting me with PRIM-gruppen and seminarigruppen i matematikdidaktik. I also wish to express my gratitude to all the students whose participation in the studies provided material for this dissertation. . In this regard I wish to acknowledge the help provided by Eva-Karin Rosén former Principal at Härnösands gymnasium, her staff and students as well as my pre-service teacher students. I would also like to thank Ulla Damber for showing me how to communicate with reviewers, to Katarina Kjellström for ironing out my queries on Nationella provet, as well as to Nina Eliasson and Maria Edlund for helping out with diverse information on PISA. My thanks also to Inger Wistedt and Lisbeth Åberg-Bengtsson for your valuable comments on an earlier version of my text and to Torbjörn Tambour for bringing clarity to the notion of dissimilarity. I also wish to recognise the contributions made by Lisbeth Åberg-Bengtsson, Per Edström and Per Åhag who were my supervisors during the initial stage of my studies – thanks for the
countless advice you all gave with regard to the PhD process. Nina Alfrey has meticulously proofread most of my work, thanks!

While the PhD process is carried out in comparative isolation, I was inspired and energised in the interaction between people. The discussion in seminarigruppen i matematikdidaktik at Stockholm University facilitated a much needed environment in which to ventilate my ideas and to be inspired by the works of colleagues. Thanks to all of you for the inspiring and valuable comments on my texts. I am also grateful to my fellow PhD colleagues at Campus Härnösand; Ann-Katrin Perselli, Anneli Hansson, Birgit Gustavsson, Catarina Arvidsson Peter Degerman and Maria Rasmusson – we may have had diverse research interests but the mere thought that there were others struggling with the basics of research provided some hope for the future. I would also like to express my sincere gratitude to the staff at Sambiblioteket [university library] in Härnösand – you are really great! In a way you have been my companion throughout this process, once again, Thanks to my friends and colleagues at the Department of Science Education & Mathematics, vaktmästeriet and IT avdelning in Härnösand, for making my stay in Norrland worthwhile! My thanks to Anders Nyström for his genuineness and especially for bestowing upon me the accolade of ångermanlåning!

Pen ultimately I’d like to thank my parents, Nyakwar Saka and Nyar Musa for believing in me, for being agreeable to my pursuing my dreams abroad and above all for 'standing in the gap', and to my late Grandparents: I hope now ‘I will get to do something more meaningful’. Lastly but not least, my special appreciation goes to my dear family; to my wife Gladys forbearing the brunt of my studies and for her support in almost single handedly taking on the family responsibilities and to my beautiful daughters Aalisa and Nina-Akeyz – no more excuses att inte följa med till lekparken!

Oduor Olande
Brandbergen, July, 2013
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# Abbreviations and Acronyms

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>2-D</td>
<td>Two Dimensional</td>
</tr>
<tr>
<td>3-D</td>
<td>Three Dimensional</td>
</tr>
<tr>
<td>CHAT</td>
<td>Cultural Historical Activity Theory</td>
</tr>
<tr>
<td>CP</td>
<td>The Collected Papers of Charles S. Peirce [these are quoted by volume followed by paragraph number]. These documents are listed under Peirce in the bibliography.</td>
</tr>
<tr>
<td>DTM</td>
<td>Diagrams, Tables and Maps</td>
</tr>
<tr>
<td>EDA</td>
<td>Exploratory Data Analysis</td>
</tr>
<tr>
<td>FC</td>
<td>First Class © [online facility used in a study with pre-service teachers]</td>
</tr>
<tr>
<td>FORTRAN</td>
<td>FORmula TRANslating system [a numerical programming language]</td>
</tr>
<tr>
<td>GIS</td>
<td>Geographical Information System</td>
</tr>
<tr>
<td>High-D</td>
<td>High Definition</td>
</tr>
<tr>
<td>ID</td>
<td>Identity</td>
</tr>
<tr>
<td>Lgr 11</td>
<td>Läroplan för grundskolan, förskoleklassen och fritidshemmet 2011 [Curriculum for the compulsory school, preschool class and the leisure-time centre 2011]</td>
</tr>
<tr>
<td>Lpo 94</td>
<td>Läroplan för det obligatoriska skolväsendet, förskoleklassen och fritidshemmet 1994 [Curriculum for the compulsory school, preschool class and the leisure-time centre 1994]</td>
</tr>
<tr>
<td>OECD</td>
<td>Organisation for Economic Co-operation and Development</td>
</tr>
<tr>
<td>PRIM</td>
<td>[PRov I Matematik] A research group for the assessment of knowledge and competences. The group is also in charge of the construction and administering of the national test in mathematics</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>--------------</td>
<td>-------------</td>
</tr>
<tr>
<td>PISA</td>
<td>Programme for International Students Assessment</td>
</tr>
<tr>
<td>SKOLFS</td>
<td>Skolverkets författningssamling [The Swedish National Agency for Education Code of Statutes]</td>
</tr>
<tr>
<td>SOLO</td>
<td>Structure of Observed Learning Outcome</td>
</tr>
<tr>
<td>SOSE</td>
<td>Studies of Society and the Environment [A study program in the Australian school curriculum which involves the study of history, geography and economics]</td>
</tr>
<tr>
<td>SweSAT</td>
<td>Swedish Scholastic Aptitude Test</td>
</tr>
<tr>
<td>MA-P</td>
<td>Mixed Arithmetic - Perceptual model</td>
</tr>
<tr>
<td>MS</td>
<td>refers to an unpublished manuscript written by Peirce. (see Bergman, 2009). This document is listed under Peirce in the bibliography.</td>
</tr>
<tr>
<td>NEM</td>
<td>The New Elements of Mathematics [a collection of Charles S. Peirce writings: these are quoted by volume followed by page number]. This document is listed under Peirce in the bibliography.</td>
</tr>
<tr>
<td>Np</td>
<td>Nationella provet [Swedish national test]</td>
</tr>
<tr>
<td>UNESCO</td>
<td>United Nations Educational, Scientific and Cultural Organization</td>
</tr>
</tbody>
</table>
1 INTRODUCTION

Graphs and graphing have been and are a part of human history and despite the ease with which graphs can be produced and accessed, the skills and abilities needed to make sense of them seem to evade our understanding. Friel, Curcio and Bright (2001), suggest that educators have a lot to learn with regards to the methods involved in reading, analysing, and interpreting information presented in data graphs and tables. This observation is shared by Aoyama (2007) and Roth (2009) among others, who observe that despite the interest and commitment to graphs and graphing, what it takes to interpret a graph remains much less clear and as such, more research is needed to explore the deeper aspects of students’ insights into graphs. The notion that we are yet to come to terms with what it takes to make sense of graphics stands out almost as a paradox given the ubiquitous nature of graphical artefacts in contemporary society. It is reasonable to assert that not a day passes without us encountering graphics of some sort through the mass media for example. It is almost universally accepted nowadays that a graphic in one form or another would accompany government reports, civil information bulletins, a company financial reports etc. Thus the power of graphics as a conveyer of information is well established. Lowrie and Diezmann (2011) observe that “The information age has provided new and increased demands on our capacity to represent, manipulate and decode information in diagrammatical and graphical forms” (p. 109). Given the widespread use and ease of access to graphics and graphing techniques, it is justifiable to explore the factors involved in making sense of, as well as in the production of graphics. This not least, as a way of providing insights into ways of supporting the teaching and learning of competencies attached to interacting with and in the production of graphical artefacts.

1 The construct graphical artefact is used in this dissertation to include all such graphics that may otherwise be considered as statistical graphs, quantitative graphics, and information graphics etc. A comprehensive discussion on the construct is done in section 2.1.4
1.1 Historical account of graphical artefacts

That graphs would take on such an important role in contemporary society and in particular in scientific endeavours, was probably not a given fact. Graphical artefacts have their origin in pre-history; a clear indication, therefore, that they did not originate with what can be referred to as ‘science’. A comprehensive chronological account of the development of graphical artefacts and important milestones can be found on the website *Milestones in the History of thematic Cartography, Statistical Graphics, and Data Visualization* (http://www.datavis.ca/milestones/). Beniger and Robyn (1978) have also outlined a plausible historical account of graphing. The history of graphics and graphing do however contain anecdotes suggesting elements of disdain and suspicion from the scientific community (see e.g. Funkhouser, 1937). Between 1660 and 1800 many devices that could automatically produce moving line graphs of temperature, barometric readings, wind speed and direction were considered inferior for analysis compared to tabular records (Beniger & Robyn, 1978). Indeed the history of graphics shows that graphics have been characterised by a period of upturns and downturns. Friendly (2005) shows that between 1600 to 1755 graphs had a period of slow but rising acceptance or popularity, reaching a peak in the period 1850 to 1900 before falling into what is considered as the modern dark age (Fig. 1.2). An important observation regarding the rise in the use of graphical artefact corresponded not only with progress but also the needs of society and the scientific community at large.

In Friendly’s (2008), chronological account of the history of graphical artefacts [he refers to these as data visualisation], eight graphical epochs are identified; these epochs centre on the motivating factors behind the development, the communication goals etc. In this dissertation the epochs are examined with focus on what can be considered as essential elements for graphing.

- The first epoch is the pre-17th century characterised by early maps and diagrams: is characterised by a general curiosity about the environment and in particular a keen interest in celestial bodies. Thus, some of the earliest graphics are connected to the heavenly bodies e.g., the 10th century illustration (Fig. 1.1) of the inclination of the planetary orbits as a function of time (Funkhouser, 1936; also Tufte, 2001). In this epoch, Ptolemy’s map in Alexandria was still used as a standard reference and
this continued until the 14th century. It is apparent that some appreciation of geometry was a pre-requisite for the development of graphics in this epoch.

Fig. 1.1: A 10th century graph with ‘planetary’ movements shown as cyclic inclination over time. Shown in descending order is Venus, Mercury, Saturn, Mars, Jupiter and the Moon (Funkhouser, 1936, p. 261)

The second epoch ranges from 1600-1699: measurement and theory. This epoch is characterised by a concern for precision in physical measurement that is, time, distance and space. There was also increased intellectual activity manifested in the growth of theory and the dawn of practical application: these are exemplified in the works of Descartes and Fermat, Tycho Brahe, Pascal among others. This period also saw the birth of demographic statistics by John Graunt and political arithmetic by William Petty. The term ‘Political arithmetic’ was adopted because the purpose was to inform the state about matters related to wealth, population, agricultural land, taxes etc.
1. Introduction

- **From 1700-1799: New graphic forms.** During this period the use of isolines and contours was realised as well as thematic mapping of physical quantities. Abstract graphs and graphs of functions became more widespread. Other developments included systematic collection of data and the use of visual forms to portray the data so that it could *speak to the eyes*. This was also the genesis of statistics theory specifically concerned with the reliability of measurements; error was perceived as deviation from regular graphed lines. Some of the technological advancement that promoted the production and dissemination of graphic works include three colour printing by Jacob le Blon and the invention of lithography by Aloys Senefelder. William Playfair [1759-1823], considered as the inventor of most graphical artefacts in common use, was also active during this period.

- **From 1800-1850: Beginnings of modern graphics** was an epoch that witnessed an explosive growth in graphical artefacts. Around this period an increasing number of scientific publications began to contain graphs describing natural phenomena: this was later followed by graphical analysis of these phenomena. It was also during this period that graphical artefacts were used to challenge theory. A French lawyer, André-Michel Guerry used maps and charts to challenge social reformers theory that crime would be reduced by increased education. The works of Charles Joseph Minard [1781-1870] illustrates the recognition that graphs had acquired when they were used for state planning. While graphical artefacts were widely used in France, in Britain there was a lot of emphasis on numbers, such that making graphs seemed ‘too much like bread making’ (Friendly 2008, p. 29).

- **From 1850-1900: The golden age of statistical graphics** is characterized by the establishment of official state offices throughout Europe. This period was also characterized with innovations in graphics and thematic cartography. The first recoded use of statistical diagram in school textbooks was in France in Pierre Émile Levasseur’s book published in 1868. Notable work around this period was conducted by Francis Galton [1822-1911] to whom when regarding his visual analysis of a bivariate distribution, Karl Pearson said, “that Galton should have evolved all this from observation is to my mind one of the most
noteworthy scientific discoveries arising from pure analysis of observation”. (Pearson, 1920, p.37; see also Friendly, 2008).

- **From 1900-1950: The modern dark ages.** This was a period characterised by lack of interest in graphical methods; consequently there were fewer graphical innovations. Attention was directed to more mathematical concerns. There was a rise in quantification and formal statistical models in the social sciences. In some ways this was a period of popularisation rather than innovation. During this period statistical graphics became widely accepted and used in textbooks, commerce, and curriculum documents etc. It was also around this period that the utility function of graphical methods in the sciences was realised. The concept of atomic number was discovered largely based on graphical analysis. Henry G.J. Moseley [1887-1915] was able to show this through a plot of serial numbers of the elements with the square root of frequencies from X-ray spectra (see Friendly, 2008).

- **From 1950-1975: Rebirth of data visualisation.** It is during this period that John W. Tukey [1915-2000] invented the graph techniques for exploring data graphically- known as EDA. Examples of some of his graphs include the stem and leaf plots and the box plot. Parallel to this Jean-Paul Benzécri developed correspondence analysis, a multivariate statistical technique summarising data into two-dimensional forms. Given that drawing of maps and graphics by hand had diminished during the dark ages, this period witnessed the automation of statistical techniques especially with the creation of the computer programming language FORTRAN in 1957. The widespread use of computers was such that by the end of this period there were modern GIS and also interactive systems for 2-D and 3-D statistical graphics.

- **From 1975-present: High-D, interactive and dynamic data visualisation:** This period can best be characterised by the increasing role of computers and software in the presentation of graphical artefact. It is a round this period that there appears to be a clear distinction between the facets of graphics – graphical artefacts for information contra analytical purposes.
1. Introduction

Fig. 1.2: Distribution [created using kernel density estimation method] of important events in the development of graphical artefacts comparing trends in Europe and North America (Friendly, 2008, p.44).

With this chronological account of the development of graphical artefacts, it can be claimed that there has been an underlying analytical component connected to the development and advancement of graphical artefact. This refers to a connection to a systematic way of sense making: a developmental progression where there is an aligning of a current encounter or experience with previously known notions or experiences converging into a coherent whole. From the very onset it is evident that the mastery of some external factor such as context together with other tools of analysis were necessary for effective interaction with graphical artefacts. Given that most of the early graphs and diagrams dealt with movement and distance it is expected that an appreciation of geometry or knowledge of the movements of the celestial bodies was necessary in order to make sense of the graphs. An example is presented in an analysis of a 10th century graph (Fig. 1.1), where after initially observing that it was difficult to reconcile the graph with the text and with actual movements of the planets, Funkhouser (1936) concluded that:
“...there is some interest in speculating upon what the maker had in mind for there is evidence that he exercised some care in drawing. There can be noticed an erasure and a correction of one of the curves in the center of the diagram. It is possible that a further study of the graph may reveal other advanced ideas in addition to the use of a grid.”

(Funkhouser, 1936, p. 262).

1.1.1 William Playfair [1759-1823]

We can only speculate about the ‘other advanced ideas’ mentioned in the citation from Funkhouser provided above, consequently, these might be such ideas that are not self evident and therefore not easily accessed. Indeed Costiagn-Eaves and Macdonald-Ross (1990) suggest that prior to William Playfair, graphic production by previous designer - scientists was characterised by a paucity of textual codes (see e.g. Fig. 1.1). It was also during the period of William Playfair, a Scottish engineer and political economist credited with inventing and publishing the line, bar and pie charts (see Spence & Wainer 1997; Symanzik, Fischetti, & Spence 2009), that we find indications of a directed focus towards a more visual interaction with graphical artefacts.

It seems that prior to this period, the burden of interaction with graphical artefacts in terms of knowing of context, tools and forms of expression was placed on the consumer of the graphs. William Playfair wanted to make his graphics self explanatory with regard to the main body of text and therefore he included adequate textual coding (Costiagn-Eaves & Macdonald-Ross, 1990). He was driven by a desire to make data ‘speak to the eyes’ (Friendly, 2008). Playfair’s approach to graphics production somehow marks a shift whereby the producers of the graphical artefacts appear to actively aid the consumer of the graphics in their sense making process. However, this does not mean that interaction with the graphical artefacts he invented were devoid of analytical dimensions, but rather the clarity he brought into interaction with graphical artefacts can best be viewed as lessening the burden of analytical demands rather than eliminating them all together. This observation is supported by a cognitive analysis of Playfair’s graphics by Spence (2006). This analysis shows how choices made by Playfair to include form and texture in his graphics, in effect, reduced the cognitive effort needed for interaction with these graphical
artefacts. The fact that the analytical dimension in Playfair’s work was not discarded is captured in his work published in 1789 - the *Linear Arithmetic* an edited revision of *The Commercial and Political Atlas* of 1786 (see Symanzik, et al., 2009). The belief by Playfair that interaction with graphical artefacts is some arithmetic of sorts was probably a consequence of the profound influence his brother, a professor of mathematics had on him as well as his exposure to the works of other mathematicians such as Leibnitz, Euler etc. (Spence, 2005). In acknowledging his brother, Playfair (1805) affirmed that it was he who taught him to know that whatever can be expressed in numbers may be represented in lines.

1.1.2 Charles Joseph Minard [1781-1870]

If William Playfair went all out to make it easier to visually interact with graphical artefact, Charles Joseph Minard, a pioneer in statistical graphic and thematic cartography, was not any different. Minard to whom Tufte (2001) attributed the best statistical graph ever produced (see Fig. 1.3), is recorded as saying that the aim of his *carte figurative* was to convey promptly to the eye an association which was not quickly apparent by numbers requiring mental calculations – to make an immediate impact of the proportions of numerical relationships apparent to the eye (see Robinsson, 1967; Funkhouser 1937). Clearly Minard was not eliminating the burden of mental calculations in interactions with graphical artefacts, rather his statements can be perceived as implying a focus on the essence of the graphics and not so much on the peripheral aspects. Robinsson (1967, p. 95) states that Minard was always concerned about the “accuracy” of his data; an observation that is confirmed by the fact that “the width of a flow line on a Minard map is always strictly proportional to the magnitude it represents” and that “he would strongly resist the tendency of the tyranny of precise geographical position to detract from the essential communication of his chosen theme”. It is therefore feasible to suggest that to Minard, graphical artefacts were not merely aesthetic entities but rather tools used to convey information clearly and accurately. Minard’s approach to the construction of graphics accordingly placed the need for civic responsibility on the producer of graphical artefacts – the need not only for clarity but also for data integrity. This might explain why, in most cases, he added the word *approximative* to his *carte figurative* (Robinsson, 1967). However, the consumer of the graphics also has a part to play regarding effective
interaction with graphical artefacts; Minard emphasised both the visual as well as the analytical dimensions intended to make his graphical artefacts *calculer par l’œil* (Friendly, 2005, p.5). Tufte (1997) suggests that the fundamental analytical act in statistical reasoning is dealing with comparison, the essence being to make intelligent and appropriate comparisons. Tufte goes ahead to show how the works of Minard [in particular the march to Moscow see Fig. 1.3] meet the *principles of analytical design*. In analytical design the focus is on constructing graphs that allows for “describing the data, making comparisons, understanding causality, assessing credibility of data and analysis” (Tufte, n.d.) that is to say, a process that assists in reasoning about evidence in the sense that “the logic of design replicates the logic of analysis” (Tufte).
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Fig. 1.3: The best statistical graph ever produced [Tufte, 2001] showing with ‘brutal eloquence’, the successive losses of the French Army in Napoleon’s Russian campaign of 1812.
1.2 Contemporary graphing practice

As shown in the preceding sections, the graphing practises of the pioneers in this field seem to have been characterised by elements of graphic rigour and civic responsibility on the part of the producer of graphical artefacts. The consumer of the graphical artefacts was [during the pioneer years of graphing] and more or less still is, expected to assume a holistic approach to data with focus on analytical reasoning. According to Tufte (2001) a graphical display should encourage the eye to compare different pieces of data and it should also be closely integrated with the statistical and verbal descriptions of a data set. For example, a line graph makes it easier to use expressions such as “lower than” or “a sharp increase” in values while a dot plot might make it easier to point out outliers using expressions such as “isolated” observation etc. Bearing this in mind, the pertinent issue going forward is a concern for graphical practice: What defines the graphing practise of contemporary producer and consumer of graphical artefacts? An insight into these concerns is significant given the expectations on citizenry to acquire such competencies that allows for effective interaction with graphical artefacts. The fact that the mass media and even some official sources do not shy away from making use of graphical artefacts as a means of communication, is probably based on the assumption that people have the necessary competencies to interact with graphics. This observation becomes pertinent given the emerging trend among data producers to avail of raw statistical data for individual interactive analysis. In the context of this dissertation, ‘necessary competencies’ [or collateral experiences] are considered as familiarity with aspects of context, subject specific tools and forms of expression needed for effective interaction with graphical artefacts.

1.2.1 Graphing in the school context

From a teaching and learning perspective the official regulator and source of advancing societal graphing practice are, without doubt, the curriculum documents and syllabuses. However, in basic school practice, ‘graphics and graphing’ does not appear as an independent subject strand but are integrated with other ‘regular’ subject strands. In the case of mathematics it is not uncommon to find the practice of graphing in the topic strands dealing with
algebra and equations as well as statistics. Indeed graphing as it is explored in this dissertation, has a bias towards statistic. Thus, in exploring how graphing practice is promoted in the compulsory school system we turn to the syllabus for mathematics in the Swedish education system. According to the syllabuses for the compulsory school system – mathematics which was based on SKOLFS 2000:135 (LpO94), where the minimum requirement (goal) for pupils at the end of the fifth school year is that they should be able to read-off and interpret data in tables and diagrams, while at the end of the ninth school year they should be able to not only interact with graphical artefact but they should also be able to interpret, compile, analyse and evaluate data in tables and diagrams (Skolverket, 2002). The criterion for the fifth and the ninth school years may well be summarised based on what can be perceived as level of analytical interaction or engagement with the graphical artefacts: a visual emphasis characterised with being able to read off values for the lower levels and a focus on analytical aspects for the higher levels of compulsory education. In the revised version of the syllabus based on SKOLFS 2008:68, a shift is made from two-levels of learning [educational] assessment to three. The inclusion of the goals for the third school year implies that focus in students learning [assessment] will now start earlier than hitherto has been the case. This division does not seem to alter the general summary mentioned earlier. The minimum goal for pupils in the third school year is that they should be able to interact with graphical artefacts at their sphere of influence involving ‘simple and everyday information’ (Skolverket, 2009). The goals for students in the fifth school year remain largely as before. There are other requirements related to graphing such as being able to deal with coordinate systems and knowledge of calibrating the coordinates. It is noteworthy that the Swedish syllabus for mathematics mentioned above does not mention design and construction of graphics, though this may be inferred in some of the wording e.g. being able to compile data.

The Swedish syllabus from the 1994 curriculum is generally open allowing for local interpretation. Boesen (2006) suggests that local interpretation is dependent upon what the individual teacher perceives as important mathematical ability. It is foreseeable that the individual discretion accorded the teacher or educator might be informed by a number of factors such as societal and industrial needs as well as assessment concerns. Given the open
nature of Lpo94 syllabus, it is envisaged that students instructed under its aegis were introduced to best graphing practices such as graphical rigour (in the production and sense making process) as well as graphical or data integrity. It is worth noting that from the year 2011 Sweden has introduced a new curriculum for the compulsory school level, preschool classes and leisure-time centres (Lgr 11). In the new curriculum document there are specifications that might be perceived as encouraging skills in graph construction: for example the core content in statistics for year three includes learning about simple tables and diagrams and how these can be used to categorise data, on relationships and change. Students at levels 4-6 are to be exposed to graphs for expressing proportional relationships, coordinate systems and strategies for scaling coordinate systems.

1.2.2 Graphing at out of school context

It is difficult to ascertain where and when students encounter graphical artefacts in out of school contexts, however, two probable sources stand out: namely, the mass media and the digital environment especially in online and computer games and gaming. Whereas in some areas of the media students may encounter graphical artefacts in a ‘static’ state, for example in printed versions of newspapers, in most cases the encounter and interaction with graphical artefacts in out of school contexts is relatively dynamic. In some elementary combat games such as those involving two-player fights, it is not unusual to find graphical indicators of fire power. In these kinds of graphical artefacts the interaction is ‘real-time’ giving information depicting instantaneous events and thus visual interaction is predominant. This implies that in a majority of out of school contexts interaction with graphical artefacts, to a large extent, involves identification of values, recognition of states etc. Whereas this might be taken as implying that one does not necessarily need to use external tools to make sense of the graphical artefacts, one might need information on the context of the game to be able to use the information effectively. Thus, for digital environment related games knowledge of context is essential for effective interaction with the graphical artefact. It is noteworthy that this demand seems to follow the pattern from earlier epochs of graphing practice [see section 1.1].
1.3 Effective citizenry

The demands imposed on citizens by contemporary society calls for a degree of knowledgeability in effective interaction with graphical artefacts created by oneself and also by others. Steen (1990) observes that numeracy and literacy [one would also include graphicacy] represent means of communication that are indispensable to civilised living ['numeracy’ as used by Steen ought to be understood in a wider perspective to include aspects of graphicacy]. Some of the reasons that make the ability to effectively interact with graphical artefacts a necessary competence include:

- The burgeoning use of graphical artefacts in the media as well as in civic reports might be perceived an appropriate indicator of the importance of graphicacy in participation in social discourse. The appearance of graphical artefacts [data], in these general societal forms of communication or sources implies that citizens are expected to understand and appreciate such condensed information (Bakker, 2004a). The assumption made on people to the effect that they are able to make sense of these graphical artefacts might be based on the understanding that i) graphicacy is one of the basic components of education and ii) that graphical artefacts are transparent for communicating information. However, this is a view that has been challenged by some researchers (Ainley, 2000; Noss, Bakker, Hoyles, & Kent, 2007).

- Engaging in meaningful economic activity can be perceived as an important characteristic of effective citizenship. Given that one aspect of effective citizenship is that of being a productive member of society, it is not uncommon therefore, to realise this by taking up employment. Graphical artefacts seen as part of statistical processes play an important role at a number of workplaces. For example some industrial manufacturing processes require that workers are able to use and

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2 Graphicacy was coined by Balchin and Coleman (1965) as a construct in the same measure as literacy, numeracy and articulacy: the purpose being to underscore the importance and uniqueness of the ability to produce and interact with graphical artefacts [further discussion is provided in section 2.2].
maintain quality control charts as well as to understand the importance of quality control processes, bank workers [economists, accountants etc.] may be called upon to explain graphical artefacts depicting development in the financial markets. This could explain how some subject specific forms of expressions attached to graphical artefacts are now part of ordinary vocabulary. Steen (1990) observed that graphic artists use words such as ‘spline curves’ for smooth models.

From a didaktik³ perspective, graphical artefacts appearing in non-educational contexts like mass media and ‘non-professional’ contexts for example, might sometimes be approached with some level of suspicion especially with regards to their informational content. However, Wainer (2010) suggests that the rate at which graphical practice in scientific journals is evolving is much slower compared to graphical practice in the mass media – in other words it is possible to find excellent graphs outside the scientifically well established spheres. This observation by Wainer highlights the importance of exposing students to appropriate and effective graphing practices. Thus, data interaction with graphical artefacts ought to be accompanied by a reflective and analytical bearing. Viewed as data analysis, interacting with graphical artefacts is more than mere description, more than just data snoopig, it is an attitude to an overall approach to data (Shaughnessy, Garfield, & Geer, 1996).

Thus, this dissertation seeks to examine how students [in a wider sense of the term] interact with commonly occurring graphical artefacts; the focus being on sense making aspects with regards to the reporting of the results of this interaction with graphical artefacts. This concern is intertwined with the essence of didaktik both as a practice and as a research discipline [praxis och forskningsfält] (Englund, 1997); the idea being to determine ways through research, in which improvements can be made to teaching and learning activity.

³ The Nordic-Germanic concept Didaktik [also translated as teaching and learning] is preferred mainly because there is no clear translation of the concept in the English language (Kansanen & Pepin, 2005), also didaktik represent a particular theoretical and philosophical educational orientation (Hopmann, 2007; Pantić & Wubbels, 2012) [see also section 4.1].
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1.4 Didaktik

The history of *didaktik* can be traced to among others Wolfgang Ratke [1571-1635] and Jan Ámos Komenský [1592-1670] both of whom claimed that their ‘method’ would make learning easier, quicker etc. (see Kansanen & Pepin, 2005). However, as a theoretical construct *didaktik* is mostly associated with Komenský, famously known as Comenius, through his *Didactica Magna*. Komenský had a noble ambition for mankind namely; to in her lifetime go to school and thoroughly learn everything that is necessary (Meyer, 2012). While the history of *didaktik* is well documented (see e.g. Kroksmark, 2007; Kansanen & Pepin, 2005; Hopmann, 1997), it is the rhetorical connection to didaktik and thus to communication (Meyer, 2012; Ongstad, 2005), that is perceived to lend itself to graphicacy in this dissertation. Taken as a way of reaching out with information or communication, graphicacy must, necessarily, be sensitive to the core of rhetoric that is, *ethos*, *pathos* and *logos*.

Before expounding on rhetoric and *didaktik*, it is worthwhile pointing out that in this the construct *didaktik* is preferred since the English word didactics does not really capture the essence of *didaktik* and is also perceived as carrying with it some negative connotation. Being didactic has also been considered as referring “... to a narrow concept of conveying instruction, often by teaching excessively” (Lindström, 2009 p.13). The other English term that is always associated with *didaktik* is education which finds its equivalent in the Swedish [Nordic] term *Pedagogik*. Ongstad, Hudson, Pepin, Imsen and Kansanen (2005) maintain that not only is *didaktik* difficult to define, but it is also “difficult to mediate across cultures, especially when the very language of explanation, English, seems almost alien to the term”, (p. 1). In comparing the teaching practices from *didaktik* and curriculum [associated with Anglo-Saxon culture] perspectives, Pantić and Wubbels (2012) posit that in the curriculum culture teachers are driven by, such concerns as what a student ought “to be able to do or know as set [out] in the curricular objectives” (p. 65) while in the *didaktik* culture “a teacher starts by looking at the object of learning and wonders what this could or should signify to the learner” (p. 65). Since it is the case that teaching and learning occurs in different contexts and involves diverse contents, it is not unexpected that there are ‘different divisions’ of *didaktik*, however, this discussion is not furthered in this dissertation. This is partly because these ‘divisions’ might in some ways be considered redundant (but not
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necessarily unimportant) in light of the observation that *didaktik* in the real sense of the concept must occur within a given context and deals with particular contents. Thus, rather than focus on the divisions, it might be prudent to focus on one of the cores of *didaktik*, namely, the fostering and the formation of the student’s self as it were, linking it to the world directed by the concept of *bildung* (Hopmann, 2007; Pantić & Wubbels, 2012).

From the history of graphicy an impression is created which suggests that graphical artefacts were mainly used for communicative purposes; this explains why Minard and Playfair wanted to make it possible to capture the essence of graphics visually. To be able to do this the graphical artefact had to realise at least three functions: *a*) display a logical order of the elements of the graphical system – making it easier for the reader to make connections with its different parts, *b*) the messenger [producers of the graphical artefact] ought to stand the test of credibility – an observation highlighted by data integrity and lastly *c*) the form and style of the graphics used ought to be appealing not only to the eyes but also to the intellect. These functions can also be extended to the teaching and learning of mathematics; while widely accepted as a logical discipline, it is still the case that credibility is paramount both for teachers and students alike but mostly on the part of the teacher. The function of appealing to the eye and intellect coincides with the aesthetic aspect of mathematics which is widely recognised. It is noteworthy that there is increasingly more research looking at issues of affect in the teaching and learning of mathematics. It is the three aspects mentioned above that find their equivalent in rhetoric viz., *logos*, *ethos* and *pathos*. This connection becomes clear when didaktik is perceived as communication (Ongstad 2005). From a communicative approach, *didaktik* as a discipline is perceived as communicating “something from someone to someone in a certain way…” (p. 69) [italics supplied]. This expression bears some similarity to the concept of sign [including graphical artefact] as defined by Peirce viz., “… something which stands to somebody for something in some respect or capacity” (Peirce, CP 2. 228) [italics supplied]. Thus, there is an apparent connection between *didaktik* and the practice of graphing or interaction with graphical artefact.
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1.5 Overview of the dissertation

This dissertation consists of ten chapters (including the introduction). Chapter 1 sets the scene, introducing graphical artefact and then providing its historical development with the purpose of showing the nature of interaction with graphical artefacts. This account is then subtly compared to contemporary society’s approach to interacting with graphical artefacts. Chapter 2 outlines the trajectory of research on graph comprehension, the aims and purpose of the dissertation, thus determining its scope; it also provides clarity on some of the terminologies associated with graphics as well as concepts that are frequently referred to in the dissertation. In particular it points out that the nature of some of these terminologies serve some specific purpose that is hopefully revealed in this dissertation. Chapter 3 presents a background of some the research dealing with relevant facets of interaction with graphical artefacts. For example ways of identifying as well as classifying levels of interaction is provided with focus on their theoretical orientation, this culminates in espousing a theoretical position and developing an analytical framework. In Chapter 4 the methodological considerations are discussed. The dissertation reports from studies where multiple pronged methods have been applied viz., statistical analysis of students’ results, video observations as well as online didaktik activity. In chapter 5 an empirical study is provided which serves as a base for the rest of the studies. A theoretical discussion connected to the findings from chapter 5 is then provided in chapter 6. Chapters 7, 8 and 9 outline the remaining studies which include a video observational study, online conversation study and analysis of students’ written solutions from Nationella provet respectively. Chapter 10 discusses the results including the dissertation’s implication for practice.
2 EXPLORING GRAPHICAL ARTEFACTS

In the preceding chapter an historical account of graphic practise has described the interplay between analytical and surface interaction with graphical artefacts. It was also indicated that interaction with some of the earlier graphical artefacts might have been more demanding or difficult to decipher probably on account of their being heavily dependent on a consumers proximity to context and other factors specific to the given graphics. However, graphical practice as exemplified by the works and ambition of Playfair, Minard etc. seems to place the analytical dimensions on graph production (Costiagn-Eaves & Macdonald-Ross, 1990; Friendly, 2007; Spence, 2006) thereby making access to information in graphical artefacts “without fatigue and trouble of studying the particulars of which it is composed” (Playfair, cited in Bertin, 1983 p. ix). In recent times and probably heightened by the proliferation of diverse means through which graphical artefacts can be easily accessed and produced, it is reasonable to maintain that the role of being a ‘consumer’ as well as ‘producer’ of graphics is becoming increasingly integrated. This claim also gains more prominence in the light of some of the data producing agents deciding to avail of raw data for public analysis.

In view of this shifting landscape in graphing practice, it might be necessary to re-examine the terminologies used in graphing practice. This chapter thus discusses some of the concepts used with respect to graphical artefacts; this is then followed by a section mapping out the research landscape involving interactions with graphs. The ultimate purpose of this chapter is to situate the present research in relation to other related research in the areas of interaction with graphics [where interaction is in this dissertation is perceived as mutually affecting i.e. graphics and the ‘consumer’ mutually influence each other in line with Peirce semiotics]. In mapping the landscape a broad entry point involved defining mathematics literacy, statistics literacy and then linking these to graphiacy. At the end of the exposé there will be an overall rationale for the dissertation as well as clarification of the relevant terminologies.
2. Exploring graphical artefacts

2.1 A matter of definitions

Generally definitions\(^4\) are perceived as playing a central role in the sense making process and as such, providing an acceptable definition of a concept, phenomenon or an object might be taken as an indicator of some level of knowledgability with respect to the concept, phenomenon or object in question. However, some constructs used in relation to ‘graphics’ are not entirely unproblematic. While ‘graphics’ might be perceived as self evident, there are differences in how the construct is defined and used. Terminology such as graphs, diagrams, graphical representations, graphical displays, inscriptions or even visual graphics might mean different things to some people, while to others they might as well be referring to basically similar entities. This is not entirely unexpected since according to Kaput (1998) this area of study is notorious for its complexity and subtlety and seems to encompass almost everything that is of interest to educational researchers.

The differences in terminology and definitions used to refer to graphics might be based on the different foci given to the entity by different researchers involved with it. For example, from an information design perspective, Mackinlay (1986) perceived bar charts, scatter plots, and connected graphs [what he refers to as graphical presentations] as sentences of a graphical language and as such are conditioned by syntactic and semantic definitions. From this point of view the thousands of graphics in use can be categorised into six graphical languages viz., Axis languages, Apposed-position languages, Retinal-list languages, Map languages, Connection languages and Miscellaneous languages. These languages link perceptual elements with particular encoding techniques e.g. Apposed-position languages such as in a line graph, and bar chart: where information is encoded by a marked set that is positioned between the axes (Lowrie & Diezmann 2007; Mackinlay 1986). The prominent element of this definition is the effective utilisation of the perceptual faculties and the structural components of the graphic system.

\(^4\) The term definition is in this dissertation used in a wider sense and is not only restricted to mathematical definitions which might be subject to certain criteria (see van Dormolen & Zaslavsky, 2003; Edwards & Ward, 2004). However, the general idea alluded to here might apply as well to some mathematical definitions.
Kosslyn (2006) on the other hand defines a graph as a visual display that illustrates one or more relationships among numbers and is also considered as shorthand for information that would require relatively more words or numbers to describe. Kosslyn seems to emphasise the conciseness of ‘graphics’ for information purposes. Bertin (1983), perceived graphics as a sign-system presenting any problem conceivable in the form of a double-entry table: serving the function of storing, understanding and communicating essential information. Given the varied use and meaning (emphasis) attached to the construct ‘graphics’ and its derivatives, we find it necessary to elucidate on some of the constructs which have some bearing to the present dissertation viz., graphical representations, inscriptions, diagrams and graphical artefacts.

2.1.1 Graphic representations

Graphic [graphical] representation is probably the most commonly used term to refer to diverse ‘graphics’. While the term might suffice in a majority of situations, it also tends to convey some epistemological signals which might be unintended. Representation is defined as “[a] thing that stands for, takes the place of, symbolises, or represents another thing” (Reber & Reber, 2001 p. 624 italics supplied). Expressed thus, this definition alludes to a one-to-one association between the ‘representer’ and the ‘represented’ [i.e. sign - referent relationship]. In the case of a graphical artefact, this clearly translates into a dyadic relationship which as it will be shown [section 3.4.1], might not necessarily lend itself to interaction with graphical artefacts as envisaged in this dissertation. While the relationship thus created might in certain circumstances be problematic, the essence of ‘representation’ as put forward in this definition is realised in the rest of the definition where Reber and Reber stipulate that in cognitive studies one makes reference to mental representation of a stimulus event which can take on various meanings depending on theoretical orientation. In cognitive psychology representation is perceived as “… the mental or encoded form of information which stands for some sensation, perception, object, or other ideas external to itself” (Matsumoto, 2009 p. 440). Reference to ‘mental’ aspects fits into the thesis that representations are cognitive structures constructed a priori, as it were, and which help in the interpretation of tasks etc. (Ciffarelli, 1993). While cognisant of the application of ‘graphical representation’ in relatively theoretical neutral sense, it is also the
case that the construct might be associated with the constructivism, thus rendering the construct rather ambiguous.

### 2.1.2 Inscriptions

Section 2.1.1, alluded the ‘connection’ between the construct graphical representation and constructivism. As a way of distinguishing between representations which exist in material form and those that exist in the mind, graphical representations are referred to as *inscriptions* in the sociology of science and postmodern discourse (Roth, 2003; Roth & McGinn, 1998). However, the construct inscription is not only limited to graphs but includes equation, statistics, tables, photographs, maps, models, charts, diagrams, graphs, formulas etc. constructed in the interest of making things visible for material, rhetorical, institutional, and political purposes (Latour, 1987; Pozzer-Adenghi & Roth, 2005; Roth, 2005; Roth & McGinn, 1998; Wu & Krajcik, 2006a). Probably, the most salient characteristic of inscription is its social accessibility thus making it liable to translation, reproduction, rescaling, transfer etc. (Roth & McGinn, 1998). The social character of the construct is further underscored in view of the observation that an inscription has no meaning in itself other than that arising in the context of other inscriptions and sign forms and that they can pass as boundary objects (Roth & McGinn). Thus, the scope of inscription is not only limited to such visual graphics that are produced manually or through the help of technology, but might be perceived as including “…all the types of transformation through which an entity becomes materialized into a sign, archive, a document, piece of paper, a trace” (Latour, 1999, p.306). This observation places emphasis on the inscription process [cascade of inscriptions] that is the multiple translations, transformation and abstraction that eventually lead to final order inscription in the form of diagrams, tables, equations etc. (Latour, 1987; see also Roth, Tobin & Shaw, 1997; Wu & Krajcik, 2006a).

### 2.1.3 Diagrams

Generally speaking, diagrams are kinds of inscriptions of some permanence in any kind of medium (paper, sand, screen, etc.). These inscriptions are mostly planar but some are 3-dimensional like the models of geometric solids or the manipulative in school mathematics. Mathematics at all levels
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abounds with such inscriptions: Number line, Venn diagrams, geometric figures, Cartesian graphs, point-line graphs, arrow diagrams (mappings), and arrows in the Gaussian plane or as vectors or commutative diagrams (category theory); but there are also inscriptions with a less geometric flavour: arithmetic or algebraic terms, function terms, fractions, decimal fractions, algebraic formulas, polynomials, matrices, systems of linear equations, continued fractions and many more. There are common features to some of these inscriptions which contribute to their diagrammatic quality as understood here.

(Dörfler, 2004, p. 5)

The definition provided above departs from the one given by Reber and Reber (2001) where a diagram is characterised as a schematic drawing that presents the essential features of a system; clearly emphasising the explanatory aspects of a diagram rather than the ‘representative’ aspect. The different approaches provided by these definitions can be explained in part, by the fact that Dörfler’s definition is formulated from a perspective influenced by Charles S. Peirce’s [1839-1914] semiotics. Within Peirce semiotics, a diagram is an icon of relationship which is aided to be so by some system of rules (see CP 4.418). It is noteworthy that while diagrams are a form of inscription, not all inscriptions which occur in scientific practices, reasoning, learning and teaching have a diagrammatic quality. The qualities of inscription that lend itself to a diagram in a semiotic sense is captured in Wu and Krajcik (2006b) process of inscriptional practices employed by scientists in scientific endeavours viz., the use of tools to construct and generate inscriptions, reading and interpretation of inscriptions, reasoning with inscriptions which could involve material and social resources and finally, a critique of the inscription to affirm its credibility. The ‘inscriptional practices’ mentioned above resonate well with diagrammatic reasoning perceived as the process of constructing representation of relationship within a given knowledge area e.g. a distribution function, or a triangle in Euclidean geometry, experimenting with them and observing the results (Bakker & Hoffmann, 2005).
2. Exploring graphical artefacts

2.1.4 Graphical artefacts

In exploring the three common constructs associated with graphical artefacts viz., graphical representations, inscriptions and diagrams, it can, without loss of generality, be said that there seem to be some theoretical perspectives informing the use of these constructs. Clearly there were connections made between the construct ‘graphical representation’ and constructivism. In the case of ‘inscription’, it seems emphasis is placed on the social cognitive dimension where interaction with graphical artefacts is perceived as a dynamic process occurring in a social sphere through which inscriptions are created and improved. From a semiotic perspective, diagrams are perceived as being attached to some set of rules that not only regulate their production but are also crucial in experimenting and reasoning with diagrams. Given that the present dissertation reports from a theoretical perspective, that is, non-constructivism and that it also does not strictly follow a ‘process’ as outlined in inscriptive practices, adopting the constructs graphical representations and inscription might be suspect. Given this development it becomes feasible to use another construct namely the application of the construct ‘graphical artefacts’. The construct graphical artefact seeks to blend some of the positive characters associated with inscription and the general notion of graphical representation. The construct diagram is inherent in ‘graphical artefacts’ by virtue of adopted theoretical perspectives in this dissertation.

In the construct ‘graphical artefact’, the term artefact is used in the sense of cultural or social artefacts: as a psychological tool which can be used to control behaviour from the outside – it is perceived as a device for mastering mental processes (see Daniels, 2008). In a talk entitled ‘The instrumental method in psychology’ given at the Krupskaya Academy of Communist Education, Vygotsky (1930) stated that by nature psychological tools are social. They are artificial formations directed towards the mastery of mental processes – one’s own or someone else’s – examples of which include: language, different forms of numeration and counting, mnemotechnic, algebraic symbolism, works of art, writing, schemes, diagrams, maps, blueprints, all sorts of conventional signs, etc. ‘Graphic’ on the other hand, is perceived as serving more than just a reservoir of information – an aspect inherent on its nature as sign. The interactions associated with graphics are not only limited to tools and forms of expression determined by a scientific domain. Thus, the construct graphical
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artefact is used to refer to such ‘graphics’ that commonly occur in educational and everyday situations. Included here is what has been referred to as statistical graphs and diagrams e.g. bar graphs, histograms, pie-charts and pictograms.

In graphical artefacts ‘graphics’ are also recognised as both a ‘direct source’ of information as well as an analytical tool in other words, as something that may be manipulated using other tools etc. in order to develop or access information. In this sense graphical artefacts may be used by individuals as mediators (Daniels, 2008). Hence, in the present dissertation the term ‘artefact’ in ‘graphical artefact’ ought to be perceived in the sense of socially constructed entities that may aid in explaining, representing and controlling our environment as well as enhancing cognitive ability (Interaction-Design.org, 2006; Macdonald-Ross, 1977; Norman, 1991). Thus, the interest in exploring aspects of interaction with graphical artefacts in sense making endeavours.

2.2  Graphicacy

The ambition of this dissertation in part includes an attempt to characterise students’ interaction with graphical artefacts. By interaction it is implied the steps taken by students [their reaction] when confronted by or when they encounter situations of graphical artefacts: what kind of tools do they use in the sense making process? Viewed in this way, and specifically, as an ‘interaction’, it is then of interest to investigate the pattern or forms of behaviour elicited with regards to tool use and forms of expression. In this regard the construct graphicacy which can be traced back to Balchin and Coleman (1965) forms an important ingredient for this dissertation. Graphicacy emerged in opposition to what may be considered as a narrow perspective of the underpinnings of a sound education where emphasis was placed on teaching literacy [reading and writing], numeracy and articulacy. To Balchin and Coleman (1965) a good education should include graphicacy – which in this dissertation is perceived as the ability to make sense of graphical artefacts due to [created by] oneself or others (Friel, Curcio, & Bright, 2001). This expression of graphicacy varies slightly from the original definition which, as it was first presented seems to emphasise what may be perceived as the communicative aspects (see Danos & Norman, 2009).
2. Exploring graphical artefacts

Graphicacy...is the communication of relationships that cannot be successfully communicated by words or mathematical notation alone./.../graphicacy spills over into literacy on the one hand and numeracy on the other without being marginally absorbed

(Balchin & Coleman, 1965, p. 947)

From the above definition it is evident that emphasis is placed on the communicative dimension attached to graphical artefacts: graphicacy is perceived as a form of communication that enhances clarity that cannot be attained by either numeracy or literacy [here literacy seems to be mostly associated or limited to reading and writing]. It is noteworthy that this view of graphicacy is relatively in harmony with the definitions of ‘graphics’ as outlined in section 2.1. Whereas clarity as such is a necessary aspect of producing appropriate graphical artefacts for communication purposes, graphical artefacts might also be used in other settings to serve more than just a communicative purpose. Bertin (1983) alludes to a dual function of graphics as a *storage mechanism* and as a research *instrument*. Hoffmann and Roth (2007) distinguish the two functions of graphs as *representational* and *epistemological*. González, Espinel and Ainley (2011) consider these functions as *informational* and *analytical*. In mathematics for example, graphical artefacts can be used as tools to enhance or attain new levels of knowledge or perspectives of a concept (see Bakker & Hoffmann, 2005; Bloch, 2003). Although clarity in respect to graphical artefact is a desirable trait, in certain cases this may deliberately be obscured by forces producing graphical artefacts. Thus, by referring to ‘making sense’ of graphical artefacts due to oneself and to others, it implicitly implies the efforts on the part of the ‘consumer’ of a graphical artefact to attain the essence for which it has been availed. This may therefore put some demand on the consumer of such a graphical artefact to be familiar with context, knowing of and accessing possible means of manipulation or transformation, the ability to effectively communicate the ‘results’ of interaction with the graphical artefact etc. In certain cases some knowledge of the processes involved in producing the graphical artefact may be necessary; for example knowledge of the sample size and sample selection methods may be necessary for sense making in the case of some statistical graphs.
2.2.1 Critical stance

Owing to the demands attached to making sense of graphical artefacts, it is reasonable to emphasise a critical disposition with respect to graphicacy. Monteiro and Ainley (2003; 2004) suggested critical sense as a skill to analyse data and its interrelations rather than simply accepting the initial impression given by the graph. Central to their study was familiarity with the nature and type of graphical artefacts used, the use of reflective questioning and adopting an enquiry approach to tasks as a way of eliciting or exercising critical sense. Using the ideas from critical literacy, Roth, Pozzer-Ardenghi and Han (2005) drew attention to the importance of being critical with regards to graphicacy. This they refer to as critical graphicacy where critical graphicacy is perceived as allowing for the “... questioning the power relations, discourses, and identities that human agents produce and reproduce using various forms of graphical representation.” (Roth et al., 2005, p. xii).

Given the ease with which one can acquire blind spots occasioned by acquired structured ways of interacting with graphical artefacts (Roth et al., 2005) the need to be critical with regard to interaction with graphical artefacts is a necessary competence for effective citizenry. Thus, care ought to be taken in teaching and learning situations involving graphical artefacts, so as to include a reflexive component that allows students to critically evaluate the knowledge claims of a particular field or media (Roth et al., 2005). In this dissertation, critical stance or being critical is embedded in the construct, graphicacy. This claim is in harmony with the intentions of Balchin and Coleman (1965) who already, at the coining of the construct identified its advantage as a means for teaching children critical ability in contrast to numeracy which they posited was too rigid or literacy which they perceived as too subjective from the start.

2.2.2 Graphical integrity

Given that taking a critical stance is considered as an essential element of being graphicate, one would view the claims by Tufte (2001), that much of the 20th century thinking about statistical graphs has been preoccupied with how some charts might fool naive viewers while other important issues such as use of graphics for serious data analysis are largely ignored as exemplifying a lopsided approach to graphicacy. However, since graphicacy involves both the
production and consumption of graphical artefacts, issues concerning data or graphical integrity are important aspects to consider in teaching and learning situations. Integrity comes with regards to faithfulness in data handling, including production, compilation and display as well as communicating the results of interaction with graphical artefacts. Thus, graphicacy can be perceived in a wider sense as incorporating ethical-communicative considerations as manifested in the need to uphold data integrity.

2.3 Graphicacy landscape

Given the multidisciplinary nature of graphicacy [research on graphicacy span a number of disciplines such as geography, engineering, design and artificial intelligence.], a narrow focus informed by the ambitions of this dissertation was presented in section 2.2. Thus, in providing an outline of the graphicacy landscape attention will mainly be given to such research that is considered to be of immediate relevance to the present dissertation, namely such research where mathematical and statistical aspects are in the foreground. In this regard this section begins by first providing a brief outline of research on mathematical literacy before moving to more specific areas of statistics literacy. Segments of the data used in the present study emanate from PISA survey which is largely concerned with use of mathematics in everyday life. Thus, in the context of this dissertation, graphicacy is perceived as an essential citizen literacy competence. This stance is taken in view of the ubiquitousness of graphical artefacts in societal communication, an occurrence that is perceived as indicating a general assumption on populace graphicacy [ability to effectively interact with graphical artefacts]. In reference to citizen literacy it is noteworthy that the construct literacy, presupposes a universality of sorts: universality with regards to subject specific domains can be perceived in at least two ways: firstly, with regards to the content of the subject, that is, common sets of subject specific competencies that citizens ought to be exposed to [especially in institutionalised basic education] and secondly, as inclusion where there is a concerted effort to provide these ‘sets of competency’ or exposure to all sections of the populace (Amit & Fried, 2002; Gellert 2004; Goldman & Hasselbring, 1997). In this dissertation focus will be on the latter.
2.3.1 Mathematical literacy

Invoking the construct, literacy, in relation to mathematics may be understood as an ambitious undertaking emanating from the importance of mathematics [its tools and forms of expression] in contemporary life. The essentiality of ‘mathematical literacy’ is captured in recognition that literacy is a fundamental human right – a foundational universal life skill with the potential to enrich peoples lives and stimulate social, political and cultural participation (UNESCO, 2005). This perspective while not entirely exhaustive in defining literacy, seems to mirror the definition of mathematical literacy when emphasis is placed on the self actualisation of the individual [i.e. a way to self-reflection, self-fulfilment and societal involvement]. This line of thought also seems to form the basis of the definition provided by OECD PISA, that is, as:

…an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen.

(OECD, 2003, p. 24)

In the definition provided above, the ‘individual’s life’ is perceived as extending to include “... current and future private life, occupational life, social life with peers and relatives and a life as a constructive, concerned and reflective citizen...” (OECD, 2001, p. 22). The definition given by PISA has attracted some criticism for among other reasons, being broad-based (e.g. Amit & Fried, 2002; Gellert, 2004). In the context of this dissertation this definition is found to be intelligible viewed from a dispositional perspective that is to say, as an attitude towards mathematics.

The broad-based approach to mathematical literacy attributed to OECD is not unexpected given the observation that in all countries of the world [i.e. the individual’s social and cultural milieu], citizens encounter umpteen tasks involving quantitative, spatial, probabilistic concepts etc. (OECD, 2003). While there might be different perspectives to mathematical literacy probably due to its complex and dynamic nature [refer to Jablonka (2003) for a comprehensive discussion on mathematical literacy], it has been suggested (Amit & Fried,
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2002) that a more general conception of mathematical literacy is needed where focus is on its most general conditions. Looking at it from the didaktik of mathematics viewpoint, this becomes pertinent with regard to the applicability of mathematical activity in the teaching and learning setting. Gellert (2004) suggests that the utility of mathematical activity ought to be determined by its contribution to the development of the learners’ mathematical literacy. However, given the different perspectives on mathematical literacy [also the span of school mathematics including e.g. probability, geometry, algebra, statistics, and calculus] it might be challenging to determine its ‘general aspects’. In the wake of a lack of clarity on what may constitute mathematical literacy Amit and Fried (2002) suggest that an approach to mathematical literacy, must never imply that only an expert can be mathematically literate: thus, basically embracing the notion of mathematics for all. They then provide a perspective to mathematical literacy based on the Greek word for mathematics mathein and ta mathe^matika which means to ‘to have learned’ and ‘things able to be learned’, respectively. In effect implying that mathematical literacy involves perceiving “…mathematics as something intellectually attainable, something truly able to be learned” (Amit & Fried, p. 502). This perspective seems to emphasise the importance of influencing the learner towards a desirable disposition essential for mathematical literacy. This viewpoint was alluded to in section 2.2.1 and is an essential aspect of this dissertation.

2.3.2 Statistical literacy

In the previous section it was indicated that mathematical literacy is multifaceted and therefore might be challenging to define. One of the challenges include is identifying aspects of mathematical literacy that might be considered as ‘general’ and thus essential for effective citizenship. On the other hand an approach advocating a focus on essential aspects of mathematical literacy would in the first instance probably identify what can be considered as ‘essential content strands’ of mathematics before specifying the essential element therein. It is noteworthy that while statistics appears as a content strand in school mathematics, it is commonly perceived as a discipline distinct from mathematics (Callingham et al., 2012; delMas, 2004; Weldon, 1986). Thus, while the exposé provided in this section may be taken as looking at literacy with respect to a particular school mathematics content strand, the
intention is rather to outline the approach to statistical literacy advocated by statistics educators.

The common research methods applied in determining statistical literacy seems to give considerable room to the application of statistical tools and forms of expression [for a comprehensive account of statistical literacy linking the same with quantitative literacy etc. refer to Watson & Callingham (2003)]. The importance of this approach is probably informed by the recognition that the study of statistics provides tools [it is noteworthy that a number of these tools have a mathematical base] that people need in order to react intelligently to the myriad of quantitative information that they are constantly bombarded with (Ben-Zvi & Garfield, 2004). Watson and Kelly (2008) highlight two definitions of statistical literacy provided by Gal (2002) and Wallman (1993) where there is emphasis on both the application of subject specific tools and the forms of expression, as well as what can be considered as possession of a critical disposition. Gal (2002) refers to the ability to interpret and critically evaluate statistical information encountered in diverse contents, the ability to communicate reaction to the statistical information encountered as well as having the right set of attitudes, beliefs and critical stance. Wallman (1993) also points out the ability to appreciate the contribution that statistical thinking can have in decision making in private, professional and public spheres. Consequently, it would appear that meeting the demands of statistical literacy requires a great deal of statistical understanding to the extent that statistical learning or instruction becomes almost inevitable (Watson & Kelly, 2008). The importance of statistics learning for statistical literacy is recognised based on for example, the observation that some of the terms used in everyday language may be at variance with statistical terminology (e.g. Watson & Kelly, 2008) while some of its ideas are complex, difficult or even counterintuitive (Ben-Zvi & Garfield, 2004). With regards to what needs to be learned, intimation is found in the approach to statistical literacy provided by Ben-Zvi & Garfields. To them, statistical literacy includes skills that may be used in understanding statistical information or research results and these include being able to organise data, construct and display tables, and work with different representations of data: aspects that are linked to interaction with graphical artefacts [also considered as graphical literacy].
2.3.3 Making sense of graphical artefacts

Before looking into the specifics of making sense of graphical artefacts [an aspect of statistical literacy with respect to interaction with graphical artefact], it is appropriate to mention that Watson (1997; see also Watson & Callingham, 2005) developed a framework that to some extent highlighted the interplay between tools and forms of expression. This framework was largely inspired by the SOLO taxonomy [see also section 3.2] whereby she identified six major levels of progression in statistical literacy. Of significance here is the progression of complexity between the levels. Level 1 pertains to basic reading of graphs, identification of simple one-to-one relationships, counting, being able to select largest number etc. Level 6 pertains to the ability to make summaries from representations, engage in critical thinking in increasingly unfamiliar and mathematical settings; there is the ability to employ proportional reasoning, rate counting, multiplication etc. [it is noteworthy that Watson also refers to a three tier model of statistical literacy involving understanding basic statistics terminology, embedding statistical terms within a real context and ultimately critical reasoning]. The SOLO model and its derivatives have been widely used within statistics education research. For example, taking a cue from the work of Watson and Callingham (2003), Aoyama (2007) operationalised a SOLO inspired framework specifically to characterise a hierarchy in students’ interpretations of graphs that is to say, their graphacy levels. In order to capture the range of possible outcomes, a mixture of tasks accompanied the graphical artefacts used in Aoyama’s study. There were tasks of the ‘informational retrieving’ type as well as ‘opinion questions’ (cf. Gal, 1998). This was necessary since tasks of the ‘information retrieving type’ may limit the application of subject specific tools [mathematical aspects] compared to for example, those items that require expression of opinion and which might allow for the application subject specific forms of expression. However, while there seem to be a concerted effort to use SOLO inspired models, Shaughnessy (2007) appears to suggest that there is reason to evaluate the merits and demerits of the model.

Generally, research on making sense of graphical artefacts seems to focus more on the artefacts per se (see e.g. Danos & Norman, 2009), that is, with design [structural aspects] or visual components. Aoyama (2007) observes that in as much as many researchers have focused on students’ ability to extract data...
from graphs etc., more research that focuses on the deeper insight into statistical graphs is still needed. In the review of research on graph comprehension, Friel et al., (2001) also make a similar observation noting that much graph-comprehension research focuses on graphs used as means of communication. This is not entirely unexpected given that the history of ‘graph comprehension’ [see section 1.1] is filled with accounts that are more oriented towards a visual interaction with graphical artefact. In their article Friel et al., showed that most research tended to identify three levels of interaction with graphical artefacts. While the review by Friel et al., also indicated that there are other factors [e.g. prior mathematical knowledge] involved in the effective interaction with graphical artefacts, considerable space is given to the three levels viz., reading the data, reading between the data and reading beyond the data. These levels of interacting with graphical artefacts [also considered as levels of graph comprehension] are linked to the type of question posed, and thereby, are seemingly linked to Bertin (1983) who also identified three levels of questioning that impact the way a graph is ‘read’. [It is noteworthy that Friel et al., (2001) took a constructivist perspective while Bertin (1983) took a semiotic approach to interaction with graphical artefact] The three levels of graphic comprehension can generally be characterised as follow: Level 1 is associated with activities involving solving a task that requires the identification of the values from a graph. For example, a task requiring the user to identify the number of Volvo car models passing through a given section of a highway from a graph of car models. Level 2 includes such questions that require moving beyond isolated elements of the graph, such that one might have to account for the distribution of car models observed on the highway on a weekly basis. Level 3 is perceived as a summative, inferential level and can be characterized by questions such as, what is your general impression of the distribution of car models observed using the highway? Based on the data, what can one conclude about say, the occupational or social economic status of the person frequenting the highway on particular days etc.? 

The centrality of the type of question posed cannot be underestimated in the interaction with graphical artefact. This is because the type of question posed can be considered as part of what can be referred to as a network of signs. These are factors associated with the graphical artefact that in one way or the other influences the trajectory of effective interaction or sense making with the graphical artefact [see also section 3.2.1]. While, the three-tier approach to
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Graphacy seems to dominate research on graph comprehension, other researchers have suggested a focus on two levels of questioning. Gal (1998) identified two levels of questions that can be posed with regard to the interpretation of graphs and tables: literal reading questions and opinion questions. Literal reading questions are related to simple activities of the kind where students point out values on the graph or compare value points. Opinion questions on the other hand focus on the quality, reasonableness and relevance of evidence used to further a thought sequence. Since the type of questioning plays an important role in interaction with graphical artefacts, it is probable that the types of questions that students are exposed to in the teaching and learning situations might impact on their general disposition with regards to interaction with graphical artefacts (see Aoyama, 2007). Research indicates that students generally adapt to certain ways of doing things by developing structured dispositions and therefore, there is a need to expose learners to different conceptions of the learning objects (Roth, Pozzer-Ardenghi & Han 2005; Nilsson, 2009). If these structured dispositions are uncritically employed this can develop into what can be interpreted as indoctrination. Also the type of questioning encountered in school situations might influence students’ perception of the subject strand. In a study conducted by Watson and Nathan (2010) some teachers observed that their students associated statistics with English [language] complaining that “...they don’t want to do the reading and the thinking and the writing cause that’s English or SOSE or something ...” (p.72).

Friel et al., (2001) observed that although research indicates that mathematics knowledge may be related to graph comprehension they “... found no analyses in which researchers considered the development of mathematics (and arithmetic) knowledge and specifically related this to the kinds of displays being used...” (p.149). However, it is observed that a model by Gillan and Lewis (1994) – the MA-P Model leaves some room for ‘mathematics’ even though it is primarily focused on the visual perceptual factors. Indeed the mathematics referred to in the model would easily fall into the category ‘calculating by the eye’, [see Section 1.1.2] given its emphasis on mental calculation. In describing MA-P, Gillan (2009) states that the method derived its name from the recognition that “…some of the processing components involved sensory/perceptual processing (e.g., visual search and height comparison) and other components involved mental arithmetic (i.e., addition,
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subtraction, division) on values of the indicators...” (p.829). This was a general prediction model intended to determine the time it would take to complete a task containing a graphical artefact, an aspect that they (Gillan & Lewis, 1994) found to be linearly related to the number of process combinations [i.e. arithmetic, visual or visual imagery] attached to the graph.

Other research seems to highlight the role of subject specific tools and forms of expression [statistical and mathematical concepts and processes are included here] in effective interaction with graphical artefacts. It was observed in a quantitative study using students’ results from a high stake examination, (Gustafsson, Wedman & Westerlund, 1992) that a subtest dealing with the interpretation of diagrams, tables and maps closely fitted or coincided with the descriptions of the overall factor (the G factor) that reflected the capacity for problem solving in new and complex tasks. In a follow-up study Åberg-Bengtsson (2005) seems to provide some clarity to the DTM subset by suggesting that it measured an analytic dimension and that the quantitative factor in DTM was related to items where calculations of one kind or another were used in addition to the reading off of values. Thus, these studies can be perceived as indicating the importance of being fluent in subject specific tools and forms of expression. Research taking a narrower focus on interaction with graphical artefacts can be perceived as indicating the importance of micro analysis on the use of tools and forms of expression with the view of enhancing learning. Callingham, Watson & Burgess (2012) pointed out that less than half of the Australian students were able to score high points on a statistics task [graphical artefact] requiring the application of proportional reasoning. Cooper and Shore (2008) considered the determination of variability and judgement of centre expressed through graphical artefacts and found that students faced difficulties depending on the different types of graph use.

It has been shown that the models of graph comprehension presented here, while acknowledging the importance of subject specific tools and forms of expression in effective interaction with graphical artefacts, did not as such accord this aspect a prominent role. As much as the SOLO model does indicate proportional reasoning at the highest level, it is perceived to be majorly descriptive and scantily on other tools and forms of expression. Also, its hierarchical underpinnings seem to be rather generalising. This observation applies also to most of the three-tier models outlined in Friel et al., (2001).
Generally, it seems that research in graphicacy would benefit from an organised approach to tools and forms of expression with respect to interaction with graphical artefact. This is based on the observation that “[s]ymbols are indispensable as means for coding the result of thinking” (van Oers, 2000, p. 136). Consequently, the extent to which students are able to manifest effective use of mathematical symbols [operators and forms of expression] in making sense of graphical artefacts might be perceived as an indicator of their ‘competence’ in using these tools and thereby reflecting how they actively and imaginatively [creatively] endow the conceptual objects encountered in their culture with meaning in the given context (Radford, 2008b).

2.4 Aims of the dissertation

Given the challenges of interacting with graphical artefacts, the purpose of the present study, as mentioned in the preceding chapters, is to identify the ways in which students interact with graphical artefacts in a sense making process. In this way the intention is to determine the nature, quality and level of interaction with graphical artefacts. In particular, and of interest here, is how students make sense of these artefacts in relation to subject specific operators or tools (e.g. addition, multiplication, and proportion reasoning) and forms of expression (discourse), namely the use of rules and conventions associated with the graphical stimuli; with focus on the students’ Critical-analytical ability in identifying and using these tools. This purpose corresponds with the purposes of research in mathematics education as a quest to understand the nature of mathematical thinking, teaching and learning and also to use such understandings to improve mathematics instruction (Shoenfeld, 2000). In this dissertation the interaction between students and the graphical artefacts (in a sense making process) is perceived firstly as an activity involving the identification of features of the graphical artefact and acting, on this initial impression and secondly the active engagement with the graphical artefact including assuming a questioning stance as well as the use of tools/operators in a Critical-analytical manner.

A Critical-analytical manner implies both the identification and selection of relevant tools/operators that further effective interaction with the graphical artefact as well as the ability to question the agencies and factors producing the
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graphical artefact (Roth, Pozzer-Ardenghi & Han, 2005). For the purposes of this dissertation, the effective use of tools/operators associated with the graphical artefact is perceived as enhancing meaningful interaction with the graphical artefacts and is further perceived as part of the sense making process. While appreciating the importance of being able to produce graphical artefacts, in this dissertation focus will nevertheless be on the sense making process from already existing graphical artefacts. Research on students’ understanding of graphical artefacts suggests that teachers should go beyond mere graph construction to discuss the meaning and the interpretation of the graphs (Shaughnessy, 2007).

2.5 Research questions

The research questions emanated from the aim of the research which is to identify, characterise the interaction of students with graphical artefacts.

1. What strategies do students use when faced with items containing graphical artefacts?
2. How are subject specific tools and forms of expression manifested in students’ interaction with selected graphical artefacts?
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3 THEORETICAL REFLECTIONS

In the preceding chapter it was shown that some of the terminologies used in relation to interaction with graphical artefacts are to some extent oriented to some epistemological or theoretical perspectives. This chapter follows the tradition inherent in scientific treatise where research activities are normally perceived as situated within the sphere of influence of some research paradigm. Thus, as an integral aspect of appropriate research practice this chapter lays a foundation for a declaration and an exposé of the world view guiding the reported research. In this way, it might make it easier for the readers to make sense of and judge the values of the analysis and discourse being espoused (Guba & Lincoln, 1994). The intention of such a declaration or discussion can also be perceived as being in harmony with and contributing to the discourse of the importance of ‘theory’ in the teaching and learning of mathematics and mathematics education (Sriraman & English, 2010). Thus, this chapter will outline the underlying epistemological and ontological basis of the present research. This is done initially with a discussion on the importance of theory in mathematics education research. This is then followed by a discussion of some relevant theories used in mathematics education research, namely constructivism and sociocultural theories. The intention is to illustrate the effects of theoretical underpinnings on potential ‘comprehension’ frameworks such as those mentioned in section 2.3.3. In discussing the sociocultural perspective a connection is made to semiotics given the role these play in forming the trajectory of the present dissertation.

3.1 The role of theory in mathematics education research

While in some disciplines the role of theory might be considered to be well established, the same does not appear to be the case for mathematics education (Fried, 2011; Radford, 2008a; Silver & Herbst, 2007; Niss, 2007). According to Niss (2007), “…it is neither clear at all what ‘theory’ is actually supposed to mean, nor what foundations theories have and what parts they play in mathematics education” (p. 1308). He also asserts that these entities are also

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5 The term mathematics education is used here in a broad sense to include didaktik of mathematics. A discussion about this is taken up in section 1.4
ill-defined and have unclear function (Niss, 2006). Indeed some researchers suggest that with regards to mathematics education one should probably talk of the ‘philosophy’ of mathematics education rather than the theory of mathematics education given that some of the widespread ‘theories’ in mathematics education do not meet the strict criteria to qualify as theory (see Goodchild, 2010). This state of affairs coupled with the multiplicity of theories producing, in some case incommensurable outcomes, has led some researchers (e.g. Jablonka & Bergsten, 2010) to suggest that this lack of connectivity between theoretical perspectives poses the risk for research in mathematics education to be detached and isolated from each other. This observation is pertinent given that research in education has been perceived as having the production of methods that work for the improvement of educational endeavours and outcomes as one of its primary goals (Lester, 2005; Schoenfeld, 2000). Thus, in this section it is deemed worthwhile to discuss the role and importance of theory in mathematics education.

In discussing the role and importance of theory a reference is made to the works of Lester (2005) and Niss (2007). Thus, the reasons why theory is essential are namely:

1. **there is no data without a theory**, however robust the data collection methods, these do not provide the reader with much information until a set of assumptions, perspective influencing the collection of data in question as well as the context are accessible or declared (see also chapter 4). In the notion ‘perspective’ is embedded the researcher’s belief about knowledge and reality.

2. a **structured set of lenses** through which phenomena may be studied providing guidance for action and behaviour and prediction of certain phenomena. This is achieved through the selection of appropriate elements, focusing on certain features, issues or problems by adopting particular perspectives etc. and providing a methodology for answering questions concerning the domain considered (Niss, 2007). Bishop (1977) posited that like spectacles, theories and constructs might help bring clarity on the subject of investigation. According to diSessa (1991) theoretical scientific understanding has the potential of providing capabilities that surpasses what can be attained by mere experience or
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intuitively-based scientific methods, hence, theory building is a linchpin in spurring practical progress.

3. Theory building calls for deep understanding, not just parochial understanding of the issue at hand. Theory helps in developing deep understanding of some important things – the big question (Lester, 2005).

4. That theory might provide a safeguard against unscientific approaches contributing to well thought out terminologies, methodology and analysis of results.

5. That it might provide protection against attacks from the outside based on perceived lack of foundations and research.

It is apparent that the function of theory outlined above includes not only the harmonising and protection of the production of knowledge but also the regulating discourse with regards to a given discipline. While this has clear advantages from a disciplinary and research perspective, it may also be restrictive and even regressive [see also section 3.5]. The connection of theory and knowledge brings with it a philosophical dimension: knowledge is not an isolated entity but is laden with perspectives, values and beliefs. The beliefs about knowledge include it being experienced as a source of empowerment; a key element leading to a number of opportunities – thus in classrooms it is not uncommon for exceptional performance in mathematics to be equated with superior mental abilities and the potential of competitive advantage and success in future career choice etc. As far as perspective into knowledge is concerned it is also acceptable to talk of knowledge as an innate aptitude or something that one must actively seek. Thus, there seems to emerge a dual function of theory; on the one hand as that of providing structure and direction from a disciplinary perspective for understanding a phenomenon and on the other hand having to do with knowledge, how it is perceived, dealt with and accessed. Assude, Boero, Herbst, Lerman, and Radford (2008) [cited in English & Sriraman, 2009] suggested that the dual perspective to theory is: a) structural– where theory is viewed as a coherent system of concepts and notion according to point 2 and 3 b) the functional view which allows for making some statement about the nature of some reality.

Thus, in exploring mathematics education research and with regard ‘to the set of lenses’ through which a phenomenon can be explored, some theories of
relevance to the dissertation are espoused. These are the constructivism, socio-cultural and semiotics perspectives: the theoretical perspectives are perceived as having a greater impact on teaching and learning generally hence the need for a brief look at their contents. In particular the discussion will be geared towards contrasting constructivism and sociocultural theories with regards to interacting with graphical artefacts. The final theoretical perspective - semiotics will then be developed as an analytical framework for the research study presented in this dissertation.

3.2 Constructivism perspective to learning

Constructivism has been promoted as a dominant theory of learning within the mathematics education community (Ernest, 2010). This theory of knowing can be traced to Jean Piaget [1896-1980]. In his *genetic epistemology*, Piaget envisaged scientific knowledge as being in perpetual evolution finding itself changing from one day to the next. He therefore, acknowledged scientific knowledge as a process of continual construction and reorganization (Piaget, 1970). Piaget’s genetic epistemology is extensive and multifaceted; it is consequently not unexpected that there are many strains of constructivism including information-processing theory, radical constructivism, enactivism and social constructivism (Ernest, 1996; 2010). However, the various forms of constructivism share the metaphor of carpentry, architecture, or construction work (Ernest). This implies that understanding is perceived as the building of mental structures (Ernest, 1996). The significant aspect of this explanation is probably the idea that this ‘construction process’ begins at a basic level using elementary means and moves successively to higher levels. This observation is alluded to by the claim that “[w]hat the metaphor of construction does not mean in constructivism is that understanding is built up from received pieces of knowledge” (p. 336). This claim also brings us to the first principle of constructivism, namely that “knowledge is not passively received but actively built up by the cognizing subject” (von Glasersfeld, 1989, p. 162). Now, this principle of constructivism can be seen as applying at two levels; the first level is purely individualistic implying that the learners construct their own knowledge and understanding based on their personal interpretation of their experience and their pre-existing knowledge. At the second level the construction takes place in the social sphere with the end products being appropriated and internalised by the individual learner; here the individual
subjects and the social sphere are regarded as indissolubly interconnected (Ernest, 2010).

While the levels, the extent and the viability of the construction metaphor can be subjected to further discussions, the general idea behind the metaphor is potent and has had a profound influence on learning. It is the case that the construction metaphor might be perceived as presupposing or demanding a logical structure or sequence in the construction or learning process. This logical structure of thinking emerges from Piaget who asserted that genetic epistemology deals with the transition from lower level of knowledge to a level that is judged to be higher (Piaget, 1970): this transition was perceived as taking place in stages. According to Radford (2008b), Piaget’s logical-mathematical structure of thinking accounts for what is supposedly a universal pattern of conceptual development. This is probably best exemplified by Piaget’s stage theory of development which has informed curriculum development for example, determining when it is appropriate for a child to begin formal education or the perceived level of maturity for given curriculum contents. In this theory development [of children] is perceived as going through four stages: The sensorimotor period (from birth to 2 years); Pre-Operational thought (2 to 6/7 years); Concrete Operations (6/7-11/12 years) and finally Formal Operations (11/12 to adult).

3.2.1 Step-wise approach to cognition and graphica

While these stages may be perceived at face value as addressing ‘maturity’ expressed chronologically, the notion contained therein has influenced other more ‘cognitive’ stages of development. In the Piaget inspired SOLO taxonomy [cf. Biggs & Collis, 1982], a similarity in the labels for the different levels can be noted. In SOLO taxonomy there are five stages: Prestructural - at this level the learning outcomes are unconnected and lack organisation providing no relevant information to the task at hand; Unistructural - here the learning outcome shows that the learner can handle isolated bits of information from the task, however the significance is not highlighted; Multi-structural - the learning outcomes show that the learner is able to identify several bits of isolated information but these are not coordinated or linked to each other; Relational - here the learner is able to process multiple data, integrating them in a whole and lastly, Extended abstract levels of cognitive engagement
which involve going beyond the subject at hand to make connections and generalisation to other concepts. The SOLO model demonstrates a stepwise progression with increasing levels of complexity at the different levels [see Fig. 3.1] an observation that in the context of interaction with graphical artefacts probably needs further justification (see Shaughnessy, 2007).

![SOLO taxonomy showing the levels of progression of learning outcome](http://www.johnbiggs.com.au/solo_graph.html)

This stepwise approach to cognition is reminiscent of a number of models used to characterise graph comprehension, as mentioned in section 2.3.3. In most of these models the number of steps are reduced to three (e.g. Curcio, 1987; Bertin, 1983; Wainer, 1992) and it is also possible to identify three types of questions that characterize the three levels of graph comprehension. While some of these models might not necessarily emanate directly from Piaget there is undoubtedly some connection. In developing her three-step model, Curcio (1987) proceeded to expand the Schema-theoretic framework into graph comprehension. Frederic Bartlett [1886-1969] is attributed with the use of
schema as a basic concept of cognition while Piaget undoubtedly popularised it. The schema theory postulates that prior knowledge is crucial for the retrieval or construction of intended meaning (Adams & Collins, 1977). It goes on to “…specify how the reader’s knowledge interacts and shapes the information on the page and to specify how that knowledge must be organized to support the interaction” (p. 5). Thus, there is some allusion to the ‘construction metaphor’ as captured in Curcio’s (1987) assertion that the effects of prior knowledge contributes to the development, revision, modification, and editing of related schemata.

The three levels of graph comprehension that emerge from research can be characterised as comprising: an elementary level focused on extracting data from a graph (i.e., locating, translating); an intermediate level characterised by interpolating and finding relationships in the data as shown in a graph (i.e., integrating, interpreting), and an advanced level that requires extrapolating from the data and analysing the relationships implicit in a graph (i.e., generating, predicting). This can be presented stepwise as depicted in Fig. 3.2.

![Three levels of graph comprehension](image)

**Fig 3.2: Three levels of graph comprehension**
Whereas there is a general correspondence for the basic level of graphical comprehension in research, the same cannot be claimed for the intermediate level of graphical comprehension. While Curcio (1987) suggests that the intermediate level involves interpretation and integration of the information presented in the graph, Bertin (1983) and Wainer (1992) seem to limit this to a part of the graph. For example, in Curcio’s model the learner may be required to describe the pattern of precipitation for a given locality in a month while in Bertin’s model the concern might be to describe the level of precipitation for the first two weeks. This observation points to the drawback inherent in what can be perceived as a linear hierarchical construct of cognitive skills more so when it is intended to depict a universal cognitive trajectory (see Radford, 2008b). The three-tier model associated with Friel et al., (2001) places importance on the type of questions posed in relation to interaction with graphical artefacts. Generally the type of question posed is an important factor in sense making especially since these have the potential of informing the choice of action to be performed and by extension, the type of observed learning outcome [since it is part of a network of signs associated with the graphical artefact].

An inherent consequence of a step-wise view to cognition is the ‘ease’ with which misconceptions and misunderstandings etc., are identifiable in the students’ solutions to mathematical tasks or interaction with graphical artefacts. This is not surprising since in this perspective to cognition that is, constructivism, knowledge is accepted as existing in the form of mental structures that should be put into use in any setting of interest (Roth, 2012). Indeed Hadjidemetriou and Williams (2002) distinguish between errors seen as flawed response to a question and misunderstandings as “part of a faulty cognitive structure that causes, lies behind, explains or justifies the error” (p.69; see also Leinhardt, Zaslavsky, & Stein, 1990). Thus, when these are not applied at the expected level of development it is then perceived appropriate to talk of misunderstanding or misconception. With regards to graphical artefacts some of these studies have identified misunderstandings and misconceptions such as: interval-Point confusion, slope-height confusion and iconic interpretation (see Clement, 1985; Leinhardt, et al., 1990; Hadjidemetriou & Williams, 2002).
In reviewing sociocultural perspectives to learning a broader view is assumed: mathematics is perceived as a cultural activity where learning is a social process through which the learner becomes progressively aware of cultural and historical forms of reflection or as form of participation; an apprenticeship in sociocultural activities, as it were (see van Oers, 1996; Radford, 2008b). Thus, the sociocultural perspective goes beyond just being associated with Vygotskij; this observation is significant given that there is also social constructivism, a strain of constructivism which credits the influence of the social sphere in cognition. However, in social constructivism the role of the social sphere is merely reduced to serving as a catalyst for an otherwise self-directed cognitive development (Cobb, Jaworski, & Presmeg, 1996).

Typical of theoretical perspectives, the sociocultural perspective to learning cannot be said to be a single strand. In a lecture at the University of Jyväskylä in Finland, van Oers (2004) claimed that there are two strands emanating from the writings of Vygotskij. On the one hand is the strand based on an anthropological interpretation where emphasis is placed on culture, symbols and interaction; here culture is perceived as a system of signs and as substance of meaning and meaning exchange. This strand is heavily connected to linguistics and is associated with Mikhail Bakhtin [1895-1975] among others. On the other hand is the action theoretical interpretation strand where focus is on action, activity, sense, goals and motives; here culture is perceived as a system of meaningful objects for action. This strand is associated with Alexei N. Leontyev [1903-1979] through the activity theory also referred to as CHAT. The emphasis on culture and cultural elements might be perceived as shifting the focus from perceiving thinking as a kind of interior life; a series of mental processes carried out in the individuals head (Radford, 2008b), to highlighting accessibility to, familiarity with and influences of elements of the sociocultural space such as cultural artefacts and forms of expression. According to Radford (2008c), sociocultural approaches are based on three principles: a) that knowledge is historically generated during the course of [the mathematical] activity of individuals b) the production of knowledge is embedded in cultural forms of thinking entangled with symbolic and material reality that informs the interpretation, understanding and transformation the life-world of the individuals and the concepts and ideas they produce c) learning is the attainment of
culturally-objective pieces of knowledge through the social process of objectification mediated by signs, language, artefacts and social interaction through the engagement in cultural forms of reflecting and acting.

3.3.1 The sociocultural space and the activity triangle

The centrality of sociocultural space or activity in CHAT is famously illustrated by the mediational triangle [activity system] of the basic structure of human activity closely associated with Engeström (1987). The thrust of the model is depicting the different aspects of the collective activity system wherein one can observe the multiple relationships within the structure and in the process, an analysis of the tension between them can be done: the tension or contradictions are understood as the driving forces for change and action or development in activity systems (Il'enkov, 1977; 1982; see also Roth, 2003). The complex nature of a sociocultural space is an indicator that some level of caution and reflection is needed before making any pronouncements on the cognitive states of individuals. Human consciousness can only be understood when all relevant factors of practical activity are taken into account (Engeström, 1987). Given the multifaceted nature of analysis using activity theory, it is not uncommon for researchers to limit the extent of their research, choosing to focus on selected aspects or levels of analysis. For example, while acknowledging Leontyev’s analytical distinction of the three qualitatively different hierarchical levels viz., Activities, Actions and Operation, Åberg-Bengtsson (1998) limited the analysis of her research to the level of action. However, this was done with the understanding that these levels of analysis are theoretical constructs that in practice, are inextricably intertwined such that some of the influences could be placed at “… either the level of activity or operations” (p. 80). This observation finds clarity in Koschmann, Kuutti and Hickmann (1998), where a distinction is made between adaptive and conscious operations. The conscious Operation originates as Actions, when these have been ‘routinised’ that is to say, practised long enough and are stable and thus they are considered as Operations. In view of the level of analysis done by Leontyev, the present dissertation can be perceived as focusing mainly on Action level, at the same time there is some clear orientation towards Operation level.
An illustration of the structure and the relationship within the mediating triangle (see Fig. 3.3) is provided using the theme of the dissertation namely, exploring the nature of students’ interaction with graphical artefacts. This implies that the subjects under consideration are the students [the term students is used here in a broader sense to include students at the schools and pre-service teachers], while the objective is to ascertain the levels or strategy of interaction with graphical artefacts. This is mediated by the tools and artefacts used by the students – the upper triangle. This action level of the triangle is connected to the social dimension through the two triangles at the base. By virtue of belonging to a formal educational institution the students [as well as the teachers] are subjected to certain ways of behaviours which include for example, fulfilling directives from the education department in the case of taking the PISA survey test, that the students keep to the course schedules and the students consent to participate in the research which might be seen as a ‘desirable trait for good studentship’. In the division of labour it is inherent in the roles of the members of the community.
3. Theoretical reflections

Fig. 3.3: An illustration of the application of the activity system for students interacting with graphical artefacts

In this illustration the upper-most triangle, shows that the subject of interest is the students, the object is the results of their discussions and response to question items containing graphical artefacts. Defining the object and the subject according to the first triangle allows for the characterising of the Activity; this is perceived as students interacting with graphical artefacts in an academic environment. This triangle is connected to a wider base where the cultural influence is realised through the rules and regulations informing the construction and to some extent the expected behaviour within the school setting. While division of labour can be perceived as involving roles such as that of a teacher, administrator, researcher and student, it cannot be ruled out that in the course of discussing the tasks that students consciously or unconsciously assigned themselves different roles, for example, in a group discussion the dominant student might control the trajectory of the discussion or signal a conclusion to a discussion. Since it was mentioned that contradiction is central to activity theory, aspects of contradiction can be
identified at different levels in this study. Some probable contradictions include what can be considered as contradictions related to the curriculum namely, the need for teachers to let some of their students participate in a research study or engage in say PISA survey test instead of following the regular lessons. Another possible contradiction at this level is in the nature of tasks encountered by the students, for example PISA claims to include performance tasks that are oriented to the application of skills or mathematical competence in realistic situations in contrast to tests focusing on mastery of mathematical concepts and ideas.

By virtue of highlighting the significance of signs, artefacts and tools in cognition (see Vygotsky, 1978; Wertsch 1985; van Oers 2002), it is reasonable to find a connection between the theory of Vygotskij and semiotics (see also Maffiolo, 1993; Smith, 1997) It has been observed that Vygotskij’s [also referred to as Vygotsky] work, the Genetic Law of Cultural Development, connects “... links in a decisive manner, human cognition to the individuals’ use of signs in activity” (Radford, 2003b, p. 49). A semiotic approach can be perceived in respect to this dissertation as allowing for a micro analysis of the process of interaction with graphical artefacts [for further discussion on this issue see section 3.5.1]. Thus, using an approach based on semiotics and hermeneutic phenomenology, Roth and Bowen, (2001) identify two mutually constitutive processes in progress while interacting with graphical artefact: structuring and grounding. In a nutshell, these processes describe a model leading to what can be considered as a chain of signification [see section 3.4.1] where depending on familiarity or otherwise, interaction with a graphical artefact can lead to the production of other signs which might then be manipulated in other relatively familiar ways. Thus, the processes can be considered, without loss of generality, as being driven by collateral experience.

3.4 Semiotics

Semiotics, the science of signs, seem to be increasingly gaining attention within mathematics education research. Semiotics has been associated with a number of philosophers and theorists such as John Locke [1632-1704]; Ferdinand de Saussure [1857-1913] the father of structural linguistics; Charles S Peirce [1839-1914] one of the pioneers of pragmatism and Günter Kress who has been associated with social semiotics. Together these scientist give rise to at least
three of the dominant strands of semiotics. It is noteworthy that some scholars (e.g. Mladenov, 2011) posit that semiotics is plural in the same sense as mathematics since there are many semiotics e.g. dyadic, triadic, and cognitive (see also Sebeok, 1994). Given that signs have been an ever present aspect of human existence, semiotics as an epistemic idea has a long history. While aspects of semiotics can be identified in other medieval scholars, it is Augustine of Hippo [354-430] who is associated with early attempts at systematising semiotics. As much as it is interesting to explore the history and development of the different strands of semiotics, this dissertation will examine Peirce semiotics. This is because, as alluded to in section 1.4, there seems to be a connection between the concept didaktik and sign viewed from a Peircean perspective.

3.4.1 Peirce semiotics

Peirce semiotics can be perceived as an improvement on the semiotics of John Locke and Ferdinand de Saussure. John Locke made a first attempt to outline a non-dyadic relation between sign and meaning. For him articulated words and ideas separately had a one-to-one relation to the entity or object in question thus in effect, eliminating mediation. This attempt was not successful, probably since Locke at this point did not use signs as a mark of transition but as an idea (Mladenov, 2011). Saussure avoided this pitfall and emphasised the interdependence of signs and their objects that is, the signifier and the signified and thus, his semiology expressed a dyadic relationship [signification] between a sign and the signified object. Saussure perceived meaning as coming out of concepts being in opposition to each other e.g. we are able to know that water is what it is because it is not fire. Consequently, sign is signified here through differential opposition.

On the other hand as a way of dealing with the issue of meaning, Charles Peirce posited that what we can know is not an object but the effects the object exhibits and it is these effects that are the sign (see Mladenov, 2011; Whitson 1997). Before getting into further discussion on meaning, it might be prudent to explore the foundational elements of Peirce semiotics – since meaning occurs in the relationship between these elements viz., sign, object and interpretant. The most commonly used definition of sign attributed to Peirce is that: “A sign, or representamen, is something which stands to
somebody for something in some respect or capacity” (CP 2.228). In this definition the triadic relationship *sign, object* [something] and *interpretant* [in some capacity] is clearly evident. This is illustrated in Fig. 3.4.

![Peirce's triadic model of sign relation showing its dynamic nature](adapted from Mladenov, 2011)

In this dissertation, Peirce semiotics is given a prominent role based on a number of factors: the first being that it puts perception at the centre of cognitive activity (Radford, 2008d; Radford 2004). It also allows for making sense of metaphorical aspects of interactions with signs. Bakker (2007) observed that Peirce semiotics is appropriate as a tool for analysing students learning due to its “*non-linearity and the possibility of stressing the dynamic character* of interpreting and making signs within the theory itself (p.14)” [emphasis in the original]. Peirce semiotics also provides valuable means of justifying the conceptual and analytical construct that is used in this dissertation.

*Symbol*

A symbol refers to its object by virtue of law; it is on the basis of an association of general ideas that the symbol is interpreted as referring to the object in question (CP 2.249). For example, ® the registered trade mark informs and
warns that a given name is protected by law against unauthorised use; for example, Red Cross-Red Crescent is associated with aid or humanitarian assistance. Words are symbols which obtain meaning based on the cultural milieu. The same also applies to emblems etc. Mathematics as such abounds with symbols. According to Sebeok (1994) symbols allow human species to reflect upon the world distinctively from the stimulus-response situation, an aspect that sets apart human representations from those of other species.

**Index**

This is a sign in which some direct non-arbitrary connection with the object can be realised or inferred. An index indicates, denotes or directs attention to its object without giving any substantial information about them (Bergmann, 2009). Thus, it is logical to assert that indices are signs with strong experiential connection (Peirce, MS 797:10 cited in Bergmann, 2009). For example smoke is an index of fire, animal tracks point to the presence of a given animal in the vicinity and so on. Sebeok (1994) includes the index finger which is used to point out and locate things associating words such as ‘here’, ‘up’, ‘there’ etc. Some educators insist on collecting students’ rough work [scribbling or scratch] papers alongside the answer sheets during examinations as a way of ‘tracking’ students understanding of mathematical concepts etc.

**Icon**

This is a sign in which some similarity is shared with the object through possessing some of its qualities. It is the only sign which involves a direct presentation of qualities that can be attributed to its object thus allowing for the manipulation of the icon as a way of getting more information regarding its object (Stjernfelt, 2000). Peirce distinguishes what he refers to as hypoicons or icon signs (CP. 2.277; see also Farias & Queiroz, 2006) that are subcategories of icons namely: images, diagrams and metaphors [this order of mention seems to bear some importance (see Stjernfelt, 2000) however, in the detailed explanation that follows this order is not adhered to for the sake of convenience]. Given that the studies presented in this dissertation have not given prominence to images *per se*, only a brief description of diagrams and metaphors will be presented. It has to be mentioned that Peirce did not dwell much on metaphors, however, the importance of metaphors in semiotics is
widely acknowledged not least within the community of mathematics educators.

i- **Metaphors**

The term metaphor comes from the Greek word *Μεταφορά* [metaphora]; *meta* meaning “across, over” and *pherein* meaning “to carry, bear”. Thus a metaphor can in respect to language, be perceived as the transfer of language and expression without compromising meaning. This is akin to locating the ‘language or expression’ in a new position to clarify visibility or understanding. In one of the two places where Peirce mentions metaphors it is defined as a hypoicon “which represent the representative character of a representamen by representing a parallelism in something else” (CP 2.277). This definition alongside that found in CP 2.222 has been considered technical (Anderson, 1984). According to Mladenov (2006), “Metaphorizing is a manner of thinking, it is not a property of thinking. It is a capacity of thought, not a quality. It represents a mental operation by which a previous existing entity is described in the characteristics of another one on the basis of some similarity or by reasoning”. (p.8).

In mathematics classrooms or discussions, it might be the case that ‘the manner of thinking’ associated with metaphors is associated with or even occasioned by a deficiency in fluency with regard to the conventions-tools, forms of expression on the part of one or several interlocutors. Thus, while at face value metaphorising might be perceived as not meeting the threshold of ‘subject specific forms of expression’ [see section 2.3], its potency as a means of creative thought and source of insight is well recognised (Anderson, 1984; Danesi, 2007; delMas, 2004; Mladenov, 2006; Presmeg, 1992; Sfard, 1994).

ii- **Diagram**

An illustration of, as well as a Peircian definition of a ‘diagram’ was given in section 2.1.3 where the iconic character of a diagram was highlighted. The distinguishing characteristic of a diagram is that it has a similarity to its object mostly through shared structural or relational qualities (Farias & Queiroz, 2006). According to Stjernfelt (2000) as long as an icon is thought of as having an entity consisting of interrelated parts and where these relations are subject
to experimental manipulation, then we are operating on a diagram. The
definition of diagram goes beyond the normal view of diagram as an
‘inscription’ to include sentences such as I HATE MATHEMATICS: this
sentence makes sense because it follows the grammar of the English language
where there is a simple relation between the subject and a predicate. Thus a
diagram can be perceived as an entity indicating relations constructed by
following the rules and conventions of, and by means of element and relations
in a given representational system (Hoffman, 2011). Sjernfelt (2000) posits that
the relations which constitute a diagram enable it to exist as an icon
(observationally) and thus it is possible to execute generally valid experiments
upon it.

**Diagrammatic reasoning**

The concept of diagrammatic reasoning seems to have a mathematical essence
being specially suited for describing the specific nature of mathematical
reasoning. Peirce claims “all mathematical reasoning is diagrammatic ...no
matter how simple it may be” (Peirce, NEM 4: 47). He then defines
diagrammatic reasoning as:

[R]easoning which constructs a diagram according to a precept expressed in
general terms, performs experiments upon this diagram, notes the results,
assures itself that similar experiments performed upon any diagram
constructed according to the same precepts would have the same results and
express this in general terms (p. 48)

Although the opening statement and even the quote cited in NEM above
proves a strong case for mathematics, it ought to be understood that
diagrammatic reasoning can be applied in diverse areas other than mathematics
such as conflict resolutions or management (e.g. Hoffmann, 2005). Indeed
Peirce refers to it as “[a]ll valid necessary reasoning” (CP 1.54, 5.162). Thus,
‘any form of valid reasoning’ that characterises diagrammatic reasoning has to
operate within the confines of some generally agreed upon and acceptable rules
or conventions. These rules can only be perceived as resulting from a process
of social negotiation within a community of practice using elements of their
cultural historical heritage.
From the definition provided above, three processes involved in diagrammatic reasoning are identified (see NEM 4: 47-48; Hoffmann, 2011; Bakker, 2007). The first process is constructing a diagram, the purpose of which might include the need to identify relationships that are considered significant (Bakker, 2007). It is noteworthy that the construction referred to in this section ought not to be confused with the production of graphical artefact which is not the focus of this dissertation. This ought to be understood in the Peircean sense which might involve mental imagination (see CP 3.363). The second process is experimenting with the diagram. Central to experimenting with diagrams is probably the notion of doing something. “All our thinking is performed upon signs of some kind or other either imagined or actually perceived. The best thinking, especially in mathematical subjects, is done by experimenting ...” (NEM 1:122 italics supplied). The third and final process involves the noting or recognition of the results and assuring oneself of the consistency and viability of the methods used in the sense making process. The whole business of doing something seems to have the intention of leading to fruitful observation (see CP 4.233). Whereas in mathematics this observation is normally achieved in written calculations, workouts, graphs, figures etc. we realise that there are other media e.g. metaphors which can lead to some level of observation.

Thus, diagrammatic reasoning is a tool for generating knowledge; it facilitates the thinking process on complex situations at both the individual and social level (Hoffman, 2011). By virtue of it being a generator of knowledge, diagrammatic thinking is perceived as related to actions of objectification that is, “actions aimed at bringing or throwing something in front of somebody or at making something visible...” (Radford, 2003a, p. 40).

### 3.4.2 The functions of a sign

Peirce distinguishes two functions of a sign based on the state or nature of its object:

We must distinguish between the Immediate Object,— i.e. the Object as represented in the sign,— and the ... Dynamical Object, which, from the nature of things, the sign cannot express, which it can only indicate and leave the interpreter to find out by... (CP 8.314 highlighted italics in the original)
The two functions of a graphical artefact have been defined as on the one hand a representational and on the other hand an epistemological function (Hoffman & Roth, 2007; Hoffman, 2011). The representational function is inherent in the definition of a sign that is, “something which stands to somebody for something in some respect or capacity” (CP 2.228). The epistemological function is underscored by Bertin (1983) who recognised the potency of graphics in pedagogy as artefacts that can introduce into all disciplines the basis of logic and the essential processes of analysis and decision making offering the possibility of revealing the intelligence of weak students. Since all thinking is in one way or another done on a sign (NEM 1: 122), it is reasonable to assert that knowledge is realised in the epistemological function of a graphical artefact when one is able to reflect on and attain the essence for which a graphical artefact indicates. Thus, the epistemological function of a graphical artefact is a means for constituting knowledge (Hoffman, et al. 2007). But how does one find out what is not provided in the graphical artefact? Peirce provides a solution; by what he refers to as “…collateral experience” (CP 8.314).

**Collateral experience**

To lay the foundation for defining collateral experience or observation [Peirce uses observation, acquaintance and experience however, in this dissertation the term collateral experience is preferred] we turn to the *Merriam-Webster dictionary* where the term is defined as: accompanying as secondary or subordinate; serving to support or reinforce. Looking at the definitions, the impression is created that collateral experience is a kind of experience inherent in a person providing support towards a given end without it necessarily assuming a prominent role. While this definition acknowledges the fact that the collateral experience might be implicit it also provides for the possibility of it being explicit rather than remaining hidden (Hoffmann, 2005; see also Hoffmann & Roth, 2007). According to Peirce, collateral experience is “…previous acquaintance with what the sign denotes” and not acquaintance with the sign system which is “prerequisite for getting any idea signified by the sign” (CP 8.179). To illustrate collateral experience, a case of normal arithmetic and modular arithmetic is used. It is almost indisputable to claim that \(10 + 4 = 14\) however, one might have some difficulties accepting a claim that \(10 + 4 = 2\), much as it is a common operation regarding calculations on a 12 hour clock.
This illustration accentuates the two aspects of collateral experience [more on this in section 3.5]. What is significant about collateral experience is that it is not only restricted to individual or personal experience. It is applicable in the social realm, since we're able to understand one another or interpret graphical artefacts due to others for that matter, if we have shared collateral experiences.

Collateral experience can be perceived as an aspect of context. In some of the research that has been mentioned [Friel et al., 2001; Aoyama, 2007; Watson et al., 2008] there is the recognition that an effective interaction with graphical artefacts may be easily actualised when there is knowledge of the conventions involved with the particular graphical artefact as well as its content and context.

3.5 Synthesis

This chapter includes an outline of the role of theory in mathematics education research followed by a theoretical reflection. The set of assumptions taken in conducting this research has largely been informed by aspects of CHAT [theory of knowledge objectification] and Peirce semiotics. This is a major departure from a majority of the research outlined in section 2.3.3 which suggests a constructivist perspective as the point of departure. That the research presented in this dissertation is influenced in a wider sense by aspects of CHAT [in particular the theory of knowledge objectification – see Radford, 2008b; 2013a] and semiotics is not entirely unexpected since, as intimated earlier, there is some parallel between the two approaches (Maffiolo, 1993; Smith, 1997) found in the role of sign. Indeed Vygotskij perceived signs [tools and artefacts] as mediating the accounts of human mental function or thoughts (Wertsch, 1985; see also Daniels, 2008). In Peirce semiotics discussed in section 3.4.3, the centrality of sign in signification is illustrated comprehensively. In exploring tool use and forms of expressions, the research presented here has as a point of departure the assumption that knowledge is a collection of culturally and historically constituted embodied processes of reflection and action (Radford, 2013a). Thus, the recognition of the applicability and use of these artefacts might reflect the extent whereby students have objectified some of the cultural means of signification (Radford, 2008b). In the theoretical perspective adopted here, learning is perceived as primarily concerned with both knowing and becoming (Radford, 2013a), a
theme that resonates well with the notion of reflective citizenry, for example. It is noteworthy that within CHAT and its derivatives, the centrality of activity is acknowledged (Daniels, 2008; Radford, 2013a). Daniels (2008) posits that “... at a very general level of description, activity theorists seek to analyse the development of consciousness within practical activity.” (p. 115). Thus, it is not uncommon for studies influenced by CHAT to be associated with practical or real time classroom teaching and learning environments. While the present research might not be strictly associated with the classroom environment, it stills falls within the theoretical realms of a CHAT study on the basis of a wider understanding of activity. Radford (2013a) refers to activity being more than people interacting within themselves and artefacts. In this perspective a student’s written solution to a task, a written theorem, is an artefact that is a trace of activity (Radford, 2013b). Consequently an investigation of students’ performance is perceived as a trace of an activity which can thus be accounted for in the activity triangle (see Fig. 3.3)
4 CONSIDERATION ON RESEARCH METHODS

Pursuing the aims of academic research calls for attention and decision making on some practical considerations, regarding the research problem. Thus, it is not uncommon in this regard to adopt an approach involving a research paradigm (methodology), methods and instruments or techniques (Olsen, 2005). In the context of the present study, the point of departure followed the idea that the research questions would direct the research approaches and data analysis procedures adopted (Lederman, 1992). Since aspects of methodology were taken up in chapter 3, in this chapter considerable space will be given to reflections on the methods and instruments applied in the research studies presented in this dissertation.

The connection between methodology, methods and technique is best captured by exploring the why and how concerns (Burton, 2002). The why concern is satisfied by the methodology which generally provides a justification of the design of the research. On the other hand method(s) goes into specific aspects or means used by the researcher when carrying out the research. Methods might refer to the procedures used in the process of data gathering as well as specific features of the scientific endeavours such as, in the case of this dissertation, the forming of analytical constructs and procedures and the application of cluster analysis or video observation (Cohen, Manion, & Morrison, 2000; Olsen, 2005). Instruments or techniques is more specific and refers to “a particular mathematical procedure or a certain method” (Olsen, 2005, p. 63; see also Bryman, 2008) for instance, participant observation, online discussions and the use of specific procedures associated with the different statistical analysis techniques.

Specific for this research study, quantitative data was availed of through the PISA survey test in the form of country item scores. Following the goal of the research namely to characterise students’ interaction with graphical artefacts, the data was analysed using the statistical method of cluster analysis. The specific instruments used alongside the cluster analysis method were the Euclidean distance for measure of similarity and Average linkage as a clustering algorithm [more about these in section 5.3.2]. The other methods included video observation and online discussions: in both cases an ‘observer-as-participant’ role was adopted (see Adler & Adler, 1987). While cognisant of
Adler and Adler’s (1987) claim that some level of ‘membership’ is almost unavoidable in observational research, a stance is taken that in order to attain ‘membership’ in a group some level of acceptance that goes beyond mere personal presence has to be attained. This is especially true for the video observations since the observer [author] was meeting the students for the first time. In this case acceptance might have been accredited due to the fact that it was the school principal who arranged with the responsible subject teachers to make available the students for the study – thus, some level of trust was accorded to the observer. Also, the construct ‘observer-as-participant’ is in the present dissertation used with a caveat given it’s seemingly field anthropological underpinnings – the observer’s role in the present study should rather be characterised as ‘passive participant’. Perceiving the observer’s roles as ‘passive participant’ is in recognition of the fact that the researchers/observers influence is in one way or another embedded in the research methods and instruments but does not necessarily include ‘membership’ of the group or object of observation in the strict sense of the word. However, the ‘passivity’ mentioned here did not limit the observer from occasionally posing questions to the participants about the activity under observation sessions. Dwyer and Buckle (2009) observe that the personhood of the researcher including her status in relation to research participants is an integral and ever-present aspect of the research process (see also Adler & Adler, 1987).

Thus, in applying a plurality of methods, the research presented here can be perceived as employing a pragmatic perspective on research where there is no denying the specificity of a research object and at the same time acknowledging the possibility of several methods that can bring clarity to the object of research. Wildemuth (1993) suggests that the methods used in a given study should be based on the research questions being addressed and also the appropriateness of a given research approach to a given study. As outlined above the present study is comprised of a ‘plurality’ of data such that a single research approach would most probably not suffice.

4.1 A tale of the two research approaches

Buoyed by ontological and epistemological assumptions that a research process is perceived as a concern to understand the world – to understand the world
A number of researchers have written about these two approaches with the intention of showing that there is not much polarity between them (Lederman, 1992; Ercikan & Roth, 2006; Bryman, 2008) However, the rationalisation strategy adopted to justify the use of the seemingly antagonistic approaches in the present dissertation is inspired by Olson (1995). Olson illustrates two levels of understanding these approaches to research: one level is based on ontological-epistemological difference and the other level is based on ‘general definition’. Viewed from a general definition or method level, it is perceived that the ontological-epistemological difference between the research approaches is somewhat diminished. This approach mirrors the operational application of these approaches as used in this dissertation. Olson posits that the general definition of ‘quantitative’ and ‘qualitative’ are closely linked to methods and also seem to indicate aspects of data gathering. Take the construct ‘quantitative’ for example which connotes counting or measuring and which might possibly be understood as involving quantities. Qualitative - connotes quality which in turn lends itself to explanations or descriptions.
Wildemuth (1993) expresses the link between methods and these research paradigms thus:

“It is true that the positivist approach, with its goal of discerning the statistical regularities of behavior, is oriented toward counting the occurrences and measuring the extent of the behaviors being studied. By contrast, the interpretive approach, with its goal of understanding the social worlds from the view point of the actors within it, is oriented toward detailed description of the actors’ cognitive and symbolic actions, that is the meaning associated with the observable behaviors

(Wildemuth, 1993, p. 451)”

The quotation above highlights the connection between data analysis and the two research approaches, thus providing room for the analysis of data both in a qualitative as well as in a quantitative manner. Thus, the research approaches reported in this dissertation are perceived as empirical methods. In the first empirical study, the success rate of students’ response to item containing graphical artefacts is explored in relation to the pre dominate tool that might be applied in the sense making process and in the second empirical study the frequency of use of tools in sense making process is quantified. Thus these empirical studies are perceived as typical of quantitative studies. For the qualitative studies it is clear that the students’ explanations of the tasks at hand are in focus.

4.1.1 Research design

Four different research methods for data collection were used in the course of the research study presented in the dissertation. The first method as it was mentioned earlier, involved data from OECD PISA survey test. The second data collection method was through video observation of the students solving PISA survey items in a collaborative setting. The third data collection method was through an online discussion with pre-service teachers. The fourth and last data collection method involved selection of students’ written work from the Swedish national test (Np). The statistical nature of the PISA data offered few choices in terms of analysis. In terms of statistical analysis, the identification of groups rather than how the data are correlated, was perceived to follow the
general goal of the study. The observational studies and the online discussion served the both to identify the supposed groups in practice and also in another context [i.e. different from a paper and pencil environment that characterised the PISA survey test]. The analysis of Np was done using some aspects of the findings from the above mentioned studies with the aim of ascertaining global application. The choice of the different data collection was perceived as providing a wider range of perspective in understanding, in the way students interact with graphical artefacts. Considering that the present research begun as a secondary analysis of PISA data, the other research or data collection methods might be understood as serving to verify or strengthen the observation made and accordingly the results derived from analysing the PISA survey data. In this respect the different data collection methods contributed to the triangulation of the research presented in this dissertation. Details of the specific methods and mode of application will be discussed in the respective chapters dealing with the individual studies.

4.1.2 Triangulation

Triangulation involves using several methods or data sources in the study of social phenomena (Bryman, 2008). Triangulation is important in research given its contribution to cross checking results. A significant factor regarding triangulation is that the uncertainty of the interpretation of a research proposition is greatly reduced when it has been confirmed by two or more studies (Webb, Campbell, Schwartz, & Sechrest, 1966). The present research characterises students’ interaction with graphical artefacts; two of the studies have been conducted using a quantitative orientation while the other two took on a more qualitative aspect. The strength of triangulation lies in its contribution to the validity and credibility of the research. Four forms of triangulation are distinguished from research (Denzin, 1970) viz., a) data b) investigator, c) theoretical and d) methodological triangulations. These methods dealt with the use of several sampling strategies on a given data set, the collaboration of researchers in the field of data gathering and interpretation, the use of more than one theoretical position to analyse the data and the use of more than one method for data gathering respectively. The triangulation method used in the present study lends itself to the methodological and specifically between method triangulation which involves the contrasting of research methods. Details of the different methods are given in the following sections.
4. Consideration on research methods
5 TAXONOMY OF STUDENTS’ RESPONSE TO PISA TEST ITEMS*

This chapter outlines the first study which indeed also served as the entry point for the research presented in this dissertation. Here focus will be placed on students’ scores from PISA. A significant aspect in this chapter is that the rationale for using PISA survey test is set out, including the factors informing the choice of items as well as the analytical techniques employed. In discussing the result special attention is given to the PISA items classification framework - a framework that is considered in many ways as representative of the three-tier graphic comprehension frameworks outlined in section 3.2. Basically, the exposé given here is intended to provide an introduction to PISA survey data *per se* followed by an explanation of some of the steps and decisions made in the course of analysing the students’ results together with a mention of the specific concerns of the study and cumulating in an outline of the research findings and the influence of the same on the overall research trajectory.

5.1 Rationale

The basis of the PISA survey is exploring how students, about 15 years old, are prepared to meet the challenges of contemporary society in other words, how they are equipped for what might be considered as effective citizenry. It is the case that at the age of 15 most students from the OECD group of countries are at the end or approaching the end of their compulsory schooling (OECD, 2003). Consequently, PISA survey has the ambition to assess the ability of the students to think critically, communicate effectively and apply knowledge learned in school to realistic situations that are likely to be part and parcel of their everyday life (OECD, 2003). An interesting aspect for this study is the assertion that PISA is not curriculum or academic oriented as such, (OECD, 2005; Sáenz, 2008) but more focused on the application of mathematical skills.

* A condensed version of this chapter was published 2013 in Educational Studies in Mathematics, Online First. DOI 10.1007/s10649-013-9493-3. entitled “Graphical artefacts: Taxonomy of students’ response to test items”
in authentic and realistic situations (OECD, 2003). This implies that unlike some other standardised tests there is no specific section testing graphicacy; an observation that is perceived as reinforcing the assumption that the focus is on the students’ interactions with these graphical artefacts regarding the tools and forms of expression at their disposal. Further some researchers have argued that the ability to understand, interpret, analyse and use graphical artefacts that is to say, graphical literacy, is correlated to general intelligence and analytical skills (Åberg-Bengtsson, 2005; Rosén 2001). Thus it is worthwhile to explore students’ ability to interact with graphical artefacts in the light of the large scale survey provided by PISA. Exploring the performance of students also has a bearing on the general discussion of the increasing importance of subject specific literacy for effective citizenry as discussed in section 2.3.

5.1.1 Research concerns

In this regard the analysis of students’ results from the PISA survey seeks to examine the nature of students’ interaction with graphical artefacts based on performance patterns on the PISA test with specific focus on the application of subject specific tools or operators and forms of expression. Given the diverse nature of graphical artefacts, and based on theoretical considerations it was deemed expedient to limit the nature of the graphical artefacts to commonly occurring graphical artefacts such as bar graphs, pie charts and line graphs. On this basis, it was envisaged that the performance may vary depending on a number of factors such as mathematical context and content (see e.g. Lowrie & Diezmann, 2011) as well as the proximity or ease of accessibility of the subject specific tools to the students’ everyday experiences or orientation. The concerns of this study were thus:

- How can the students’ interaction with items containing graphical artefacts be characterised?
- What patterns of performance can be identified for these items from subject specific tools and form of expression perspectives?
5.2 Empirical matters

The PISA study that forms the basis of this chapter focuses on students’ results from the Nordic countries. The choice to zero in on the Nordic countries was based on a number of factors including the cultural and geographical proximity between the countries, the largely structural similarity of the educational systems [although there are some discrepancies after upper secondary education (see e.g. Stenström & Leino, 2008)]. Another favourable aspect was the existence of several earlier studies indicating similar patterns of performance for these countries in science and mathematics in large scale international standardised test results (see Olsen, 2006; Olsen & Gronno, 2006; Lie & Roe, 2003; Kjørnsli & Lie, 2004). The outlined factors also offered a considerable advantage from a statistical analysis perspective such as those concerned with aspects of sampling. With regard to statistical analysis it is worth pointing out that since PISA is an international comparative study some statistical research concerns had already been dealt with. The OECD PISA boasts of a rigorous statistical methodology; “strong quality assurance mechanisms for translation, sampling and data collection as well as the state of the art technology for data analysis” (OECD, 2006, p. 7; see also OECD, 2003). This observation served to instil some level of confidence in dealing with PISA data. Thus, the pertinent decision made with regard to PISA survey was the selection of items to be explored in the present research.

The data available for the purposes of this research included students’ results for 2000, 2003 and 2006, with reading, mathematics and science as main domains of focus, respectively. However, based on the complexity of a multiple-disciplinary analysis with regards for example, to the diverse tools and forms of expression involved in interaction with graphical artefacts from the various domains, the pursuance of this broad based approach was abandoned in favour of items from mathematics subject domains. The data comprised of the students’ success rate for a number of items containing graphical artefacts. Since these success rates [p-values] contained information on the overall achievement for the countries and the overall difficulty of the items (Olsen, 2005) – this information was deemed to be sufficient for the purposes of this study thus, it was decided not to transform them further. Also transforming the data further by standardisation for example, might not necessarily be
feasible and sometimes has the effect of dampening the results (Kaufman & Rousseeuw, 2005; see also Dudaite & Elijio, 2008, September).

5.2.1 The PISA data set

The final data set comprised of a total of 19 items from PISA survey test from 2003, these are shown in Table 5.1. From this table the items are identified with the same item ID or codes used in the PISA survey for ease of identification with other research and reports dealing with aspects of the OECD PISA surveys. This ID consists of the unit number and a question number where for example the letter ‘M’ in task M155Q04 identifies it as task number 155 from mathematics domain while the numbers Q04 indicates that it is the 4th item or question of the given task. It is noteworthy that although the test results in the present study are assumed to be independently distributed, PISA survey has parameters in place that connect these items in terms of what it refers to as cognitive competency, level of item difficulty etc. (OECD, 2003). Thus in Table 5.1 there is reference to competencies associated with the different items.

The PISA cluster of competencies shown in the third column in Table 5.1 are based on the kind of cognitive demands or processes through which test items are encountered. This can be perceived as the cognitive processes which should be activated to connect the real world in which the problem arises with the mathematics needed to solve the problem posed (OECD, 2003; Sáenz, 2008). In particular reproduction cluster of competency, involves the reproduction of known knowledge such as identifying the point of intersection of a line with the x-axis, determining the largest sector of a pie-chart or the gradient of a straight line etc. In connection cluster of competency, the problem solving is at a level that does not necessarily follow simple routine. There is more independent problem solving strategy even when standard procedures are employed to solve non standard problems. An example of tasks fitting into this cluster of competency is task 8 from the Swedish national test for 2009 [see Appendix II] where there is among others the selecting and using strategies for solving non standard problems. Finally the reflection cluster of competency includes an element of reflection about the processes needed or applied to solve a problem. This demands an understanding of the tasks and the factors
Table 5.1: Selected items classified using aspects of PISA framework

<table>
<thead>
<tr>
<th>Item</th>
<th>Item format</th>
<th>Competency</th>
</tr>
</thead>
<tbody>
<tr>
<td>M302Q02</td>
<td>Closed constructed response</td>
<td>Connections</td>
</tr>
<tr>
<td>M302Q01</td>
<td>Closed constructed response</td>
<td>Reproduction</td>
</tr>
<tr>
<td>M438Q01</td>
<td>Closed constructed response</td>
<td>Reproduction</td>
</tr>
<tr>
<td>M150Q02</td>
<td>Closed constructed response</td>
<td>Reproduction</td>
</tr>
<tr>
<td>M150Q01</td>
<td>Closed constructed response</td>
<td>Reproduction</td>
</tr>
<tr>
<td>M155Q01</td>
<td>Open constructive response</td>
<td>Connections</td>
</tr>
<tr>
<td>M155Q02</td>
<td>Open constructive response</td>
<td>Connections</td>
</tr>
<tr>
<td>M828Q02</td>
<td>Open constructive response</td>
<td>Connections</td>
</tr>
<tr>
<td>M155Q04</td>
<td>Complex multiple choice</td>
<td>Connections</td>
</tr>
<tr>
<td>M467Q01</td>
<td>Multiple choice</td>
<td>Reproduction</td>
</tr>
<tr>
<td>M438Q02</td>
<td>Multiple choice</td>
<td>Reflection</td>
</tr>
<tr>
<td>M150Q03</td>
<td>Open constructive response</td>
<td>Connections</td>
</tr>
<tr>
<td>M192Q01</td>
<td>Complex multiple choice</td>
<td>Connections</td>
</tr>
<tr>
<td>M828Q01</td>
<td>Open constructive response</td>
<td>Connections</td>
</tr>
<tr>
<td>M179Q01</td>
<td>Open constructive response</td>
<td>Connections</td>
</tr>
<tr>
<td>M828Q03</td>
<td>Short response</td>
<td>Connections</td>
</tr>
<tr>
<td>M302Q03</td>
<td>Open constructive response</td>
<td>Reflection</td>
</tr>
<tr>
<td>M513Q01</td>
<td>Open constructive response</td>
<td>Connections</td>
</tr>
<tr>
<td>M155Q03</td>
<td>Open constructive response</td>
<td>Connections</td>
</tr>
</tbody>
</table>

around it, creativity, an ability to generalise, justify and communicate solutions and mathematical matters both orally and in writing (OECD, 2003; Sáenz, 2008). The characterisation of items by PISA using the competency framework (Sáenz (2008), has referred to these as levels of theoretical complexity) is reminiscent of the hierarchical development ranking from simple to advanced forms of cognition as mentioned in section 3.2. Indeed Conway and Sloane (2006) suggest that situated cognition, Piaget’s constructivism and Realistic Mathematics Education (RME) have influenced PISA design of mathematics literacy study. Thus, there is an indication that PISA competency levels are constructivism oriented.

Within the PISA competency framework, it is not entirely unexpected that items from the reflection cluster might prove to be more challenging than items from say the reproduction cluster. This, since the reproduction cluster is
perceived as involving demands that are not as challenging as those of the reflection cluster. While PISA attaches focus on item difficulty on a set of competencies, this study focuses item difficulty on the students’ familiarity with and consequently their capacity to identify and effectively use specific tools and forms of expression. In this perspective the tools and forms of expressions that are familiar and easily accessible might probably not pose as much of a challenge as those that are unfamiliar or those rarely encountered or never used.

5.3 Method

The analysis of students’ scores for the selected items and countries was done initially using simple graphing methods. This was basically a way of getting a general orientation of the data, namely the high and low points allowing for the individual investigation of the sets of observations concerned. After this initial investigation a more subject specific analysis was conducted using cluster analysis.

5.3.1 Graphical analysis

In statistical analysis simple graphics are considered as appropriate for first time analysis given the visual powers of graphs to provide a quick feel of the data. Tufte (2001) posits that well-designed data graphics are usually the most powerful method for analysing and communicating statistical information. Feder (1974) contrasts numerical methods with graphical methods arguing that while the former are designed to yield specific answers to rigidly defined questions, the latter are less confining and facilitate understanding the relationships reflected in the data; their displays often uncover features of the data that were totally unanticipated prior to the analysis. Thus, plotting a graph can be perceived as a first step in the exploratory means of understanding the data. With this in mind, the items were plotted in a graphic profile diagram as illustrated in Fig. 5.1 revealing a pattern of items recording high respective low success rate scores. This observation necessitated the need to characterise these items initially by re-plotting them to create another profile diagram based on maximum and minimum values.
Fig. 5.1: A profile diagram showing the response pattern from PISA 2003 for the Nordic countries based on success rate. A line graph is preferred because it provides a vivid picture of performance differences rather than depicting a continuous variable.

The items plotted in Fig. 5.1 [see Fig. 5.2] were then rearranged in ‘descending’ order [based on the average value] using the maximum and minimum values for the specific Nordic countries thus revealing a trend that indicated the presence of groupings in the results. A preliminary analysis of the items revealed a pattern that coincided with the kind of subject specific tools needed to effectively interact with the task or solve the problem. Thus, Fig. 5.2 was perceived as being consistent with the characterisation presented in Table 5.2 that is, in terms of items perceived as requiring Identification (I) contra those requiring a Critical-analytical approach (CA).
5. Exploring students’ performance from PISA results

Table 5.2: Categories [levels] of interaction with graphical artefacts

<table>
<thead>
<tr>
<th>Identification</th>
<th>Critical-analytical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identifying or reading off values from the graphical system</td>
<td>Critical-analytical interaction with the graphical system: a global rather than a local perspective to data, mental rotation etc.</td>
</tr>
<tr>
<td>Manipulation of aspects of the graphical systems using elementary operators e.g. addition and subtraction</td>
<td>Multi-referencing, creative use of mathematical concepts forms of expression and integrating ideas from other subject domains.</td>
</tr>
<tr>
<td>Application of terms and concepts that are non problematic from an everyday perspective.</td>
<td>Appropriate use of argumentation and communication skill to express reflections from the graphical system.</td>
</tr>
</tbody>
</table>

For each item a minimum and maximum p-value was recorded and plotted in a line graph giving the range of performance from the selected countries: Denmark, Finland, Iceland, Norway, and Sweden (Fig. 5.2). Generally from Fig. 5.2, it seems that the items that are considered as having potential for Identification approach (I) registered higher success rate than items perceived as having potential for Critical-analytical approach (CA). It is also shown that the inter country difference in performance tended to increase for items classified as requiring CA approach [see also Fig. 5.1]. Items M302Q01 and M302Q02 which registered least inter country difference are typical Identification type questions where visual perceptual dimensions are used to read off values from the graphical artefact. Items M155Q03 and M828Q03 from items classified as requiring CA approach also registered relatively low inter country differences; these items involved the calculation of percentages. It can also be observed that most of the items registering larger inter country difference are from the ‘Critical-analytical’ group of items. These included among others items M150Q03, M179Q01, M513Q01 and M467Q01. The first three items required some form of explanation that is, the communication of observations made, while item M467Q01 involved calculation of probabilities.
This observation suggested the presence of possible grouping of the data an observation that necessitated further statistical analysis.

Fig. 5.2: Items success rate showing maximum and minimum score for the items indicating distribution of the items into categories that is, whether these are Identification (I) or Critical-analytical (CA) type items. A dotted line graph is preferred since it provides a vivid picture of item performance differences

5.3.2 Cluster analysis

The graph of items success rate plotted according to perceived item potential solution approach namely, Identification (I) contra Critical-analytical (CA) as shown in Fig. 5.2, indicated the presence of ‘groups’ in the data characterized by comparatively ‘higher’ and ‘lower’ success rates. This observation warranted further statistical analysis using agglomerative cluster analysis. Cluster analysis was employed initially as an independent means of identifying the occurrence of natural items grouping from students’ performance and further, as a way of controlling the viability of the identified two groups that are perceived as characterising interaction with graphical artefacts. As an interdependence technique, cluster analysis explores structure among a set of variables or observations by placing the most similar variables or observations into a number of heterogeneous groups with high degree of internal homogeneity. Generally cluster analysis is used to identify the existence of natural groups in a defined set of variables, entities or objects on the basis of the similarity of their
characteristics. It is noteworthy that the reference to ‘hierarchical’ in cluster analysis is based on the procedure of constructing a hierarchy of treelike structures rather than depicting order of preference or importance. Hierarchical cluster analysis can be contrasted with non-hierarchical cluster analysis where the objects or observations are assigned to predetermined clusters.

One characteristic of cluster analysis is that it does not make prior assumptions on the number of groups or clusters in the data. This observation was of importance for this research given that as shown in section 2.3.3 different previous studies on interaction with graphical artefacts or graphical comprehension identify different levels and groups. Thus, the application of cluster analysis on the data might be perceived as serving the function of testing, with the further possibility of confirming or discarding the notion of the presence of three natural groups [this level of grouping seems to be the most dominant] in interaction with graphical artefacts. As suggested earlier, cluster analysis groups individuals or observations so that observations in one cluster are more similar to one another than they are to observations in another cluster (Hair, Black, Babin, & Anderson, 2010) Since cluster analysis is a multivariate statistical technique, it is worthwhile to state that in this dissertation, the individual item is perceived as a single observation with the countries’ scores (p-values) as the different dimensions of the observation. Thus, the data can be characterised as involving 19 observations of a 5-dimensional variable that is, the country item scores for Denmark, Finland, Iceland, Norway and Sweden.

There are two major types of hierarchical clustering methods: agglomerative and divisive methods. In this study the agglomerative method was employed [default for hierarchical cluster analysis (hclust) in R-package]. In the agglomerative method the individual data are at first perceived as own or as a single cluster, these are then subsequently merged step-wise [bottom up] into groups until a single cluster comprising of relevant single observation is formed. The divisive method on the other hand works in the reverse order, beginning with all observations as a single cluster and then separating this successively into groups until individual clusters are formed. The models or groupings in cluster analysis are based on measure of (dis)similarities or the proximity of one item.
relative to another item. The choice of a measure of dissimilarity must satisfy the following conditions:

Let \( x_i, x_j \in \mathbb{R}^r \). Dissimilarities usually satisfy the following three properties:

1. \( d(x_i, x_j) \geq 0; \)
2. \( d(x_i, x_i) = 0; \)
3. \( d(x_i, x_j) = d(x_j, x_i). \)

Some of the proximity measures include Manhattan distance, Minkowski distance, Mahalanobis distance and the squared Euclidean distance. The proximity measure used in this study is the classical Euclidean distance, since it is the most frequently used and the most natural distance function. This is defined as follows:

Let \( x_i = (x_{i1}, \ldots, x_{ir})^T \) and \( x_j = (x_{j1}, \ldots, x_{jr})^T \) denote two points in \( \mathbb{R}^r \). The Euclidean dissimilarity measures is defined as follows:

\[
d(x_i, x_j) = \left[ \sum_{k=1}^r (x_{ik} - x_{jk})^2 \right]^{1/2}
\]

The cluster algorithm used was the average linkage procedure. In this procedure, the similarity of any two clusters is the average similarity of all individuals in one cluster with all individuals in another. The advantage of the average linkage is that it is not affected by extreme values and is considered to be relatively robust (Everitt, Landau, & Leese, 2001; Hair, Black, Babin, & Anderson, 2010).

Considering that cluster analysis yields clusters at different levels, the determination of relevant cluster level was motivated by the desire for generality and simplicity rather than desire for details (Romesburg, 1984). Common to most statistical methods some knowledge of the items or
observations used is of vital importance at least as a way of controlling the reasonability of the clusters formed. This condition was fulfilled from the graphical analysis which indicated, as mentioned earlier, the presence of two sets of success rate groups based on students’ performance.

5.4 Results

For the p-values of items used in this study, two major clusters are identifiable by general inspection. These are clearly determined when the dendrogram is cut at about height 70, giving the cluster formed at the node at approx height 48 and approx height 57 (Fig. 5.3). These clusters will subsequently be referred to as cluster I and cluster II respectively. Specifically cluster I includes the items merging at node 48 that is to say, starting from the ‘top’ (far left) of the dendrogram, and is comprised of items M155Q01 moving down to item M302Q02. The remaining items comprised of items M155Q03 down to item M192Q01 form cluster II.
5.4.1 Stability of clusters

Since there is a tendency for the interpretation process of the results from cluster analysis to be subjective, Olsen (2005) suggests a set of criteria focusing on the stability of the clusters formed as a first step in ascertaining validity. Following this suggestion, the clusters formed for the items were subjected to the criteria of (i) internal stability – this is related to factors such as the distinctiveness and internal cohesion of clusters, and that the clusters ought to be approximately the same size (ii) stability under selection of technique – that
the clusters observed ought to be comparable with other studies undertaken on similar kinds of data sets and (iii) face value – that the clusters ought to have some cognitive value. The criterion for internal stability of the clusters was deemed to be fulfilled both in regards to external isolation and internal cohesion. The criterion for external stability was perceived as remotely suggested based on some of the studies suggesting a two way classification of student interaction with graphical artefacts (see e.g. Gal, 1998; Ben-Zvi & Arcavi, 2001) – as well as the application of the categories for Identification contra Critical-analytical approach [see Table 5.2].

The selected technique was also found to provide solutions that were not very different from those achieved through Ward method with Euclidean measure of proximity. The removal and addition of some of the observations, an exercise suggested by Olsen (2005) also did not affect the clusters formed: in particular items M150Q02, M155Q02, M155Q03 and M179Q01 were removed since the students solutions for these items included mixed scores that is to say, full and partial credit awards [see appendix III: Fig. III a]. Similar clusters were also, to a large extent, observed at the national level for some of the countries; this observation was taken as implying clusters stability.

Regarding internal cohesion, the results were also assessed for uncertainty using AU (approximately unbiased) p-value and BP (bootstrap probability) [see appendix III: Fig. III b]. The AU yield values are less biased than those from the conventional BP, thus preference is given to the AU values. It is noteworthy that Fig. 5.3 was produced using Average linkage method with Euclidean proximity measurement. AU p-values are a better approximation to unbiased p-value than BP values computed by normal bootstrap re-sampling. p-value is a value between 0 and 1, which indicates how strong the cluster is supported by data: with \( n_{boot} \) pegged at 10000, [i.e. the number of bootstraps made]; it was observed that the two major clusters of items formed were strongly supported by the data. For a cluster with AU p-value > 0.95, the hypothesis that the cluster does not exist is rejected at significance level 0.05. Thus, there is strong indication that the clusters formed are probably authentic; for both cluster I and II the AU p values are observed to be greater than 0.99. The results from cluster analysis were also controlled for standard errors of the AU p-values by plotting a p-value versus standard error graph - the values were
Students’ narratives from graphical artefacts

less than 0.008 indicating that the clusters are not largely influenced by random errors (Suzuki & Shimodaira, 2006).

5.5 Commenting the PISA study

The aim of exploring students’ performance on items containing graphical artefacts included characterising their results. The observations from graphical analysis suggested a pattern of performance characterised by varying levels of success rate based on factors such as tools and forms of expression involved. This observation seems to lend credence to the suggestion of there being a two-way classification of modes of interaction with graphical artefacts. Thus, it can be claimed that the I/CA model seems viable given that the graph of students’ success rate on items containing graphical artefacts delineated the two groups and subsequently the approaches. It was also evident that items that required action with [calculations] ‘problematic tools’ like percentage and probability yielded low scores. This observation viewed from a perspective of ‘tools and forms of expression’, is consistent with observations from other research where students experienced difficulty with proportions, percentages and probability (e.g. Adjiange & Pluvinage, 2007; Noelting, 1980; Jones, 2005).

The analysis conducted using cluster analysis provided results that confirmed that these groups are probably authentic thus they are not overly influenced by random factors. This observation not only bolstered the validity of the I/CA model but also seems, in one way or the other, to challenge some cognitive models based on what might be considered as a step-wise model of interaction with graphical artefact [see Friel et al, 2001; also section 3.2]. For example, in the PISA framework outlined in section 5.2.1, it might be expected that items from the connection cluster of competency would form own groups, however, based on the cluster analysis these are found in both I as well as CA clusters. In this sense it can be said that cluster analysis seeks to bring some clarity to the PISA data in terms of data reduction. The data reduction occasioned by cluster analysis might also be perceived as an indicator that focusing on tool use and forms of expression has potential for providing a clearer characterisation of students’ performance and consequently, cognition.

In general, the clusters seem to provide a suitable fit for the categorisation of interaction with graphical artefacts involving aspects of Identification on the one hand and Critical-analytical approach on the other hand. In a previous
study involving graphical artefacts, Åberg-Bengtsson (2005) was able to determine a subset of SweSAT that measured analytical dimension that is to say, items that involve some degree of calculations. While this study partially confirms this finding, it is also the case that the observations made in this study go a step further to show the existence of differential performance based on the nature of calculation involved. Thus, while operations involving addition is regarded as calculation of sorts, the observation made in this study is that performance tends to be higher for most students’ when interaction with graphical artefacts involves some additive operations. While these kinds of calculations might have been captured in Åberg-Bengtsson’s study, as part of analytical dimension in this study such interaction dealing with easily accessible and familiar tools and forms of expression would be considered as involving Identification approach. The matter of accessibility to and choice of tools is probably best captured in the observation in the two items M155Q04 and M467Q01 [generally classified as requiring a Critical-analytical approach] which display the highest success rate range that is, the difference between minimum and maxim scores for the Nordic countries. Based on the characteristics of items classified as requiring a Critical-analytical approach, the high success rate range on these items may be attributed to the need, on the part of the students, to make informed choices from a range of varied possibilities [as opposed to where there is a single dominant tool needed for effective interaction with a graphical artefact]. These could be the selection of a solution from a multiple choice question or the selection of appropriate subject specific tools and problem solving methods.

The analytical importance of using both the I/CA model and cluster analysis is realised when one considers that the suggested classification of items given in Table 5.2 is based on empirical results which are, in most cases, purportedly generalisable. However, considering cultural disparities caution ought to be taken when making these generalisations (Radford, 2008b). Thus, from the results provided in the present study, some of the items classified as requiring a Critical-analytical approach are clustered with those deemed as requiring an Identification approach [see Fig. 5.3]. While this can be perceived as a problematic nature of classification (Kozielecki, 1986; see also Engeström, 1987), it provides an opportunity to look closer at the items in light of the categorisation criteria. Item M155Q04 [population pyramids] was classified as
CA on account of its demanding the consultation with several graphical artefacts \textit{[aspect of multirefering]}, however most of its task questions were such that they required the identification of values and trends. The question was also of a multiple choice format. Item M467Q01 was classified as CA since it required calculations with proportions; this was also a task of a multiple choice format. A possible explanation for this ‘discrepancy in clustering’ is that the item format might have reduced the burden on some of the choices the students had to make, in other words they might have had a scaffolding effect in terms of application of subject specific tools and forms of expression which contributed to the higher performance. The ‘difference’ between results from cluster analysis and the proposed construct can also be seen as revealing the dynamics or the interplay of the students’ approach to interaction with graphical artefacts. The two approaches to graphicacy, viz.: Identification and Critical-analytical, ought not to be seen as mutually exclusive. Whereas the proposed I/CA model is ‘static’ in as far as empirical evidence is concerned, it seems that applied together with cluster analysis it might be possible to determine which subject specific forms of expression, tools and operators that have actually been integrated into the students’ repertoire for engaging with graphical artefacts. Thus, in a case where say test items requiring the application of proportional reasoning are found to cluster with items requiring Identification approach one might have to re-examine the justification of considering proportional reasoning as a problem area for students with several years of schooling. This observation can also be perceived as depicting an advantage of using cluster analysis in combination with the proposed taxonomy.
5. Exploring students’ performance from PISA results
6 THE ANALYTICAL FRAMEWORK

In chapter three, a brief exposition of constructivism, sociocultural and semiotics theories was provided where, in effect constructivism was contrasted to the latter theories. In the wake of the observations presented in chapter five in which two levels or categories of interaction with graphical artefacts were determined from students’ results, the present chapter returns to the discussion on theories and theoretical frameworks. The intention being to provide a theoretical basis for the observations made in chapter five as well as establishing a theoretical base for the analytical framework that will be employed in the rest of the dissertation. While it is true that the sociocultural and semiotics theories have, to a large extent, influenced the trajectory of the present dissertation including laying the ground for what is considered as the analytical framework used to analyse the data, a stringent application of the same cannot be claimed. Eisenhart (1991) posits that the selection of a theory as a basis for a research framework implies that one “… is choosing to conform to the accepted conventions of argumentations and experimentation associated with the theory” (p. 205). This implies that there are some restrictions in formal theories that the research is bound to conform with. A complete adherence to a given theory may also diminish the possibility of being open to other impulses from research especially if these are not addressed in the theory (Becker, 1993 see also Eisenhart 1991; Lester, 2005). Thus, being entirely bound to a single theory might impede multi-methods validation of data that is, triangulation (Eisenhart, 1991; Lester, 2005). It is also noteworthy that an analytical framework is not aimed at explaining or generalising the phenomenon per se but rather offers an approach that can be used by researchers to generate explanations in answering specific research questions (Stanley, 2012).

6.1 Subject specific tools and forms of expression

Within the sociocultural theoretical perspective the role of tools and artefacts is acknowledged as playing a crucial mediating role and subsequently also in cognitive transformation and development. The use of signs leads humans to structures of behaviour that break away from biological development and create new forms of a culturally-based psychological process (Vygotsky, 1978 see also Sebeok, 1994). Wertsch (1985) suggests that Vygotskij’s theoretical
approach can be understood by recognising its three interrelated general themes of which the theme he posits as analytical and preeminenting the others is “the claim that an adequate account of human mental functioning must be grounded in an analysis of the tools and signs that mediate it” (p. 15). While Wertsch seems to distinguish between tools and signs, this dissertation adopts a broader meaning of tools as things that can be consciously used, modified, transformed etc. by another with resulting effects. Vygotsky (1981, p. 137) exemplifies “artificial devices for mastering mental processes.” – psychological tools and their complex systems as including language, algebraic symbol systems, work of art, diagrams, maps, schemes, conventional signs etc. Indeed in the given example, sign is inferred as index, symbol and icon. This is perceived as indicating a semiotics element in Vygotskij’s theoretical perspective. Radford (2008b) refers to these as semiotic means of objectification and posits that the investigation of students’ interaction and use of semiotic means of objectification is a methodological way of accounting for learning.

From Peirce semiotics it is recognised that all our thinking is performed upon signs of some kind or other; with the best reasoning, especially in mathematical subjects, being done by experimenting upon a diagram or other schemes (NEM 1:122), thus there is, as outlined earlier [section 3.5], some level of harmony within these theoretical perspectives. It was mentioned earlier that a sign [as semiotic means of objectification] has two functions which are realised through collateral experience [section 3.4.2]. Now, collateral experience can be perceived as operating on two levels or dimensions determined by whether the object of the sign is immediate or dynamic (CP 8.314). The case of immediate objects the interaction with these signs [graphical artefacts] is relatively non-problematic, while in the case of dynamic objects there are deliberate efforts to make sense of signs [graphical artefact]. Now, Peirce uses collateral experience in a way that might be considered to be rather fuzzy; in CP 8.314, one gets the impression that collateral experience is limited to dynamical objects wherein sense making can only be achieved by collateral experience. However, in CP 8.183 it is evident that “no sign can be understood…unless the interpreter has “collateral acquaintance” with every Object of it”. In clarifying the application of collateral experience, we return to the research conducted by Roth and Bowen (2001) mentioned earlier, where they posit that interaction with
Students’ narratives from graphical artefacts

graphical artefacts can be characterised as *structuring* and *grounding*. Structuring entails identifying relevant signs and tools mostly by visual inspection that might be used for effective interaction with the graphical artefact. Grounding on the other hand involves the linking of an inscription [a graphical artefact] with a network of relationships in the meaning making process. Grounding can be perceived as being experience based; experience with the demands imposed by the graph, in other words familiarity with the conventions attached to the graphical system at hand, identifying the appropriate signs, tools e.g. the need to apply mathematical concepts or operators and forms of expression in interacting with the graphical artefact.

### 6.1.1 Identification and Critical-analytical approach to graphicacy

The basic principles behind structuring and grounding can be simplified and made more accessible and thereby relevant as part of school discourse [the other reason for abandoning the concepts coined by Roth and Bowen (2001) because within Peirce semiotics the concept *ground* takes a meaning connected with the principles determining how a sign represents its object (CP 2.228) while *structuring* from the account provided by Engeström (1987), appears to have some constructivism underpinning] by perceiving them as respectively involving identification and critical-analytical approaches to interaction with graphical artefacts. Identification approach is characterised by focusing largely on perceptual elements and using less demanding tools, while Critical-analytical approach is perceived as involving evaluation of the graphical system, active engagement with subject specific operators and forms of expression. Thus, ‘Identification’ (I) can be perceived as pointing to identification of values from graphical artefacts e.g. pointing out the month with highest rainfall in a graph of annual precipitation as well as identifying with certain tools and forms of expression associated with a given graphical artefact. Critical-analytical (CA) can be perceived as focused on critical stance, reflexivity and mathematical or subject specific manipulation, transformation etc. [of graphical artefacts].

### 6.2 I/CA approach to graphicacy

While graphing practice in authentic situations is desirable (Gal, 2003; Rumsey, 2002; Watson, 1997), it is not uncommon in compulsory schooling that learning with regards to graphical artefacts remains largely theoretical. Thus,
students are largely expected to interact with theoretical [mathematical] tools and forms of expression wherein their ‘competence is ascertained’. The focus on tools and forms of expression implies that to some extent students’ graphicacy might be evaluated through fluency with subject specific tools and forms of expression (Radford, 2008e; Vygotsky, 1978). Research shows that students generally have difficulty with certain topics [strands], a factor that might impede effective graphicacy. For example, some students even after several years of schooling still have problems with ratio and proportions (Adjiange & Pluvinage, 2007; Noelting, 1980), probability concepts, (Jones, 2005), aspects of ‘multireferencing’ (see. Guthrie, Weber, & Kimmerly, 1993), visual-spatial skills [including mental rotation] (Stavridou & Kakana, 2008) and the ability to explain their strategies (Diezmann & Lowrie, 2009). Thus, for graphical artefacts involving some of these ‘tools’ it might be expected of students to be critical with regard to tools selection as well as displaying effective analytical skills. In terms of Peirce semiotics it is reasonable to posit that students’ collateral experiences with some of these ‘tools’ might not be ‘immediate’ or readily accessible and subsequently some high degree of focus is needed. Hoffman and Roth (2007) refer to these kinds of situations as involving ‘knowledge in focus’ – a form of knowledge that is ‘yet’ to be internalised. Given that ‘tool’ use in interaction with graphical artefacts is largely experience based, it is logical to find a connection between identification and critical-analytical approaches to graphicacy within Peirce’s conception of collateral experience. The relationship between these is given in the model of interaction with graphical artefact given in Fig. 6.1. In the model, a connection to collateral experience is also shown, in developing the model, an understanding that collateral experience manifests in two ways (CP 8.183; 8.314) as well as a possibility of the lack (Bergmann, 2009; Hoffmann & Roth, 2007) of it is also taken into account.
The model suggests that in learning situations, ‘tool’ use in interaction with graphical artefacts is largely experience based. This model is connected to Peirce triadic model through what is perceived as the indexical property of a sign implying that a sign is not necessarily neutral as it might be supposed. According to Roth (2008) a sign or symbol cannot be considered in isolation because it is always bound up with intentions, motives and the object of actions. A time series graph on the financial pages of a newspaper illicit some effect on the reader leading perhaps to deep concentration or skipping the pages altogether. This follows the notion that “signs are inherently connected to the kind of experience and action they are capable of prescribing” (Bergmann, 2009, p. 252). Thus this model is connected to the triadic model as shown in Fig. 6.2.
The indexical character of graphical artefact is best appreciated when we consider that educational tasks given to students occur in what can be considered as a network of sign systems. For example in a task containing graphical artefacts this network might include the question item, any accompanying explanations as well as the figure per se. These contribute in providing directions for cause of action. Barthes (1977) uses an example from the advertising world to show how a number of factors such as linguistic, symbolic, and literal meanings contribute to aid the effective interaction with a graphical artefact [in this case a picture] and greatly reducing its polysemy, namely the risk of the information being interpreted differently. Thus, in a network of sign systems even the ‘hints’ that normally accompany some school tasks can be seen as contributing to the effective interaction with graphical artefacts.
7 GRAPHICACY: EVALUATING STUDENTS DISCUSSING A ‘MEDIA GRAPH’*

In this chapter the video observation study is presented. This study was necessitated by the findings from the analysis of students’ results from the PISA survey study outlined in chapter five. While making individual assessments of the items, it was observed that some of these items showed some intriguing results. For example item M192Q01 - Containers, indicated some gender differences while item M179Q01- Robberies, showed general low success rate being perceived by the author as a common school task which ought to have attracted higher success rate scores. This necessitated the need for further exploration of the nature of students’ response to task M179Q01 [task M192Q01 was not explored further because it was not yet in the public domain and was replaced by trial task M465Q01]. This was done by simulating an assessment situation in a collaborative setting in contrast to an original paper and pencil test environment. The expectation from the study was to identify possible explanations to the intriguing observations made from the test scores. The general outline of this chapter is to provide a justification for selecting the item in question, to provide some background information from the PISA survey and to offer a deeper analysis of items using, the so called double-digits [see section 7.2.1]. These codes will then be used parallel to aspects of I/CA in characterising students’ interaction with graphical artefacts in the analysis of the video observations.

7.1 Rationale

As it has been pointed out that students’ success rate on some of the items from the PISA survey test were, in some ways, intriguing and thus it became pertinent to try and find explanatory factors behind the observed performances. From a sociocultural perspective, it might seem natural to attribute part of the students’ performance to contextual factors viz., in this

* A derivative of this chapter was published 2013 in the Nordic Studies in Mathematics Education (NOMAD), 18(1) pp. 5-30, with journal article title: Making sense of a “misleading” graph
case the paper and pencil nature of the PISA survey. While a paper and pencil test environment has its advantages, there are also some drawbacks inherent in it, such as revealing only a narrow range of students’ know-how and with certain skills such as oral argumentation and persuasion being overlooked. Thus, it was desirable to diminish the restrictive aspect of the PISA test by including a more open collaborative environment. In this way students would be encouraged to verbally express themselves, sharing ideas and tool use and including all forms of expression while solving the problem at hand.

Apart from the potential of providing more insight into students’ know-how, communication is an integral aspect of mathematics literacy. In the definition of mathematical literacy provided by OECD PISA, the integral role of personal involvement in using mathematics and solving mathematical problems are alluded to; “through communicating, relating to, assessing and even appreciating and enjoying mathematics” (OECD, 2003 p. 25 italics in original). The integral nature of being able to communicate mathematics (including verbal expression) is appreciated when the functional nature of mathematics literacy is understood (OECD, 2006a) and since effective citizenry demands social participation. Within the context of the Swedish curriculum, the importance of verbal communication in mathematics is manifest in the inclusion of a verbal (oral) section in the Swedish national test (Np) and more importantly, the ambition to promote verbal mathematics communication is explicitly stated in the national curriculum document. In the Swedish compulsory school system one of the goals of mathematics teaching is to ensure that the pupils develop their ability for logical reasoning, to draw conclusions as well as explaining and justifying their reasoning or thinking both orally and in writing (Skolverket, 2009). While PISA is not curricula oriented as such, reference is made to national curricula affording it importance with respect to informing the teaching contents of the mathematics classroom; a fact that might have some effect on students’ mathematical literacy as manifest in the results of an assessment and testing situation. With regards to the use of mathematics in real-world settings PISA acknowledges that this is “based on the skills learned and practised through the kinds of problems that typically appear in school textbooks and classrooms” (OECD, 2003 p.24).
7.1.1 Research concerns

At the core of the present study is item M179Q01-Robberies (Fig. 7.1). This item was isolated for closer analysis based on what was considered as it being a relatively common area of concern in the topic strand of statistics. Also it was situated in the personal domain, that is to say, items perceived as dealing with aspects of everyday life (OECD, 2003) and it also mirrored a case from the media typifying what might be considered as a ‘misleading diagram’. Thus, the item was considered apt in fostering statistical thinking (Gal, 2003; Rumsey, 2002; Watson, 1997). This task (M179Q01) called for a reaction to information attached to a graphical artefact appearing in a newspaper indicating the occurrences of robberies [a typical school mathematics approach to instruction involving a similar graph in provided in appendix IV]. It is of interest for this study to explore the approach and depth of analysis employed by some Swedish students in a collaborative environment as they solved the task. Here, the ability to select appropriate and optimal operators and being able to communicate solutions effectively is integral and being perceived as indicating elements of Critical-analytical approach. To achieve this, the following questions guided the investigation:

- What forms of expression do students use when faced with a statistical/mathematical task containing graphical artefacts?
- What is the nature of the students’ graphicacy, as manifested in their written and verbal responses to the given task?

By posing these questions it is intended not only to explore the levels of graphical comprehension but also to identify the possible factors that may influence the quality of the students’ solutions to typical items containing graphical artefacts.
7.2 Empirical matters

Before discussing the choice of method used, the selection of students and items as well as general study execution, it is deemed appropriate to provide some background information occasioned by secondary analysis of results of PISA double-digit coding system on item M179Q01.

7.2.1 The double-digit coding system

The test item under investigation was of the constructed open-ended format and as such there was scope for free-responses. As with the case with constructed open-ended or free-response items, students have the possibility of using diverse tools and forms of expressions including strategies for solving a mathematical problem. Thus PISA utilised the double-digit coding which was
developed as a means of obtaining more information from the students’ written solutions in addition to the normal correct or incorrect credit awards reflecting the complex and multistage processes involved in mathematical thinking (Allerup, Lindenskov, & Weng, 2006; Lie, Taylor, & Harmon 1996). In the double digit coding system, the first digit indicates the extent to which the response is correct while the second digit indicates the content of the response (OECD, 2003).

The free-response format of the item (M179Q01 Robberies), required the students to make a case for or against the claims made by a journalist’s interpretation of the graph (Fig. 7.1). For a convincing case, a student needed to use optimal arguments, possibly accompanied by the appropriate use of mathematics operators and forms of expression. The OECD average success rate for this item was about 30%. This figure – 30% was taken as indicating that a considerable number of students might have had difficulties providing acceptable solutions to the task. Slightly less than half of the Swedish students provided satisfactory solutions to the task. With a success rate of about 46 % Sweden was among the top OECD countries (see OECD, 2010). Since the aggregate national success rate scores given above does not give much information about the nature of the students’ responses, the double-digit coding came in handy: the credits were awarded as shown in Table 7.1 and it is also possible to observe the tools and forms of expression of choice employed by the students.
### Table 7.1: Characteristics of responses for item M179Q01 as provided by OECD, (2009)

<table>
<thead>
<tr>
<th>Full credit:</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Code 21:</td>
<td>Solution states that the statement is NOT reasonable based on the observation that only a small portion of the graph is shown.</td>
</tr>
<tr>
<td>Code 22:</td>
<td>Solution states that the statement is NOT reasonable and uses arguments based on ratio or percentage increase.</td>
</tr>
<tr>
<td>Code 23:</td>
<td>Solution states that trend data is required before judgement can be made.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Partial credit:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Code 11:</td>
<td>Solution states that the statement is NOT reasonable but lacks supporting explanation. It focus ONLY on an increase given by an exact numerical figure but does not compare with the total.</td>
</tr>
<tr>
<td>Code 12:</td>
<td>Solution states that the statement is NOT reasonable with correct method but with minor computational errors.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>No Credit:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Code 01:</td>
<td>The solution contains a NO with insufficient or incorrect explanation.</td>
</tr>
<tr>
<td>Code 02:</td>
<td>The solution contains a YES, focuses on the appearance of the graph and mentions that the number of robberies has doubled.</td>
</tr>
<tr>
<td>Code 03:</td>
<td>The solution contains a YES with no explanation, or explanations other than those in Code 02.</td>
</tr>
<tr>
<td>Code 04:</td>
<td>Other responses.</td>
</tr>
<tr>
<td>Code 99:</td>
<td>Missing.</td>
</tr>
</tbody>
</table>

Following the discussion on Identification/Critical-analytical approach put forward in chapter five, the credit awards for codes 21, 22 and 23 are considered here as indicating some level of Critical-analytical (CA) approach. This is supported by the nature of the solution provided which indicates aspects of effective communication of the solution, use of subject specific tools and forms of expression (using ratio, mentioning trend etc.). The scores on codes 11 and 12 are perceived as involving an Identification approach. While the ‘minor computational errors’ are not explicitly mentioned, it is
possible that these may involve the use of subject specific tools such as ratio or percentages. Fig. 7.2 shows the percentage distribution of the solutions by codes from Sweden [the horizontal axis represents the code].

![Graph showing distribution of students' responses from Sweden based on double-digit coding](image)

**Fig. 7.2: Distribution of students’ responses from Sweden based on double-digit coding**

From Fig. 7.2 it can be observed that comparatively higher numbers of students provided solutions that were scored for code 11. The characteristic feature of the solutions given for code 11 was the lack of explanation accompanying the provided solution; an observation perceived as indicating an Identification approach to the task. The second most frequent solution for Sweden is code 21 (19 %). Though yielding full credit, the solutions on code 21 seem to be devoid of subject specific operators and forms of expression and are to a large extent, also inclined towards an Identification approach. These solutions question the production or presentation of the graphical artefact (data integrity) as seen in highlighting the structural aspects of the graphical artefact. Significantly, solutions scored for code 11, though perceived as based on an Identification approach, indicate an appreciation of number sense through the use of elementary arithmetic (subtraction).

The general success rate for solutions obtaining full credit (codes 21, 22 and 23) showed that a majority of students gave explanations that were not
accompanied by the use of subject specific operators and forms of expression as observed in the low response rate scored for solutions on code 22 and code 23. These codes implied the use of mathematical operators and knowledge of statistical concepts respectively. Whereas the double-digit codes [see Table 7.1] provide information about the solutions that were put forward, it still misses the dynamics involved in providing the solutions. This has the effect of limiting the kind of analysis that can be done on the nature of the responses. Thus, in opting for an observational study it is also of interest to observe if the pattern of response and strategies extracted from the double-digit coding is replicated in the transcript of the students’ conversations.

7.2.2 Video recording

Having seen the distribution of the students’ solution strategies, it is worthwhile admitting that the importance of preferring a collaborative environment presupposes the difficulty that the quality or extent of mathematical competence ‘ascertained’ from written works or grades entails. The visibility of mathematical competence and accountability is realised in a setting where two or more persons working on a mathematical problem carry out their mathematical reasoning through discussions (Greiffenhagen, 2008). The benefits of video observation are well documented (see e.g. Stigler & Hiebert, 1999; Ulewicz & Beatty, 2001; Hindmarsh & Heath 2007). Generally observational data enables the researcher to look at what is happening in situ, to see things which might otherwise be unconsciously missed, to make discoveries that the participants might not freely talk about, to move beyond perception-based data and to access personal knowledge (Cohen et al., 2000). For example, with video observation there is almost unlimited possibly to re-enact the recorded observation in the process thus gaining more insight into obscure aspects of the observation. The possibility of cross referencing and re-interpreting the study by colleagues or in a peer review setting is easily facilitated.

The advantages of video observation over simple observation and interviews are substantial; however, the success of a video observation is to a large extent dependant on skills associated with technology as well as some element of media planning. Prior orientation with the research environment in a broader sense is often necessary “in order to be able to decide where to place the
camera and microphone” (Heath & Hindman, 2002 p. 116) so that the most relevant aspects of the activity are captured. In conducting the video observational study, two video cameras were used. One of the video cameras was mounted at a fixed angle allowing an overview of the students’ activities. The other was handheld allowing for zooming in on students activities involving some paper and pencil calculations and focusing on particular students as they spoke [turn taking]. An audio recording instrument was also placed on the working table to ensure proper recording of the students’ verbal utterances.

A review of the recorded material was made [watching the video recordings as well as listening to the audio material], after which a strategic decision was made to limit analysis to the verbal aspects of the research material. As a result of the review a decision was made to explore item M179Q01 closely; part of the interest in this item has already been mentioned, but it is important to emphasises that the nature of this item was such that it was possible to include everyday perspectives in the discussion, thus opening up the possibility of observing the interplay between everyday perspectives and also the school mathematics perspectives.

7.2.3 Students selection

As mentioned earlier, the test environment characterised by collaboration among students was expected to allow for deliberation leading to consensus in the problem solving process. The study was initially intended for students at the upper level of the compulsory school, primarily as an attempt to conform to the sampling criteria of PISA. In Sweden the PISA survey is usually conducted when the students are in the last year of the compulsory school and they are about 15 years of age. However, due to the timing of the study [the study was conducted in the month of October] a decision was made to focus on first year upper secondary students. From a research point of view, this was not considered as a handicap given that the students had not started learning topics dealing with statistics and data analysis and neither had they been in contact with the question item used in the observational study prior to the investigation. On the other hand, having qualified for studies at the upper secondary school, it was assumed that the students had met the necessary and sufficient mathematical competency required for solving the question item.
7. Graphicacy: evaluating students discussing a ‘media graph’

The aim of mathematics teaching as outlined in the Swedish syllabus (Skolverket, 2009) is that at the end of compulsory schooling students should have developed numerical and spatial understanding, and further they should understand basic statistics concepts and methods for collecting and processing data and be able to describe and compare important properties of statistical information. At this level they should be able to interpret, compile, analyse, and evaluate data in tables and diagrams.

The selection of the students for the study was done with the help of one of the school principals. The principal ensured that the teachers of mathematics were informed of the intended study and the possibility of their being called upon to supply some of their students to take part. It was also through the principal that a request to include students from different study streams was realised. The decision to include students from the different streams was made based on what is generally perceived of their varied interest in mathematics. It is reasonable to assume that, students’ who have selected streams with mathematics and science orientations might show some liking or preference for these subjects. With this in mind, students were selected from the Aesthetic vocational stream [program] (EST), Applied sciences theoretical stream (NV), Child vocational recreational stream (BF), the Technical vocational stream (TE) as well as Applied science-biology (NV-spets). The ‘spets’ stream is a special national program of study offered at selected centres for students who are interested in intensive theoretical studies. Since the students were unknown to the observer [author], their selection for inclusion in the survey was done with the help of their respective teachers. The basic criterion for selection was a desire that the teachers involved should to select students with the acceptable levels of interpersonal skills necessary for group work in other words, the selected students ought to be able to function in a collaborative setting. The rationale for imposing this condition was to ensure the chances of having some meaningful discussions within the groups.

The students were assigned to groups of three except for one group where the unit consisted of only two students. A total of nine boys and eight girls were involved in the study. The students had access to several tasks [see section 7.2.4] with the observer ensuring that they worked on one task at a time. Basically the students received a task from the observer whenever they felt they had exhausted or solved the task in hand. In this way no undue pressure was
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put on them. Initially, two groups of students were picked for closer analysis. However, this was later extended to include three other groups of students. The initial group consisted of students from the EST stream (3 male students) with the other group taken from the TE stream with 1 female and 2 male students. Though not explicitly expressed, the initial idea was to contrast the two groups based on a perceived notion that some study programs have students with a particular affinity and liking for mathematics. An extended group of students from the NV streams (3 male and 3 female students respectively) as well as students from the BF stream (3 female students) was also considered for further analysis. While all the partaking students had met the minimum criteria needed to solve the task in the survey, as mentioned earlier, it is not uncommon for students admitted to the TE stream to be highly motivated in mathematics compared to those admitted to the EST stream. This observation, as mentioned earlier, was initially used as the basis for including students from these vocational streams. However, it was observed that the nature of conversation for the groups of students generally displayed similar dynamics, and so this line of reasoning was not pursued further. In working with the items, the students were free to discuss the test question, brainstorm and come up with an agreeable solution which would then be submitted verbally or as written text. The expectation from this collaborative environment was that it would contribute to an improvement on the quality of solutions provided in the PISA survey test items.

7.2.4 Item selection for observation study

Out of the initial 19 PISA survey items that formed the basis for the PISA study discussed in section 5.2.1, a further selection consisting of 5 tasks with a total of 9 items were included in the video observational study. The motivation for selecting these items varied and ranged from their perceived importance in an everyday perspective to instances of intellectual curiosity: in this regard the curiosity was a reflection on possible aspects that might lie behind the less impressive students’ performance. In particular item M150Q01 could be solved without reference to the graph thus, the interest to explore the extent to which the graphical artefact might have been a distraction. Item M179Q01 was considered as consisting of integral aspects of the topic strand statistics however, the performance seem to reflect the students’ incapacity to provide comprehensive solutions to the task. Item M153Q01 included aspects that
required making convincing arguments involving subject specific tools and forms of expression; item M465Q01 [used instead of item M192Q01] indicated some level of gender effect since the performance of female students was generally observed to be lower across the board. Even though all these items provided interesting perspectives into the students’ interaction with graphical artefacts, in this video observation study, only the methodological illustration is provided for item M179Q01 (Fig. 7.1). This item, as mentioned earlier, contained such aspects that seem to resonate well with the aims of PISA that is, the application of mathematics in everyday situations and the statistical literacy perspectives (Gal, 2003; Rumsey, 2002; Watson, 1997). Specific for the research represented here is that the items offered the opportunity to interact at different levels thus giving it an edge compared to the other tasks with regards to further the general goals of this dissertation.

7.2.5 Transcription process

The importance of the transcription of recorded material from the observational studies is crucial for analysis and reporting research. Whereas some researchers consider the actual embodied activity of the observational study as constituting data (e.g. Button & Lee, 1987; Psathas & Anderson, 1990) others (e.g. Zimmerman & West, 1975; Ochs, 1979) perceive the recorded material as the researcher’s data. In the case of Zimmerman and West (1975) the potency of their claim is backed by the experience that they discarded the ‘recorded’ materials [for securing anonymity of interlocutors] and used solely the transcriptions for analysis. However, Ochs (1979) maintains that transcriptions are not only the researcher’s data but they are also a reflection of the researcher’s theoretical goals, choices and definitions. In this study the view on transcription is that it is part of the data analysis process and therefore the video and audio materials are perceived as constituting data. However, transcriptions might in some special situations be considered as data in the absence of recordable material, in which case some reflections on methodological and theoretical bias ought to be accounted for. Embedded in the transcriptions are a number of decisions which might influence the outcome of the analysis process this is an observation that even Ochs (1979) alludes to when she observes that “transcription is a selective process reflecting theoretical goals and definitions” (p. 44).
In the course of this study, two distinct transcription formats were used, reflecting the theoretical goals of the study viz., semiotics and the effects of strategic adjustments that are part of conducting an academic research. The first set of transcriptions was done with an orientation towards the Jeffersonian transcription (see Appendix I) model [the Jeffersonian method is a system of keys and codes that was developed by Jefferson to characterise conversations] to capture aspects such as conversational prosody and ‘peripheral’ sounds. The second set of transcriptions followed a simple top-bottom convention for recording conversation without highlighting other aspects of conversation mentioned earlier. The decision to use different methods was informed in the first instance by the realisation that aspects of the Jeffersonian method, while important might have a clouding effect and thus hamper access to the essence of the study, especially for persons not well versed with its symbolic system [see Appendix I]. Secondly, while the transcription was oriented towards the methods of conversational analysis, it was apparent [and also based on the aims of this research] that it would be difficult to adhere to the strict conventions of conversation analysis, an observation that might legitimise the desire to consider analysis of the video data as more of an analysis of [verbal] interactions rather than conversational analysis. The transcription and the coding process were facilitated with the help of the transcription software InqScribe®. Significantly the software enabled the inclusion of timeframe in the transcriptions and reworking the Jeffersonian codes. This applied to a greater extent to the first set of transcription [transcription with the Jeffersonian codes] rather than for the second set of transcription. However, it is noteworthy that the codes given from the software had to be adapted to the Jeffersonian ‘format’. In the Jeffersonian transcription system, the dynamics in the conversation is highlight. While this is missing in the simple basic top-bottom transcription, explanatory notes have been included to elucidate some parallel activities of interest.

7.3 Students’ discussing a ‘misleading’ diagram

The video observations were in a way meant to augment the findings from the double-digit codings [section 7.2.1]. In presenting the results an analysis is made using the I/CA model [including aspects of Peirce semiotics] with emphasis on tool use as well as forms of expression; this is then contrasted with aspects of a three-tire model of graph comprehension found in Friel,
Curcio and Bright (2001). An attempt is also made to provide a code score based on PISA double-digit codes for the general conclusions of each group. Four student groups from the Swedish school system were included in this study. The first group was comprised of girls from the Child care Recreational Program while the second and third groups comprised of students from the Natural Science Program. The fourth and last group consisted of students from the Aesthetic Vocational Program.

Each member of the group was given a copy of the question and any such material that would be necessary to work out the answers to the question items e.g. scribbling or scratch paper in case there was a need to make mathematical inscriptions or computations. Since the emphasis of the study was placed on collaboration within the group, there was no time limit placed on the students as they solved the problem questions; they were encouraged to work at their own pace. Of the four groups, the students in discussion group I and IV opted to discuss the question without simultaneously and independently writing down the answers, while the students in group II and III discussed the task and independently made some annotations on the worksheet provided. For groups II and III, the annotations [inscriptions made on original graphs and written solutions] are attached at the appendix IV. The purpose of the appendix is to avail of the full range of responses from the participating students for each item under investigation.

7.3.1 Discussion group I

This group comprised of three girls from the Child Care Recreation Program. At the start of the video observation, they unanimously agreed to tackle the task together as a group. It is observed that one of the girls seems to have considerable influence on the group.

Based on the PISA double-digit codes the general conclusion for this group namely that the increment was large may not have received any credit being scored at Code 03. However, analysis of the discussion shows that the students put forth some valid observations and reasonable arguments as they engage in the task.
In analysing this transcript the two-way classification mentioned earlier viz., I contra CA approach, is employed. At the very beginning of the transcription the students seem to be guided mainly by visual perceptual concerns lifting out values from the graphical artefacts. This is akin to reading the graph and is typical of an Identification approach. While there seems to be a general desire to expedite the discussion, Helen, makes a statement in turn 2 that might be taken as disagreeing with the observation made by Karolin in turn 1.

[1]. Karolin here it is 507 and up here it is about 516 (pointing at the bars), that is a lot of robberies …in a year, it is quite a lot

[2]. Helen yeah…It is difficult, …however,

However, Helen does not further develop her thoughts and thus the groups concern with Identification approach becomes even more pronounced further into the discussion [turn 12-17]. Here the students literally try to outdo each other on who is reading the graph correctly while pointing out its structural aspects [as if the key is in deciphering this component]. Alongside the structural components, the students have also been able to establish the difference in robberies for the two years as 8 [by simple arithmetic]: this still falls within identification since simple subtraction is considered as non problematic for many students at this school level. There is also some form of proportional comparison made in the sense that robberies of the magnitude of 500 are considered to be a lot in a year. It is worth noting that this is not explicitly pitted against the 8 robberies that signify that increase. Consequently, from a subject specific perspective it is reasonable to claim that the students are yet to confidently invoke proportional reasoning.

[12]. Karolin but it is about eight …eight

[13]. Helen but even this bit here …(aligning the apex of the bars with axis values using a pencil) this one is to here

[14]. Linn No...!

[15]. Helen yes! it is this …that there is up to here… (showing the units alignment on the y-axis)[as if demonstrating how the graph should be read]

[16]. Karolin yes … eight, sixteen
Helen’s response to the observer’s question [turn 21] on their proposed solution to the task in turn 22 makes sense given her response in turn 2. She seems to have a hard time accepting the trajectory that the group’s solution is taking. However, in turn 23 and 25 Karolin brings up another comparison dimension that concerns the nature of the robberies. This dimension is not related to the graph and can be considered as reflective of a reality outside the task at hand. Indeed this is considered in part, an intention of the PISA survey, that is, to test how students are able to apply mathematics learned in school to realistic situations outside school. It is also significant that this comparison dimension comes after the proportional comparison [which was not really developed in the discussion] and as a way of convincing Helen who seems to have some misgivings. By taking into consideration the nature of the robberies it might be claimed that the students are in a way, reading beyond the graph. It is significant to note the change of context from being a task item to be executed in the framework of the school environment into an item of personal and societal concern.

Although these students show some number sense, they do not seem keen to effectively apply specific mathematical or statistical tools and forms of expression to clarify their dilemma. Thus it is not entirely unexpected that they turn to personal-emotional (value-laden) explanations. Although Karolin’s personal-emotional argument seems to be acceptable to the group, Helen still insists on looking at the differences between the data sets pointing to the fact
that the data available covers just two consecutive years. However, it is when Karolin insinuates that previous robberies were much fewer that Helen situates the task at a point that may be perceived as invoking some mathematical/statistical reasoning, thus vindicating their final conclusion (turns 31-35). At these turns it appears an assumption is made that the number of robberies before 1998 was normal or ‘constant’. At this point the students’ interaction with the graphical artefact moves a notch higher and serves to justify their conclusion.

[31]. Helen these are values from 1998 and 1999
[32]. Karolin yes it is quite a lot!
[33]. Observer Okay
[34]. Helen the question is if it was the same …it was probably the same then
[35]. Karolin I don’t know how much it was before…it was probably less
[36]. Helen no … I don’t think that… this is what is normal. I find it hard that think it was less than 1999
[37]. Karolin it seems like …..(makes some inaudible comment)
[38]. Helen Yes

The discussion point for this group was characterised by minimal application of mathematical methods and forms of expression. The overriding argument seems to be the personal perspective of robbery in relation to the number and nature of robberies at a given duration. However, an observation made by Helen in response to Karolin’s assertion gave room for an assumption that bore elements of statistical thinking and which made the group’s conclusion reasonable and thus feasible. This assumption though not explicitly expressed, was characteristic of higher level statistical reasoning; demonstrating albeit in an obscured sense, consciousness of ideas of trend analysis.

The overall discussion was dynamic, and in terms of Curcio’s (1987) levels they successively progressed from reading the data to reading between the data and further toward reading beyond the data- they read values from the graph, made comparisons and questioned the nature of the robberies and appreciated the need to situate the data set in relation to previous years. In terms of Critical-
analytical approach vis-à-vis Identification approach, the group’s discussion was at the interface with a strong focus on the latter. Whereas they showed number sense and appreciation of statistical reasoning, it seems that their grasp of these forms of expression was not firmly grounded. It is worth noting that the final solution provided by the group might have scored an incorrect solution.

7.3.2 Discussion group II

In this group there were three boys from the Natural Science Program. The boys seemed to be getting along quite well but even here there appeared to be a respected opinion that the rest followed. Whereas this group discussed the task together, when it came to giving the solution they decided to write separate answers (see in the appendix IV).

In this discussion group the case seems to be settled when Johan says that this is a classical case of misleading diagrams, in turn 4 and 5. This is an observation that is probably based on experience with similar diagrams. It ought to be noted that at the same time Johan points out the ‘proportional’ difference in robberies for the two years—thereby pointing out what might be perceived as doubling the effect of the graph [15 and 7] then declaring that the actual difference is just 7.

[4]. Johan watch this! (aligning the apex of the bar with the axis using a pencil to highlight the gap between them) I think this is say 15 this is 7; there are 7 more robberies in a year [pause]

[5]. Johan it is this kind of…It is a classical case of misleading diagram

However, in the succeeding turns Alex seems to make a significant comment that seems to call for some caution in Johan’s use of the term ‘misleading diagram’. He rightly takes the perspective that the diagram is correct but there is something more to it. This observation can be taken as implying that the diagram is suspect.
The group seems to arrive at a consensus that the conclusion by the reporter is incorrect with Johan providing an explanation in turn 16 which has a remarkable similarity to an example from BBC (see Fig. IVa in appendix IV) which points out that the diagram is intended to give the impression that the increase is twice as much.

This group showed some sensitivity to mathematics in their expression as observed when Amis commented on Johan’s statement which the latter immediately corrected (turn 17-18). However, they probably did not find it necessary to use mathematical and statistical tools to motivate the solution. They seem to focus on the visual perception aspects of the graph (i.e. the initial appearance), this is best captured in Alex’s written response “No, the robberies increased by just 7-8 every year but the presentation makes some to think that there are twice as many robberies in 1999” (see Fig. IVd in appendix). Indeed it can be claimed that while there was some proportional reasoning, this was not really based on the number sense such as 7 out of 515 but on the graphical artefacts that is, depicting a doubling of the values.
The approach taken by this group’s interaction with the graphical artefact falls into an Identification approach in as much as the students seem to relate this graph to a class of graphs that are ‘classically misleading’. This implies that the students may be familiar with the rhetoric accompanying similar graphical artefacts from their historical and socio-cultural heritage. Based on this, the students were able to doubt the structural presentation of the data concluding that it was not a viable diagram given that it is intended to provide a misleading visual impression. Clearly the general conclusion from this group was that the statement given by the journalist was not reasonable, and thus it would yield a score for code 21. The solution from this group might also highlight some of the drawbacks of written tests. In the absence of further information, written solutions do not always provide a clear picture of the students’ comprehension of concepts.

7.3.3 Discussion group III

This group comprised of three girls from the Natural science program. Here the discussion was not as vibrant as in the other groups, save for one girl, Susanne who led the discussions. Towards the end, she invited the other members in as a way of seeking consensus. One of the girls, Petra remains quiet throughout the session. However, in the end she provides written solutions that indicate depth in understanding (see Fig. IV e in appendix IV).

This group begun by deciding on a question to work on from an array of questions.

1. Susanne: we take the first one…that is, this one (shuffling the papers)
2. Observer: which one?
3. Susanne: Robberies
4. Susanne: I think this is not …reasonable, that is the axis is broken, actually if it started at zero (making marks on the bars as if extending them to the x-axis) they’d be the same, so it is not a huge increase at all as they claim. Here it appears as if there is a double increment and that is not the case. [pause]
5. Susanne: that is what I’d say is correct
It seems as though the students in this group were generally in agreement that the diagrams had been truncated to make the difference appear larger as captured in turns 4 and 9. However, Marias comments in turn 9 cast a shadow on her actually following the discussion, a fact that is confirmed in her written response (see Fig. IVg in appendix).

The students then proceed to suggest ways of correcting the graph and go on to alter the graph (see Figs. IVe - IVg in appendix). This approach to ‘correct’ the problem with the diagram also shares some similarity to the approach used by the students in discussion group II: Susanne and Maria include the x-axis (to highlight the origin) in their diagram, without paying specific attention to the scale, while Petra just extends the bars. Clearly the correction is done to the graphs to diminish the visual effects. However, technically, it can be claimed that Petra adheres strictly to the convention of graph construction as pertaining to axis intervals. By starting from a perceived point of origin as the others had done and with the given scale, the graph would yield much longer bars than what they have reproduced.

Whereas the students questioned the production-presentation of the graph, hence indicating reading between the data and some elements of reading beyond the data, two of the students seem to miss out on one of the basic elements of graphicacy which would be perceived as reading the data that is, making appropriate graph construction. Their final solution is probably influenced by prior experience with similar illustrations, thus clouding their
critical stance. Consequently, the solution provided is perceived as based on an Identification approach emanating from a reproduction of previously encountered cases. This solution would also be scored at code 21 following PISA double-digit codes.

7.3.4 Discussion group IV

The transcript for this group defers from the other groups following the Jeffersonian transcription method. The purpose of presenting this transcription is to show the dynamics [including aspects of tempo] of the discussion that are not evident in the basic transcription methods. The group comprised of three male students; John, Tim and Peter from Aesthetic Vocational Program. At the onset John and Tim dominated the discussion with Peter having a negligible role in the group. This was not entirely unexpected given that he was considered less proficient in mathematics compared to the other two. The discussion begins with John reading the question, however, Tim seems to prefer a direct approach, that is, providing the solution directly as observed in the tempo of his turns (turn, 6, 9 12, 15). Indeed in the sequence that follows there are quite a number of overlaps and interruptions.

The discussion begins with the group deciding on the task to work with followed by John reading the task out loud.

[4]. John:  "hhh (.) = (Reads) The graph shows that there is a huge increase in the

[5]. number of robberies from nineteen ninety eight (.) to nineteen ninety-nine

[6]. Tim:  [>here i <:s >

[7]. Number of robberies<

[8]. John:  Yes (0.1) precisely (continues to reads) the number of robberies

[9]. Tim:  [Approximately

[10]. five hundred and seven, five hundred and nine
[11]. John: nineteen o eight (> yes< nineteen ninety eight it was abo(0.1) ut
seven ()
[12]. Tim: [yes,
[13]. John: five hundred and seven, five hundred and eight something > ohhh<
( ) and
[14]. like
nin- ninety- nine it was around five hundred and sixteen something
that( )o hhh
[15]. Tim: [ yes (0.1) it is-
[16]. John: >it is not that mu[ch< i thin-
[17]. Tim: >|No <(0.2) an[ (0.2)d > “the state(.)me(0.1)nt is<
there is a
[18]. huge Increase <

At the end of this sequence it seems the students are poised to conclude that the statement was not a reasonable characterisation of the state of robberies. The students discussion presented above seem to be basically based on visual inspection and number sense. The prominence of number sense is highlighted in turn 28 where John suggests that if the value was higher then maybe the increase would be considered huge.

[28]. John: >it was in the range for-()fif- twenty, thirty, forty robberies< then it would
[29]. have been different () but I don’t think it is a huge increase.
[30]. Tim: No () neither do I.
[31]. Peter: But it is:: a rath::er big: increase
[32]. John: Yes, yes but it is becau[-
[33]. Peter: >|but(().and: since it lies above five hundred
(0.1) one
[34]. has to count:: (0.2) yes
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[35]. John: Yes

[36]. precisely one has to count the number in total

[37]. Peter: so it is not that much

[38]. Peter: so(0.2) ehh:: “It is not a reasonable interpretation”

In turn 31 Peter seems to bring a new dimension to the discussion by asserting that the increase is large however, viewed in terms of the total number of robberies then it is not much [turn 31 and 33]: it is fair to claim that Peter seems to suggest that the numerical value alone might not say much in the absence of a comparison with the total. The reasoning promoted by Peter indicates some level of proportional reasoning which is disguised in everyday forms of expression. An interesting observation is made when the students are pressed further to justify their solution.

[55]. O But(0.1) Ehm:: Okay (.4) is it just that, that you believe ehm(0.1) is the

[56]. Explanation why there are:: (0.1) or are there other explanation?

In all fairness it seems the question posed by the observer [O] was taken to imply that the suggested explanation to the solution was not really plausible. It is thus not strange that the students resorted to what can be considered as a ‘standard answer’ to the task. The explanation in the following sequence seems to reveal more on the structural elements of the graphical artefact. It is also noticeable that the students do not use subject specific [mathematical] language in their explanations e.g. “if one had several numbers at the side” implying extending the y-axis

[63]. Tim: It appears of course(0.2) This bar appear larger but then it is not from the

[64]. Beginning of:

[65]. John: yes=Prec(0.2) That is(0.1) it appe-it appear larger because they have here drawn

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a little diagram (.) If it had been a (0.3) larger diagram (.) then would this(0.2)

if one had several numbers at the side so: (0.1) number of robberies

one had several to choose from then this distance would be less and then it

would also appear smaller (0.3)

While the discussion points from this group seem to follow the same trajectory as for the other groups, it is the dynamics provided by the Jeffersonian transcription method that clearly distinguishes it. Tim interrupts and overlaps John’s turn implying that the solution is obvious. While one expects John to provide some mathematical or statistical explanations at the end of the reading he seems to have a solution at hand that merely uses simple number sense such as 7 is less than 30 or 40. Indeed it is Peter who seems to make attempts at what can be considered as proportional reasoning however, probably because he is considered ‘less proficient’ he is not able to forcefully further his reasoning. At the end of the day the students seem to adopt an explanation touching on the production of the graphical artefact that is, its structural components. It is also interesting that it is Tim who readily offers this opinion which is not contested by the other students with John volunteering further explanations.

7.4 Commenting graphicacy: evaluating students dialogue on a test item

The aim of this study was to explore students’ interaction with a graphical artefact. The contents of the graph under investigation touched on aspects of everyday life as well as mirroring a case from the media. This was also typical of items considered to exemplify ‘misleading statistics’. Since this item was embedded in the topic strand statistics, it was expected that the students would employ some statistical forms of expressions or mathematical language and processes in solving the task. In exploring the students’ solutions, at least two observations were made with regard to interaction with the graphical artefact: presumably in the first instance there is the interaction with the item as a
school task and secondly as an artefact in an everyday situation. This observation can be perceived as indicating the role of contextual influences on students’ interaction with graphical artefacts (see e.g. Monteiro & Ainley, 2007; Roth 2009; Nilsson, 2009). The contextual influence can be understood as part of the triadic sign system through which the interaction with a graphical artefact is mediated semiotically by aspects of collateral experience as well as by the user’s situation within the society and culture generally (Whitson, 1997). Thus, while the focus of this study is on tool use and forms of expression, it is accepted that contextual factors might influence the use of tools as well as forms of expression.

The solutions were analysed with regards to tools and forms of expression as well as on the construct based on Critical-analytical approach to graphicy. In refocusing the double-digit rubrics according to the I/CA framework, it is revealed (see Fig 7.2) that the percentage of students providing solutions indicating content or subject knowing (codes 21, 22 and 23) and where there is an indication of the use of mathematical concepts or language were considerably low. In this study, the use of mathematics and statistics operators and concepts as well as questioning the factors associated with the graph were taken as an indication of a Critical-analytical approach. The construct Critical-analytical approach is consistent with the notion of statistical literacy perceived as constituting critical questioning, knowledge of some statistical and mathematical concepts and terminology, as well as basic communication skills (Gal, 2002; Rumsey, 2002). The importance of mathematics is also highlighted by Friel, et al (2001) who posits that mathematical knowledge and experience are among the qualities necessary for graph comprehension Thus, the low response score on codes 21, 22 and 23 [19%, 11% & 1% respectively as shown in Fig 7.2] was probably an indication of the difficulties the Swedish students had in applying a Critical-analytical approach in determining solutions.

Based on the three codes that is, code 21, 22 and 23, it is reasonable to assert that for solutions obtaining full credit three strategies were employed. Code 21 involved a visual-structural strategy with some arithmetic and where the focus was on the presentation of the diagram perceived as misrepresentative, code 22 involved the appreciation of mathematics tools of proportional reasoning [percent, fraction, ratio etc.] and code 23 involved aspects of stochastic reasoning as manifested in the mention or insinuation of the lack of trend data.
Thus evaluating the discussion groups, it can be suggested that the initial discussion in group I involved the use of visual perception strategies and the use of simple arithmetic e.g. in determining the difference in the number of robberies to be about eight. While some level of ‘proportional reasoning’ is alluded to, from the transcript [turns 15-17] it seems uncertain as to whether the numerical value of each robbery is considered per se or whether it is being related to the total number of robberies for a specific year. In group IV, there is a clearer application of proportional reasoning [turns 31-38] however, on further prodding the students readily turn to an explanation that is based on visual perception and concerns structural elements of the graphical artefact. Thus, it can be claimed that the students seem to have some degree of difficulty in appropriately relating proportional reasoning to the task. This observation follows similar findings in Australia made by Watson, Callingham and Donne (2008) where proportional reasoning seem to hinder many students from achieving higher scores in stochastic tasks. It is significant that Watson, et al., suggested that it is difficult for students to move over to the application of proportional reasoning regardless of context.

Given the documented students’ encounter with proportional reasoning, it is therefore not surprising for students to move over to other problem solving strategies and tools. In group I it is observed that when the discussion took a determined trajectory focusing on the social and personal implication of the robberies that the discussion gets into what can be considered as aspects of acceptable ‘statistical reasoning’. In the discussion between Karolin and Helen the stage is set for considering the occurrence of the robberies at hand as being exceptional from the trend hitherto. This assumption justifies their solution and the conclusion which would otherwise not be considered correct according to PISA double-digit coding. However as much as this group’s discussion brings up points that resonate with the notion of trend [thus stochastic thinking], this remains peripheral and is not really developed and used effectively to guide the group’s discussion. With uncertainty in the application of subject specific operators and forms of expression, the personal-emotional dimension then dominates the discussion and appears to form the basis for the final conclusion. In the other groups and especially group II and III, there was a clear focus on the visual-structural elements of the graphical artefacts. In
explicitly expressing their solution the students explanations seem to portray a reproduction of facts rather than a Critical-analytical approach.

From a visual-structural perspective there seems to be the general impression on the part of the student that the task is intended to hoodwink the reader. While this might not entirely be untrue, it is the reasoning provided that is in some ways wanting and probably points to elements of acquired blind spots or prejudices that hinder them from a more critical reflection of the task. The focus on data integrity [whether or not the graphical artefact is produced with the intention to deceive] is not entirely unexpected if the students are exposed to these methods in their teaching and learning environments. According to Roth, Pozzer-Ardenghi, and Han (2005), students generally adapt to certain ways of doing things by developing structured dispositions. These dispositions generate patterned perceptions and with it the field of possible patterned actions and this leads to indoctrination. Whereas developing structured disposition may be desirable, in the absence of critical sense, indoctrination may lead to acquired blind spots and prejudices. In group II, Alex [turns 6-8] seems to question the purported misleading nature of the graph, unfortunately this discussion point is not picked up on by the other students in the group. When the observer posed a question in discussion group III [turns 8-9], it is noted that Maria was not able to adequately articulate her theory or solution [turn 7]. These occurrences might be seen as indicating that some of the students in this study have acquired blind spots or prejudices that hinder them from addressing divergent thoughts or defending their solutions away from standard forms of expressions. Indoctrination occurs not only in teaching and learning environments but also through school based instructional methods. These structural dispositions can also be acquired through other learning resource materials, e.g. textbooks. Thus the ‘misleading’ dimension of a graphical artefact is to a large extent dependant on the indoctrination process – if it encourages a Critical-analytical stance or an Identification approach to the graph. Some researchers suggest (see e.g. Nilsson, 2009; Parnafes & diSessa, 2004) that one way of avoiding ‘blind spots’ is the adoption of teaching methods that expose learners to different conceptions of the learning objects. In the case of this study, for student’s focussing on the initial impression from the graph, the misleading aspect was the data presentation, while those interrogating the production of the graph, the information presented on the
Students’ narratives from graphical artefacts

graph were insufficient to make a clear judgement. This observation is perceived as highlighting the assertion by Tufte (2001) that much of contemporary thinking about statistical graphs has been preoccupied by how these can be used to deceive naive readers to the detriment of serious data analysis.

The present study sought to investigate students’ forms of expression when faced with a graphical artefact in a collaborative setting. From the observations made, it can be claimed that the students showed general awareness of graph production and the presentation facet of graphicacy. There was also an extensive use of everyday forms of expression, though this was expected given the setting of the video observation and is not entirely undesirable. It was also observed that students, albeit to a limited extent, incorporated mathematical and statistical methods and forms of expressions [such as proportional reasoning and the notion of trend] in their discussions. This is a pertinent observation since the effective use of mathematical tools and forms of expression is considered as part of statistical literacy. The findings from this study also emphasise the significance of a Critical-analytical approach with respect to graphicacy (see also Gal, 2002; Rumsey, 2002; Watson et al 2008). It was observed that students seem to uncritically reproduce solutions from similar cases thus exposing their vulnerability on slight adjustments to the tasks. Given that students are able to access diverse graphical artefacts from different sources, it is important that teaching and learning are designed to foster a Critical-analytical disposition.
7. Graphicacy: evaluating students discussing a ‘media graph’
8 METAPHORICAL DIMENSION IN GRAPHICACY

The studies presented in chapters five and seven were conducted based on the assumption that hinged on the requirements of the curriculum documents that the students taking the tests or collaborating in an assessment situation had the necessary pre-requisite knowledge to engage in the tasks at hand. The studies presented in this chapter deviates slightly from this assumption since they are a result of short lectures developed and taught by the author within the framework of an elective pre-service elementary school teacher students [these will inter-changeably be referred to as pre-service teachers and students] on-campus course in the didaktik of mathematics [generally perceived as mathematics education; see section 1.4]. The lectures were based on the topic strands geometry, statistics and probability. These were spread over a period of three days for the on-campus sessions. Each of the topic strands was allocated four hours of teacher led sessions with the rest of the time allocated for student to student interaction as well as for independent reflection. The lecture sessions were made up of a small group of pre-service teachers consisting of eleven female and two male students. This study targets a different set of actors (i.e. students and pre-service teacher students) and is perceived as a positive undertaking to enrich the overall research study environment and to provide an arena for reflection on professional learning (Watson et al, 2008). In presenting this study factors illustrating the design and aims of the lecture will be outlined and will clarify the purpose of the study viz., pre-service teachers’ interaction with graphical artefact regarding the use of subject specific tools and forms of expression at their disposal.

8.1 Rationale

The underlying principle for conducting this study was to gain some leverage concerning some of the factors necessary for effective interaction with graphical artefacts. This was in fact meant as a way of ascertaining the viability of the assumption that the students have been exposed to the necessary subject specific tools and forms of expression that might be indispensable for effective interaction with a graphical artefact. It should also be noted that these pre-service teachers having qualified for advanced studies, were assumed to have met the general knowledge requirement expected of students who have progressed through the compulsory school system. The assessment
environment following the lectures was also collaborative involving asynchronous online discussions. This implied that the students were required to write down their reflections and suggested solutions for the tasks. Unlike a normal paper and pencil test environment, the online environment allowed for comments and discussions on the contributions made as well as offline discussions which could then be posted online after possibly being refined. Thus it was expected that before the students made a contribution online, they ought to have reflected on the task and their suggested solutions. It is these written discussions that formed the basis for the present study.

The importance of communicating mathematics is well established (Borasi & Rose 1989; Clarke, Waywood, & Stephens, 1993; Emig, 1977). These researchers suggest that writing down mathematics is a learning strategy that allows for in-depth understanding of the mathematical concepts in question. According to Borasi and Rose (1989), restating concepts and rules in one’s own words can facilitate student internalisation since they are not just content to manipulate symbols successfully but strive to create their own meaning for symbols in order to express them in words and on paper. This aspect is perceived in this study as a fundamental element in the development of the Critical-analytical stance with regard to learning therefore it was deemed as having a clear pedagogic-didaktik function.

8.1.1 Research concerns

With regard to the Critical-analytical approach to interaction with graphical artefacts and given that the students had been exposed to the necessary pre-conditions for engaging the task [see section 8.2.1], it became pertinent to explore:

- How pre-service teachers use the statistical tools or operators at their disposal to make sense of the graphical artefacts.
- The nature of online conversation of the pre-service teachers while discussing the task on graphical artefacts.
8.2 Designing the study

As mentioned earlier, this study builds on a series of lectures led by the author and two other colleagues, each dealing with different topic strands. In keeping with the typical open nature of the Swedish curriculums where focus is on ‘participatory goal fulfilment’ (see Boesen, 2006), this course allowed for local interpretation and time allocation. Thus, while the lectures were traditional in keeping with the notion of ‘lecturing’, other adjustments were effected to cater for deeper engagement with the learning objects.

8.2.1 The lectures

The background information preceding the study was a series of lectures. The lecture of interest for the task at hand was on descriptive statistics and included defining statistics and the statistics process, a recap on the measure of central tendency and a mention of measures of dispersion as well as data presentation including the merits and de-merits of some of the data presentation methods for example, graphs and tables. Other aspects included in the lectures were examples on how to introduce basic statistical concepts in a teaching context as well as the identification of some of the problem areas documented in research. Some aspects of EDA were illustrated, being specific to statistics and also relevant to this study. In EDA, data is explored using graphical techniques where the focus is on meaningful investigation of data sets with multiple representations with little probability theory or inferential statistics (Bakker, 2004a; Prodromou & Pratt, 2006). A data set was used to illustrate aspects of data analysis, the benefits and disadvantages of a histogram, stem-and-leaf diagram as well a boxplot, including conversion within the diagrams. The rationale for this was inspired by the assertion that “there is no mathematical thinking without using semiotic representations to change them into other semiotic representations” (Duval, 2008 p. 39).

8.2.2 Solving the task

As mentioned earlier the outline of the lecture was basically traditional. Thus, to cater for some of the limitations of the teacher lead lectures and as a way of promoting the learning process, the open nature of the course was exploited. The students had access to an online resource, namely the FirstClass ®
[henceforth FC], an e-learning resource normally reserved for distance studies but was incorporated here even though this was an on-campus course. In availing of the online resource the ambition was to stimulate an explorative attitude towards learning, promote a critical and analytical stance, infuse creativity in learning and extend the interaction space among students as much as it was possible. As a way of promoting these ambitions, the students received three extensive tasks from the topic strands statistics and probability at the end of the teacher led session. These tasks were intended to provide the students with material that would also help them revise some of the concepts covered in the lectures. The students were randomly divided into two online discussion groups. Following the documented advantages associated with writing down mathematics in one’s own words (e.g. Borasi & Rose, 1989; Clarke, Waywood, & Stephens, 1993; Emig, 1977), a condition imposed for the online discussions was that the participants were not to use audio or video conferencing, that is, the discussions were to be predominantly written accounts.

8.2.3 The task

The task used in this study was considered as having CA potential. The item involved the comparison of the variability of examination scores for two classes presented in graphical artefacts [histogram]. The pre-service teachers were to determine which of the two groups of pupils’ scores had the largest variation. The task was deemed as appropriate as a vehicle for promoting statistical literacy and ‘statistical reasoning’, where statistical reasoning is perceived as the way people ‘reason’ with statistical ideas and make sense of statistical concepts as well as being able to explain statistical processes (Gal, 2002; Garfield, 2002; Garfield & Ben-Zvi, 2004). The task is reproduced in Fig. 8.1.
The following graphs show the distribution of exam scores for two classes. Comparing the exam scores from the two classes, one could infer that:

a) Class I has greater variability than Class II.
b) Class II has a greater variability than Class I.
c) Class I and Class II have equal variability.

Which alternative do you think is reasonable? Explain.

**Class I**

**Class II**

---

**Fig. 8.1: The assessment item - adapted from Cooper and Shore (2008)**
Although the task was of the multiple choice format type, it was evident that a superficial approach to the data might not have been fruitful. The two data sets were such that they shared similar approximate measures of centre as well as a basic measure of spread namely, the range. The most visible [striking] difference was the shape of the graph, consequently it was expected that the visual dimension would have a major influence in the students’ manner of interaction with the graphical artefact [indeed discussion on the tails of the bell shaped features would provide the simplest solution].

8.3 **Students online conversation**

Given that the students’ online interaction in FC was a type of asynchronous learning, the data generated from this interaction was re-arranged chronologically ending with the latest posts for ease of reading. Thus, while the data that is posted first would normally be below the body of the discussion charts, in presenting these discussion the first posts were made to appear first or placed at the top while the latter chats were moved to the bottom of the chat-list. In this way it is easier to follow the train of thought of the discussions. Other factors such as time and date were included to indicate the intensity of the discussion. It is also noteworthy that the students had several tasks up for discussion; comments and discussions around these tasks have been removed and appear here in the form of three dots (...). However, in some of the logs references may be made to these other tasks in the course of discussing the task at hand. The full transcript of the online logs for the task at hand is provided in appendix V.

**8.3.1 Online log for group A**

The data collected from this group seem to indicate some awareness albeit frail, of aspects of measures of dispersion on the part of the students. At the very beginning of the group discussion Eva [post number A2] mentions the two facets of variability viz., range and spread claiming that the range is the same for both histograms while the spread is larger for class II. She then proceeds in log A4 to operationalise [define] them.


Hello yes, I thought that in diagram 1 there were many in the middle
of the range while those in diagram 2 were more evenly spread, but I think the range is the same for both [diagrams].

Eva’s use of the range here is rather suspect given her expression which seems to imply that the range suspends the data rather than being an interval. The next log follows after two days when Helny not only provides another explanation of the range but also introduces the expression “cognitive difference”.

[A5] 23/11 17:03 The tasks – Helny
Hello! Eva and all members of this group!

…

The task on range.

I interpret the task: the range extends from having 50 correct up to and including 95 correct and which is apparent for both graphs…

that is the range is of the same size. Alternative c

However, if one considers the number of students: “cognitive difference” then it is class two which has the largest variation. Based on [the observation] that every column [the tabular frequencies] contains an increasing number of students. Class 2 students have many 55 and 65 correct and 85/95 correct which implies that there is a large number of students at different cognitive level.

(hence I agree with Eva)

…

Who are [the members of] in this group?

Does someone have another suggestion?

Helny
This form of expression though camouflaged in pedagogical terms is largely informal considered from a statistical perspective given that this student [Helny] is mostly making a comparison of between-groups variability of the two graphical artefacts. In this log there seems to be a connection through comparison between visual and mathematical [number sense] aspects of engaging the task. Helny’s contribution seems to go beyond providing a general pronouncement of the magnitude of observation but also makes a comparison between the bins.

At this point the group seemed to be satisfied with the online contributions, such that an attempt by the tutor [the author] to engage them to try out a conversion to a box plot, is largely overlooked. The contribution by Karin in log A9 introduces the notion of ‘distribution’ and probability [chance]. This log is perceived as a modest attempt to organise the thoughts from the group members in more statistically correct forms of expression. Her explanation on probability seems to point towards an intuitive concept of variability 

unlikeability given her concern with how much the individuals’ scores differ from each other (see Kader & Perry, 2007). However, her analysis of the distribution of the grades for the different classes reveals that this is an area that she may not be well acquainted with.

[A9] 23/11 22:10 The variation task and lottery – Karin

Variation task

Concerning the task on variation I perceive that alternative b) is the correct answer. Just like Eva and Helny said, here the distribution of the results has been more dispersed in Class II. Since most of the students are at different levels. In Class I the probability is higher that if a student asks a classmate what their grades is, then it will be 75. The interesting thing with both diagrams is that the number of grades still is very evenly distributed. Like in Class I that there are exactly as many with 55 and 95, then 65 and 85 and the rest at 75. The same applies to Class II, that the numbers are exactly evenly distributed from middle score. Interesting…Then it would be alternative c) because the variation is then actually the same in both classes, since they are “evenly” distributed. Or? Shall I still point out that I am sticking to alternative b), since there is a slightly larger distribution anyway.
Karin’s contribution above can also be perceived as treating the bins as consisting of homogeneous scores. While this observation may be perceived as indicating a shaky grasp of histograms, in this case it is reasonable to accord her the benefit of the doubt given that it is not uncommon to make this assumption when converting to a boxplot. However, her attempts at providing an explanation using statistically correct forms of expression are not really successful and so she decides she is sticking to alternative b). Her uncertainty with mathematical/statistical explanations gradually drives her to seek for consensus that is solution by acclamation.


Just as you point out on the task on variation of course it depends on how one perceives this thing, about how we view variation. Since we all interpret it in the same way, that the answer is b), so we are lucky that all interpret it in the same way. Although we are still discussing the issue of interpretation…yeah yeah, kind of confusing. What I want to highlight is that it is a question of [personal] interpretation as regard our perspective on variation. End of story.

The discussion log from this group shows that the entry point for solving the task was through an elementary measure of variability – the range. It was also evident that the students were more comfortable with informal forms of expression and were rather shy to engage in ‘advanced tools’ such as the boxplot when engaging the task. However, many group members showed some awareness of some statistical concepts, yet they were not effective in using these tools and forms of expression to explain their suggested solution. A case in point is Eva who was not able to develop the use of boxplot though she seems to suggest the need to use statistical methods to bolster the solution. Much as the students did not use analytical methods nor were they entirely critical, it is evident that they interrogated the diagrams and were reflective in as far as the visual dimensions were concerned; they also attempted to ‘reach out’ to subject specific tools and operators but were however not sufficiently confident to use them.
8.3.2 **Online log for group B**

The activities of group B were majorly characterised by individual effort. There was some comparative delay in engaging in the tasks however, after a few online exchanges the group provided some interesting thoughts in their discussion. Similar to group A the entry point for the discussion centred on the range as observed in Linn’s log in B2 where she begins by referring to another task that was included for the online discussion.

[B2] *21/11 16:11 graph of variation – Linn*

Well it would now be appropriate to get started with this one [meaning the task] then.

I have not read that Danish article [referring to one of the other tasks ] I cannot manage today. However, Johana and I looked at this one [task] on graph of variations earlier today.

Quite spontaneously, we thought that option a) was most appropriate considering that there was such a big difference between those who scored 75 and the rest.

But then one can of course consider the range [of the scores], of course it extends from 55-95 for the two classes. This could imply that it is alternative c) that is correct.

It seems the duo ‘spontaneously’ considered the visual aspects of the graph and they were tempted to conclude that class I had the largest variability. However, being aware of the range as a measure of variability they settled for option ‘c’. Petra in B4 does not seem impressed by their suggested solution and so she provides what can be considered a ‘repair’ contribution.
Hello!

I had a slightly different thought to yours, the way I see it, there is a larger variation in class II.

It became easier when I thought of it as measuring different colours on students’ clothes in two classes. Then it is clear that there is a greater variation in Class II.

Regards Petra

This solution seems to resonate with most of the group members as being ‘concrete or hands-on’. The explanation given by Petra seems to resonate with school practice too where elementary school teachers normally have to keep a watchful eye during outdoor activities. This explanation may be seen as not just referring to within class differences but leaning more towards a most likely colour to be observed from the group of students [the intuitive notion of unalikeability]. Erik in B10 generally agrees with the group’s contributions but introduces an explanation that seems to involve the mean and which he considers is ‘logical’.

Hello all!

…

Diagrams:

As regards the next task on the results of the two classes, I think even in this case it ought to be very logical that it is option B, that class II has a larger variation than class I. This I explain by that the majority in class I have obtained an average score [it is not clear if he refers to the mean] while class II has a more spread and even score. There are more in class II with a higher and lower score which also entails there being greater variation.
Now, I do not know if you have time to reflect on what I have posted since probably the online discussion group winds up soon. But if you can and have time to read through it and want to comment on and discuss, please feel free to send your views to my inbox here at FC.

See you tomorrow in the student text lesson.

With kind regards

Erik

While it may not be clear if Erik in the above log is talking about the mean, the general impression is that he makes a connection between variability and a measure of centre. While the suggested solution might be perceived as devoid of precision in the application of subject specific forms of expression, it comes close to the standard approach to variability in statistics teaching and learning involving the mean. As if building on the thoughts provided by Erik, Charlotte in B12 provides a ‘visual’ connection between the mean and dispersion in her illustration of the shotgun and pellets. Her explanation may be perceived as showing an understanding of the statistics concepts from an everyday perspective. In the case of Charlotte it seems the everyday forms of expression dominate in relation to statistical operators and forms of expression. Much as she appears weak in terms of analytical methods (statistical calculations), she does however make a plausible link between variation seen from a statistics perspective [i.e. deviation from the centre or mean] and visual display of dispersion.

[B12] 27/11 22:26 Now FINALLY I have embarked on this ... anyone able to read? – Charlotte

Hello!

Sorry that I am so late with this work. *I am ashamed* I am in the processes of sequentially completing my assignments (I still have some piled up...) and now it is time for this [the online discussion].

Since I have been away from mathematics lessons, I received assistance from my (quite humorous) husband [with the task].
Perhaps I have been too explicit in my explanation - but that is because it is too hard to just use text when one has to discuss these kinds of “problems”

The attached document “The spread in the results” is about the task with the graphs. In the document I explain how I settled for alternative B as the most correct.

…

…

I hope that you are able to read this one now [referring to her post]… I am aware that I have tendency to be verbose…

Have a good time - now I’ll have to do some abs

~*~ Charlotte ~*~

[NOTE: the explanation below was given as a separate attachment]

**The spread in the results:**

One way to discuss this is to perceive the results as pellet marks from a shotgun [aimed at] on a shooting target, where every student is assigned a pellet that hits within an area on the target. Then the tables could appear as follows:
Fig. 8.2: Charlotte's characterisation of the variability for the scores
Something that people talk about in connection with shotgun shots is the spread. This indicates how the pellets are scattered from the barrel. A narrow barrel means that one gets a little pellet spread and thus more pellets end up in the middle of the target. If one would liken the classes with a shot from a shotgun on a target then, it can be perceived that class II has a larger spread as more hits are further away from the centre of the target. If a similarity is made between variation and spread [of the pellets on the target board] then class II will have more variation than class I.

8.4 Commenting metaphorical dimension in graphicacy

This study reports on the online discussion from a group of pre-service teachers discussing a task on variation expressed graphically. The focus of the study was on the students’ use of statistical tools and forms of expression that they had been exposed to [among others from lectures taught by the author]. Thus, it was anticipated that the students would apply some of these methods to solve the task. In particular, the students were prompted to consider converting the task from a histogram to a boxplot. However, this was not picked up on; the students seemed to rely on visual strategies focusing on elementary statistical concepts [it is noteworthy that a plausible solution could also be provided by discussing the shape of the graphs] such as the range, which was not optimal. While this is consistent with the observation made by Bakker (2004b), namely that students’ generally lack the conceptual understanding for analysing data using the statistical techniques they have learnt, it might be possible that there are other underlying factors involved. That Petra’s post in B4 is considered as “concrete” is probably an indication of external influences in the problem solving process. It is possible that the pre-service elementary school teachers in this study were intent on providing a solution that would resonate with elementary school pupils, thus projecting their future role and as such, ‘complicated’ mathematical and statistical methods were not accorded priority.

While it might have been possible to manifest a Critical-analytical approach to problem solving without resorting to ‘complicated mathematical and statistical methods’, these two groups of students employed a strategy that can be
characterised as an Identification approach; there was a general application of visual strategies and elementary operators e.g. subtraction in the range and use of everyday language. As expected, this strategy did not lead to plausible explanations and yet the everyday nature of their explanations did not appear to hamper advanced discussion on statistical reasoning, in this case. This observation seems to deviate somewhat from the claim made by Bieehler (1997). While the solution given from group A seemed to have been arrived at by consensus or popular acclamation [it seems that the group members were not able to express their solution convincingly], group B provided some interesting explanations based on what can be considered as metaphoric explanations. The use of metaphorical explanations or thoughts provided room for discussing the different facets of variability. In this case the metaphorical thought brought out what has been referred to as an ‘intuitive’ notion of variability – unalikeability (Kader & Perry, 2007). The students had not been introduced to the notion of unalikeability and as such, it can be said to be a consequence of their experience. The importance of experience in the metaphor cognitive creating function is rooted in abductive reasoning since the first premise of abduction is perceptual judgement (Sørensen, Thellefsen & Moth, 2007). Thus this case shows a trajectory of the students’ interaction with the task using an Identification approach and then using metaphorical thought as a pole vault (Mladenov, 2006) to make a connection to what might be considered as advanced statistical reasoning.

In the two metaphors that is, of the clothes of different colours [henceforth T-shirt for ease of reference] and Shotgun, it is observed that there is assumed collateral experience at play. It is noteworthy that some issues need not to be discussed such as the underlying notion of the school yard where students with different colour T-shirts are to be found. In the shotgun metaphor there is a deliberate attempt to explain the shape of the shotgun barrel and its effect on the shooting range. Regarding diagrammatic reasoning it goes without saying that the students created a diagram. It ought to be noted that a diagram can be made in the mind, on paper, on a computer screen etc. (cf. Dörfler, 2004). A process that can be influenced by familiarity with tools and other experiences, (see Bakker, 2007) in this case their everyday experiences seem to have been a dominant factor. It might be that the dominance of everyday ‘methods’ over subject specific tools and forms of expression was probably occasioned by
difficulties in experimenting with the already provided graphical artefact or diagram. Given that diagrammatic reasoning calls for experimenting with a diagram, this can be said to have been actualised when the task was carried over by the metaphors. This allowed the students to ‘visualise’ the task in a different light and even allow for comparison in forms of expression. The metaphor of children with different coloured T-shirts might have appealed to the imagination e.g. picturing a group of children playing in a school yard with a teacher keeping an eye on them. This metaphor also seems to convert the task from quantitative data to involve what can be considered as categorical data. In this sense it can be claimed that the data is transformed to a level that is intuitively manageable (see Kader & Perry, 2007). In the metaphor of the shotgun, a significant experimentation act is without doubt the superimposing of the pellets on a Cartesian plane, in effect anchoring the illustration back to the task. Here the concept of variability as word was related to a ‘tangible’ illustration from the shooting range. Since the students provided these as ‘evidence’ for their solution it is reasonable to assume that they reflected on the experiment and were convinced that the results were viable.

A reflection borne out by this study is recognising the importance of metaphorical thoughts and explanation in statistics teaching and learning. While it was anticipated at the onset that these students might use subject specific tools and forms of expression in solving the task, it is the application of metaphorical thought that gave rise to new perspectives on variability.
8. Metaphorical dimension in graphicity
9 GRAPHICACY: ANALYSING STUDENTS’ WRITTEN SOLUTIONS

Generally speaking, school mathematics practice seems to accord considerable emphasise to students’ written work in the teaching and learning of mathematics. This is probably manifest in the notion of ‘doing mathematics’ which, for a large part, has been taken as involving working out mathematics tasks or problems in exercise books etc. Hence, it would be reasonable to assert that one way of exploring how students use mathematical tools and forms of expression would be in the written accounts of their mathematics problem solving activities. One such activity would no doubt include students written accounts as recorded from assessment and test environments. Within the Swedish context Nationella provet [the Swedish National test (Np)] is perceived as appropriate material for exploring students’ use of tools and forms of expression.

The importance of a study involving the use of students written solutions from Np in mathematics for grade 9 was also informed by the generally accepted assumption that education at the compulsory school levels serves to prepare the students for adulthood and, by extension effective citizenship and also for further studies. Based on the latter, Np may be considered as a high stake test implying a directed level of preparedness on the part of the students and teachers prior to taking the test. Thus, unlike the previously presented studies which were not entirely obligatory, it is expected that students do not approach Np in a casual manner. Since findings from the previous studies indicated that there was near absence of Critical-analytical approach with regard to interaction with graphical artefacts, it became interesting from a research perspective to examine the same in the framework of a high stake test such as Np. Also unlike the PISA survey results, this study explored actual written solutions from a sub sample of Swedish students from the Np. The study is presented by first providing a table characterising [in terms of students’ scores] the selected items from Np. This is then followed with a more detailed analysis of one of the items with respect to tool use and forms of expression. This analysis is based on the understanding that students use of tools and forms vary and have different consequences on the final outcome of interaction with graphical artefacts [see sections 3.4.2, 3.5, and chapter six].
9.1 Rationale

While national testing in Sweden is designed to serve the purposes of supporting teachers in assessment and grading students in relation to the curriculum goals and thereby assisting in ensuring national equity [fairness] in assessment (Pettersson et. al., 2010), generally national testing in Europe serves a vital role in the evaluation of educational systems, informing educational reforms and policy implementation, as well as offering a local comparative instrument (Eurydice, 2009). By this it is implied that the results from the national examinations may be used to explore the effects of curriculum documents on teaching and learning, compare the performance of students from the different segments of the population, informing policy makers etc. Some researchers, (e.g. Boesen, 2006) have suggested that centrally administered tests can influence instructional practice dictating, for example, how much time is set aside to topic strands such as statistics, geometry and algebra. National tests might also be used to explore the effect of instruction and has the potential of providing insight on students’ fluency with mathematical concepts and tools. However, examinations might also be perceived as situated in an abstract context in relation to some tasks especially the so called authentic tasks, such as tasks that are perceived to mirror everyday realistic situations. Thus, one of the many ways of utilizing examination performance would be to perceive them as indicators of students’ potential to operate with given subject specific tools and forms of expression at some level of abstraction. While this study is not entirely concerned with assessment issues per se, some of the assessment tenets mentioned above are deemed appropriate in the evaluation of students’ graphicacy and the environments informing the same. From a research perspective, this study also provided an opportunity to illuminate aspects of the proposed framework involving Identification contra Critical-analytical approach to interaction with graphical artefacts.

9.1.1 Research concerns

Coming from the background where, in the previously presented studies, there was a comparative scarcity of students’ written solutions and thus mathematical computations, this study provides the opportunity to closely examine students’ written solutions. The aim being to explore the way in which
students interact with graphical artefacts and, with special focus on identifiable tools and forms of expression from their written solutions, to task items from Np. In this regard it is intended to outline the most dominant or accessible tool or forms of expression used by students when solving a selected task and further, to analyse if some insights can be gained regarding strategies while interacting with graphical artefacts. This analysis is largely going to be based on the Identification contra Critical-analytical framework (I/CA). Thus the concern of this study is:

- What insights into the dynamics of tool use and forms of expression applied are revealed using the suggested classification framework in students’ interaction with graphical artefacts?

### 9.2 Empirical matters

Nine tasks containing relevant graphical artefacts were selected from the Swedish national test for the years 2007, 2008 and 2009 [see the appendix II for some of the items]. These were then classified based on the pre-determined framework of analysis that was presented in chapter six, that is, whether they elicited potential for I or CA approach. The same procedure was conducted by two independent expert researchers in science education and mathematics education respectively. There was general agreement in the classification done independently by the researchers even though each took a different approach in applying the proposed framework. The first approach involved looking at the overall complexity of the item vis à vis the proposed framework, while the other approach involved identifying the different ‘characteristics’ satisfied by the items based on the proposed framework. The minor differences noted, especially on two items, were agreed upon after some discussion and thus requirement on explanation and justification of solution was given prominence over other solution factors. Consequently, these items were classified as having potential for CA approach. The classification process characterised the items as shown in Table 9.1.
Table 9.1: Categorisation of items based on approaches to interaction with graphical artefacts including students’ p-values. The task numbers given in the second column are not reflective of the task Id as provided in the national test.

<table>
<thead>
<tr>
<th>Year</th>
<th>Task No.</th>
<th>p-value</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>1</td>
<td>0.62</td>
<td>CA</td>
</tr>
<tr>
<td></td>
<td>2 a</td>
<td>0.94</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>2 b</td>
<td>0.20</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>3 a</td>
<td></td>
<td>CA</td>
</tr>
<tr>
<td></td>
<td>3 b</td>
<td>0.51</td>
<td>CA</td>
</tr>
<tr>
<td></td>
<td>3 c</td>
<td></td>
<td>CA</td>
</tr>
<tr>
<td>2008</td>
<td>4</td>
<td>0.63</td>
<td>CA</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.74</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>6 a</td>
<td>0.75</td>
<td>CA</td>
</tr>
<tr>
<td></td>
<td>6 b</td>
<td>0.22</td>
<td>CA</td>
</tr>
<tr>
<td>2009</td>
<td>7 a</td>
<td>0.92</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>7 b</td>
<td>0.37</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>8 a</td>
<td></td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>8 b</td>
<td>0.52</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>8 c</td>
<td></td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>8 d</td>
<td></td>
<td>CA</td>
</tr>
<tr>
<td></td>
<td>9 a</td>
<td>0.78</td>
<td>CA</td>
</tr>
<tr>
<td></td>
<td>9 b</td>
<td>0.51</td>
<td>CA</td>
</tr>
</tbody>
</table>

From Table 9.1 it is observed that items classified as having potential for CA but which were conducted orally, that is items from section A of Np [i.e. tasks 1 & 4], seem to score relatively better with success rate in the years 2007 and 2008 being greater than 60 %. It is noteworthy that the oral examinations or testing were conducted in a collaborative environment which can be perceived as being flexible allowing for students opportunity to express themselves more effectively. A collaborative setting not only provides a linguistic advantage but can also be perceived as allowing for some level of peer scaffolding. Items 6a and 9a also recorded relatively higher success rate scores even though they required students to provide written explanations to their solutions. It is noteworthy that the mathematical tools needed to solve these items were simple arithmetic and some aspect of elementary spatial sense for 6a and 9a respectively. Items 2b and 7b considered as Identification type based on the
demand on elementary graph skills for item 2b [cf. Shaughnessy, 2007] and simple arithmetic, recorded low success rate score: some skills with coordinate systems were required for these items.

9.3 Method

As mentioned earlier, nine tasks formed the basis of the present study. Out of these a question item – task 9 from year 2009 [see Fig. 9.1] was selected for further analysis. The factor informing this decision was based on the nature of the task as well as the general pattern of students’ success scores. With regard to the performance pattern, while the task was classified as having potential for CA on account of it requiring explanations on a suggested solution, there were comparatively more students who provided correct response for part a of the task (78%) compared to those with a correct score for part b of the task (51%). Also, save for the framing of the question, this task would have passed as a case of a misleading graph. With permission from PRIM gruppen – Stockholm University, the students’ written solutions were then made available to the author for purposes of analysis. In the analysis of the written solutions special focus was placed on what can be considered as the presence of dominant tool use. Based on the discussion in chapter five, attention was given to the use of tools such as proportional reasoning, fraction, etc. on the one hand and visual perceptive approach, the use of elementary arithmetic procedures etc., on the other hand.
9. Analysing students’ written solution from Nationella provet

The diagrams show how the medals were distributed for some different countries at the Olympics in Peking in 2008. Determine whether each of the claims made below is true or false. Explain your reasoning.

![Diagram of medal distribution for Australia, Spain, and Great Britain]

a) Great Britain won more gold medals than Australia. (1/0)

b) Spain won more silver medals than Australia. (0/1)

Fig. 9.1: A task having a visual-perception orientation but which also requires some mathematics ideas for effective interaction with the graphical artefact.

As alluded to earlier, the motivating factor for selecting this item was hinged on the observation that it provided an opportunity to combine visual perceptual with ‘Critical-analytical’ components. Whereas for part a it was entirely possible to successfully interact using a visual perceptive approach [spatial sense], the second part that is, part b might require more than the application of these visual perceptive faculties. The students’ solutions explored were selected at random from a number of responses from Np. These were then classified for purposes of illustration with respect to tools and forms of expressions. A total of 115 written solutions [a sub sample] were randomly selected from a sample of 2544 students’ solutions.

9.3.1 Classifying forms of expression

The 115 students’ solutions were then classified with respect to two instances. Firstly, four categories of forms of expression, that is, whether the students used mathematical, everyday, graphical or short answers namely, responses devoid of explanation contrary to the requirement of the item. It is noteworthy that in the present study the phrase ‘everyday’ refers in principle, to a solution that includes an integral part of everyday exchange (Sfard, 2008), which is descriptive in nature and where the descriptive aspect is given prominence as
Students’ narratives from graphical artefact

contrasted to a solution involving mathematics symbols, symbolic manipulation and calculations etc.

The following figure illustrates the type of solutions from the four categories outlined above [only for solution giving correct responses].

1. Mathematics

![Mathematics Solution](image)

Fig. 9.2: A student's solution showing use of mathematics form of expression

This solution shows some understanding and use of mathematical tools and forms of expression. Here, there is the use of fraction, percent and multiplication, the solution also compares values. There an extended use of approximation as well as an accompanying explanation. It is important to note that this solution shows the application of several tools; an observation that can help in understanding the numerical value behind Figs. 9.8, 9.9 and 9.10
2. Everyday

![Image of a student's solution showing use of everyday form of expression](image)

**Fig. 9.3: A student's solution showing use of everyday form of expression**

While this solution passes as using everyday forms of expression it is the case that there is some mathematics involved: for example there is the comparison of area [spatial sense] etc. Thus, it is important to emphasise that everyday form merely denotes the dominant form of expression and not necessary that a solution is devoid of mathematics.

3. Short answer

![Image of a student's solution providing a short answer for 9a](image)

**Fig. 9.4: A student's solution providing a short answer for 9a**
This is a typical short answer solution. It might be deemed controversial to award credit for this solution but it can only be speculation that this was largely influenced by the response given in part b) of the question.

4. Graphical

![Image of a student's solution showing use of graphical form of expression]

**Fig. 9.5: A student's solution showing use of graphical form of expression**

This solution seems to have been mainly focused on the visual perceptual aspect. This is evident in the way the pie-charts have been drawn to highlight the difference in the shadings of the gold sector for the Great Britain and Australia.

9.3.2 Classifying tool use

In the second instance, the 115 students’ solutions were also classified with regards to *tools used*. Here consideration was given to mathematical operators and symbols for example multiplication, division, percentages etc. These were recorded as ‘used or potentially used’ when mentioned or written and not necessarily on actual application. Other solutions were considered as manifesting aspects of visual comparison.

With regard to tools use, seven approaches to solving the task were observed. These were labelled as *i) Visual comparison*, where there was a focus on
reporting what is considered as a strategy largely based on visual perceptions for example, when a solution is reported with words like “it is evident that the gold sector for Great Britain is larger than that of Australia” ii) *Fraction*, this is when the solution contained fraction symbols for example 1/3 or expressed in words such as “about half of Spain’s medals were silver” this approach was also used for solutions containing iii) when students use words such as the area is twice as much as iv) *Percentage* this was considered when the word percentage or the sign was used in the solution v) *Critical*, here recognition was given to such solutions which indicated a ‘questioning’ of the production of the graphical artefact for example, when a solution included explanations such as “the area containing silver for Spain appears larger however ...” vi) *Multiplication* and vii) *Division*. Other reported strategies and tools that fell outside these seven approaches were considered as miscellaneous [others].

Fig. 9.6: An illustration of a student’s solution showing the application of fraction and percentage tools in interaction with the graphical item.

Fig. 9.6 shows a student’s written solution. The general impression created by this solution is that the students used mathematical forms of expression: there is a formula probably indicating division of a whole into parts or just defining a fraction, in this case the 33% might thus be perceived as a result of this division operation [it is noteworthy that the claim that Great Britain took 8/10 gold medals as indicated in Fig. 9.6 is disputable]. However, since this occurrence can only be inferred, the solution is recorded as applying or reported using % and fraction and not division. It is worth noting that at this
level there is a thin line between division, ratio and fraction. [For further discussion on this see e.g. Pantziara & Philippou, 2011]. The solution given in part b – is considered as a short answer though it might be the case that this student considered it as a continuation of a ‘discourse’ started in part a: Therefore, this task scores full credit on item a but unfortunately yields no credit on item b in accordance with the official marking scheme. Fig. 9.7 also shows item b of another student solution which is classified as using everyday form of expression [it is noteworthy that everyday forms of expression do not necessarily mean they are devoid of mathematical reasoning].

![Image of a student's solution](image)

**Fig. 9.7:** An illustration of a student’s solution showing an approach characterised with application of a more explanatory strategy using everyday forms of expression.

In this case it is evident that this particular student resorted to explanation that was based on observable aspects of the item: there was a use of everyday language, visual comparison. The one aspect that is also alluded to in this written solution is the element of questioning the production of the graphical artefact as supported by the opening statement “one can be led to believe it is true...” Thus this solution is also perceived as indicating some level of critical questioning. However, this solution yielded only partial credit.
9.4 Results from exploring use of tools and forms of expression

From the general success rate provided in Table 9.1 it was observed that a relatively high number of students had no difficulties with item 9a of the task, recording a success rate of 78%. However, for item 9b, the second part of the task, only about half the number of students in the sub sample were able to provide acceptable solution as indicated by the success rate that was pegged at 51%. For item 9b, the students might have had to display some knowledge or appreciation of division, multiplication, proportional reasoning etc. To get a picture of the strategies employed or insinuated by the students, as well as the forms of expression employed, we provide a breakdown for this based on the classification procedure for the students’ solution outlined in section 9.3.1. (see Table 9.2). This table shows the forms of expression used in reporting the solution for item 9a.

Table 9.2: Forms of expressions used in interacting with the graphical artefact given in item 9a

<table>
<thead>
<tr>
<th>Form of expression</th>
<th>Everyday</th>
<th>Maths</th>
<th>Short answers</th>
<th>Graphical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct responses</td>
<td>89</td>
<td>95</td>
<td>32</td>
<td>100</td>
</tr>
<tr>
<td>Incorrect responses</td>
<td>11</td>
<td>0</td>
<td>60</td>
<td>0</td>
</tr>
<tr>
<td>Non credit awards</td>
<td>0</td>
<td>5</td>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: values for forms of expression are % of subsample (n=115). The values for scores award are based on % for respective forms of expression.

From Table 9.2 it can be observed that for item 9a a large number of students responded with what was considered as everyday form of expression, with up to 89% scoring full credit or providing what was considered correct responses, leaving only 11% providing incorrect solutions. Although only 15% of the students provided a solution that was perceived as mathematical, it seems that a majority of these students also received full credit for their solution. Given the demands on the item, to provide explanation, it is not surprising that of those providing incorrect solutions a majority was recorded as giving short answer solutions.
A look at the strategies and tools preferred by the students is given in Fig. 9.8.

![Bar chart showing distribution of tool preference and strategies used for solving item 9a](chart.png)

**Fig. 9.8: Distribution of tools of preference and strategies used for solving item 9a**

In terms of the subject specific tools used in interacting with the graphical artefacts, it was observed that quite a large number of students’ preferred a visual approach involving using aspects of the pie graph such as touching on size of its sectors. While Fig 9.8 shows some students making use of the mathematics tools of fractions, percentages and division, the comparatively higher number of students solution with visual comparison is perceived as an indicator that there was not as much concerted effort to provide explanations using mathematical tools and forms of expression.

The general pattern of forms of expression was basically the same for item 9b as for item 9a (see Table 9.3) except for the notable differences in percentage levels. Here about 50 % of the students preferred to use everyday forms of expression in dealing with the item, with about 30 % choosing what was considered as mathematical form of expression. The effects of using a ‘mathematical’ strategy was clearly evident here with about 95 % of those
students scoring full credit compared to only 29% of those taking an everyday approach. Also it is observed that apart from the students providing short answers, the students using graphical strategies had a rather higher rate of incorrect responses.

Table 9.3: Forms of expression used while interacting with graphical artefact given in item 9b

<table>
<thead>
<tr>
<th>Form of expression</th>
<th>Everyday</th>
<th>Maths</th>
<th>Short answers</th>
<th>Graphical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full credit</td>
<td>50</td>
<td>32</td>
<td>13</td>
<td>4</td>
</tr>
<tr>
<td>Partial Credit</td>
<td>29</td>
<td>91</td>
<td>0</td>
<td>25</td>
</tr>
<tr>
<td>Incorrect responses</td>
<td>43</td>
<td>4</td>
<td>13</td>
<td>0</td>
</tr>
<tr>
<td>Non credit award</td>
<td>27</td>
<td>2</td>
<td>79</td>
<td>75</td>
</tr>
</tbody>
</table>

Note: values for forms of expression are % of subsample (n=113). The values for scores award are based on % for respective forms of expression.

The low success rate reported for 9b might be attributed to the demand of the task that seemed to require the need for the students to apply some subject specific tools in effective interaction with the graphical artefact. Following the results shown in Fig. 9.9 it can be claimed that there were a relatively large number of students using fractions, division and percentages in solving the task. There is also some indication that while mathematics tools and forms of expression is appreciated in performing the task there is still some element of visual-perceptual influence.
For item 9b, and based on the distribution of use of mathematics tools and forms of expression, it seems a relative larger number of students engaged in some form of calculation on the total number of medals for the different countries. This was expected given the demands imposed by the task. Of the tools used in reporting and calculating the solution, fractions, division and percentages were prominent, in that order. Though this item would be perceived as having the characteristics of a misleading diagram, only a handful of students gave results that indicated that they questioned the production of the artefact, is here indicated in the bar as ‘critical’. However, it is noteworthy that this was not a demand of the task. An analysis based on the quality of credit awarded also revealed a number of differences regarding the level and quality of interaction with the graphical artefact (see Fig. 9.10).
Fig. 9.10: Comparison of the frequency of students using the different strategies, tools and forms of expression based on credit awarded for solutions to item 9b

From Fig. 9.10 it is evident that most of the solutions scoring full credit involved some manipulation of mathematical symbols [mathematical calculation] and where fraction and division seem to have been the prominent tools and forms of expression. For the solutions providing incorrect responses and partial credit, a majority of students’ solution strategies seem to involve visual comparison. The solutions providing incorrect responses also seem to indicate difficulties with fraction and percentages. Similar to the solutions yielding no credits, the solutions scoring partial credits also indicated minimal use of mathematical tools and seemed to rely more on aspects easily accessible from the graphical artefact.
9.5 Comments on students solutions from *Nationella provet*

It is recommended that students need to be exposed to relevant subject specific tools and forms of expression with the goal of enforcing effective interaction with graphical artefact (NCTM, 2000; Skolverket, 2002; 2009). However, the observations from student solutions used in the present study, especially regarding item 9b, show some preference for strategies that are visual perceptual oriented. In Fig. 9.10 it can be observed that compared to the students who scored full credit, a relatively large number of students scoring partial credits and those giving incorrect responses, favoured ‘visual comparison’. The results for those scoring full credit probably emphasises the importance of being able to effectively apply subject specific tools and forms of expression in interaction with graphical artefacts. It was observed that most of the students’ scoring full credit also seemed to report solutions and use strategies involving mathematical tools such as division and fractions. It can also be observed that quite a number of students giving incorrect responses used the mathematical tools fraction and percentages [for this item, as mentioned earlier, visual comparison would not suffice]. This is probably indicative that these could be challenging tools for them to manipulate or work with in interaction with graphical artefacts.

It was mentioned earlier that the task was generally classified as having potential for CA. However, it was observed that 78% of the students managed to obtain correct scores for part a. While this observation was not entirely expected by the definition of CA, in exploring the use of tools and forms of expression this study reveals that a simple demand such as the need to provide explanation to a solution [as was the case for task 9], might not solely determine the level of engagement with a graphical artefact. This seems to suggest that tool use may have more influence on ability to interact with a graphical artefact to the extent that when the appropriate tool of choice is used, explanation might not necessarily be that challenging. Thus for items 6a and 9a the relatively high success rate may be attributed to the simplicity of tool use – simple arithmetic [addition] and spatial sense for items 6a and 9a respectively.

It seems that the students participating in the test favoured forms of expression that can be referred to as everyday forms of expression [see Tables
9. Analysing students’ written solution from *Nationella provet*

9.2 & 9.3]. While this might suffice in some cases, it should be considered that in the long run they might not provide stable solutions. Probably the preference for everyday forms of expression might explain some of the not so encouraging success rates for items explicitly referring to subject specific forms of expressions e.g. calculation of median [items 6b and 7b] and operations in the coordinate system e.g. item 2b and to some extent item 7b. Thus the present study can also be perceived as highlighting one of the strengths of the I/CA framework namely, that it can help in identifying the areas of strengths and weaknesses in students’ written work.

The I/CA framework has the potential of providing valuable indicators for developing or improving classroom practice. This is more important with regards to fostering the Critical-analytical stance. In characterizing Critical-analytical approach, the importance of being able to select appropriate tools and forms of expression is paramount. Thus, while the general conclusion from the present study would be that students that are well versed with mathematical tools and forms of expression performed relatively better than the others, it is also the case that given the task at hand it was observed that a number of students may need help for example, in the use of proportional reasoning and the coordinate system while interacting with graphical artefacts. The observations made in this study seem to follow findings from other studies (e.g. Callingham, Watson, & Burgess, 2012; Noelting, 1980; Watson, Callingham & Donne, 2008) which indicate that students might have problems interacting with graphical artefacts where proportional reasoning is a prerequisite.
10 SUMMARY AND DISCUSSION

The aim of this dissertation has been to provide a researched account characterising the ways in which students interact with graphical artefacts. The interest in exploring this aspect finds justification in the fact that graphical artefacts pervade our world and at the same time it seems that research is yet to establish exactly what it takes to make sense of graphical artefacts (Roth, 2009). Some researchers (e.g. Wu & Krajcik, 2006a) posit that little is known about how students progressively develop their inscriptional practices [graphicacy] or the features of the learning environment that support them in this process. This notwithstanding the observation that graphicacy is a well researched area with its own theories of graphing practice as exemplified by the works of Playfair, Bertin, Tufte among others.

The present dissertation can therefore be regarded as a contribution into the understanding of and a research into the characterisation of making sense of graphical artefacts. It was shown in sections 2.3.3 and 3.2 that a number of researchers investigating this area have, to a large extent, relied on a constructivist theoretical approach where cognition is perceived to ‘reside in the psyche’. Thus, research using other theoretical perspectives would greatly contribute to the corpus of the current knowledge base in interaction with graphical artefacts. The approach taken here focuses on ‘external’ aspects viz., on the means used by students in the sense making process, and in particular the tools and forms of expression that emanate from the educational environment and how these are applied when confronted by graphical artefacts. The historical review of graphing practice provided in section 1.1 was intended to compare graphing practice in the past with contemporary society as well as providing insight into the historical role of tools and forms of expression in graphicacy [graphing practice]. From the historical account in chapter one, it was shown that there has, apparently, been a concerted effort to make use of the visual perceptual faculties in graphing practice; the intended aim of the graphical artefact was to be made clear and visible. It seems this aspect of graphing practice is still prominent with regards to interpretational interaction with graphical artefacts. It is the case that a majority of graphical artefacts intended for societal consumption tend to appeal to the visual
perceptive faculties. It would be envisaged that the dynamic nature of modern society, as well as the increasing complicated nature of the tools that can be accessed with ease, might be reflected in the nature of contemporary graphing practice. Easy access to diverse material tools etc. has heralded a practice where it is not uncommon for raw data to be made freely available for private analysis and this calls for the need to approach graphing with a Critical-analytical demeanour.

This chapter addresses the research observations from the four studies in the process delineating the concerns of the dissertation and its general utility to the didaktik of mathematics and statistics. The first concern was on the strategies used by students, where strategy is perceived more as a dispositional rather than a set of routine methods, and their approach to graphicacy. By this is implied not only the attitude that students bring with them when interacting with graphical artefacts but more importantly, the orientation of this interaction. The second concern was specific on the application of subject specific tools and forms of expression in interaction with graphical artefacts. These concerns have been divided into four themes guiding the overall discussion. The four themes are graphicacy and tool use, graphicacy in context, categorising graphicacy and lastly, curriculum documents and graphicacy. These themes are covered in depth in the different chapters of this dissertation, for instance chapter five launched the idea of categorising graphicacy with respect to tools and forms of expression; this was a slight departure from the common method of categorising with respect to the type of question posed. Chapters seven and eight dealt with graphicacy in context that is to say, public and professional contexts. Tool use was not only more pronounced in chapter nine but was the general theme that was also touched upon in chapters five, seven, and eight. This chapter ends with a mention of some of the limitations of the study as well as suggestions to future research.

10.1 Graphicacy and tool use

With regard to graphicacy and the expression ‘tool use’ it is prudent to clarify that this assumes two prominent dimensions: firstly, tools as used in the production of the graphical artefacts and second, tools used to interact with and facilitate the sense making process. The former includes such tools as might be used in designing graphical artefacts including the competences thereof and the
Students’ narratives from graphical artefact

latter might include a variety of tools based on the context and experiences attached to the graphical artefact at hand. While cognisant of the fact that in some cases knowledge of the methods used in the production of a graphical artefact might be integral to the sense making process (see Hoffmann, 2011), this study focused on students’ interpretation of graphical artefacts produced by others that is, readily existing graphics (Friel et al, 2001). To achieve this and considering the context wherein the study was situated it was natural to turn to evaluation and testing documents such as the PISA survey test and the Swedish National Test (chapters five & nine). The basic assumption of situating a study within the school environment is that the research subjects are fully prepared and equipped with the prerequisites needed to accomplish that which is required of them. This assumption finds legitimacy in the requirements and recommendations listed in the policy documents (curriculum & syllabuses). An observation that implies that the findings presented in this dissertation should be evaluated in the light of the contents of these policy documents.

Exploration of students’ results from the PISA survey indicated that the students’ interaction with graphical artefacts would, based on the performance scores, be characterised into two groups, these have been referred to as performance due to items requiring Critical-analytical versus those requiring Identification approaches. It os probably not unexpected that performance scores would be characterised into two groups given that it is the nature of assessment that some items will be challenging for most students while others might be experienced as rather simple to deal with. Thus in the context of this dissertation major factors contributing to the challenging nature of test items are perceived as: ease or otherwise of access to the subject specific tools, as well as, what can be considered as fluency in using these tools in interacting with the graphical artefacts. Though this might be considered as an obvious observation, research into interaction with graphical artefacts seems to focus on the visual perceptive or structural elements (Lowrie & Diezmann, 2007; MaxKinnlay, 1986; Tufto, 2001) and the type of questions posed (Bertin, 1983; Friel et al, 2001; Gal 2002). Thus, when an analysis of tool use was done using the Np, it was observed that the results generally corresponded with the expectations from the tools involved, in other words, when the task went beyond reliance of visual perceptive factors not as many students were able to
score full credit. This was not an isolated instance but a general observation in the groups formed from PISA analysis [e.g. section 7.3], for example it was observed that students score for some tasks requiring the application of mental rotations, the use of ratio and proportion were relatively lower (see also Watson, Callingham, & Donne, 2008). However, in instances where the tasks contained some ‘scaffolding element’, the success rate was comparatively higher. Thus, it is reasonable to assert that scaffolding aspects of a task such as multiple choices and oral [discussion] tasks ease access to or lighten the burden of applying subject specific tools and forms of expression. Indeed, these can be perceived as forming ‘a network of signs’ guiding the way the in which graphical artefact is to be interpreted. Thus the observation made from this study supports the graphicacy model put forward by Gillan and Lewis (1994) which incorporates some subject specific tools, albeit at a ‘mental’ level viz., addition and subtraction, implying such tools as could be applied without difficulty. However, the present study goes a step further to a micro analysis were subject specific tools are identified including their influence on effective graphicacy. In chapter eight, it is observed that students were not fluent in using relevant [appropriate] subject specific tools and forms of expression attached to variability expressed graphically, however, even in these cases there was the invoking of metaphorical thoughts: a tool that seems to have the potential of opening up new ways of understanding variability. Thus, it can be claimed that the students chose to use those approaches that were close at hand, with most of these tending to be of the Identification type.

10.2 Graphicacy in context

The PISA survey test, while providing an entry point to the research study covered in this dissertation, could but only provide a limited range of students’ solution methods. To broaden the score of students solution methods, other manifestations [methods covering students’ solutions] of the same were employed such as the double digit code provided by PISA, the inclusion of discussion groups and diversifying to other contexts other than those associated with the PISA survey test. The discussion sessions provided more opportunity to capture the depth of students’ strategies of interaction with graphical artefacts providing flexibility to restructure and readjust their explanations (see Hoffmann, 2011). This is vividly illustrated in a discussion involving a PISA survey test task [see section 5.2.1]: the final conclusion of the
Students’ narratives from graphical artefact

group discussion would not have received any credit based on PISA credit award criterion however, based on the nature of the discussion from the group, coupled with a pragmatic outlook, it would be reasonable to award credit based on the exhaustive nature of the discussion incorporating aspects of statistical thinking. The positive effect of allowing for an oral discussion of tasks was also observed with some of the items from Np which, while posing a potential challenge to students, provided relatively higher score; a factor that can be attributed to the oral-discussion items.

The inclusion of an oral-discussion method of data collection allowed, in part, for the exploration of the extent to which mathematical forms of expression was incorporated in verbal discussion. However, it was observed that, the effect of ‘everyday’ forms of expression was predominant in the students’ discussions. For example in section 7.3.1 it was observed that personal-emotional aspects were a prominent factor, this was probably due to the context of the question which was largely based on the public sphere but with a personal touch. The solution suggestions and forms of explanations provided by the pre-service teachers seem to have in a way been somewhat influenced by a desire for a ‘pedagogical explanation’ – the [metaphorical] explanation provided using students with different T-shirts was endorsed as being ‘concrete’ probably in contrast to the statistical methods and explanations:

That students seemed to shy from use of subject specific forms of expression was consistent with the findings from both PISA survey test using double-digit codes and from the Swedish National Test. Thus, from the perspective of the study it is possible to posit that the students’ solution was dominated by everyday forms of expression with very minimal subject specific forms of expression. This is not unexpected given that from a semiotics view point, contexts may form part of a network of signs that influences the modes of interaction with graphical artefacts. From a semiotics – ‘artefactual’ perspective on cognition it is not entirely unexpected that students use everyday forms of expression in interaction with graphical artefacts: it is natural that when faced with a task that needs to be executed that one resorts to those tools that are easily available and those that one is familiar with. This was also observed when the pre-service teachers resorted to other means of explaining their solutions, abandoning the subject specific tools that they had access to through the lectures and which might have provided a more robust explanation. The
10. Summary and discussion

use of non subject specific tools and forms of expression were not only limited to instances of interpersonal interaction, but was also noticeable in written tests. In section 5.4.2 a task from the Swedish National Test [item 9a and b] that, though perceived as having potential to be challenging to the students, registered some relatively encouraging success levels pegged at 78% for part a and about 50% for part b. A closer analysis of the tools used by the students revealed that for item a, about 72% of the students used everyday forms of expression and 89% of these received correct score awards, while only 15% used mathematical tools and forms of expression and 95% of these scored correctly. This feat was not entirely replicated in part b where of the 50% using everyday forms of expression only 29% received full credits while of the 32% using mathematical tools and forms of expression 91% received full credit awards. Thus, it is reasonable to posit that these observations show the robustness of mathematical forms of expression in interacting with graphical artefacts. The observation that students seem to apply less subject specific tools and forms of expression, seem to follow the claims by Bakker (2004b) that students generally lack the conceptual understanding for analysing data by the tools they have learnt. However, it cannot be categorically stated that students turn strictly to everyday forms of expression, indeed Bakker (2004b) refers to a case where students are provided by a set of techniques which were applied both unreflectively and non critically. In the case of discussing PISA items, it is observed that students provided solutions that indicated a similar train of thought. The approach associated with these solutions would, without loss of generality, be considered as not being occasioned by a reflective or Critical-analytical stance to the task. This non Critical-analytical interaction with a graphical artefact seems to indicate acquired blind spots or indoctrination that prompt the students to react to some structured disposition when dealing with the tasks (Roth, et al, 2005). It is noteworthy that indoctrination does not only occur in the teaching and learning environment through e.g. teacher led instructions but can also be acquired through other learning resource materials, e.g. textbooks. The observations made in section 5.2 seem to mirror the explanation given in appendix IV a. This occurrence might be an indicator that students have been structured into particular ways of engaging with graphical artefacts which might primarily include focusing on the physical features to the detriment of a Critical-analytical approach to data. This indoctrination contradicts the view of learning as open-mindedness and
whose outcome is not necessarily the construction, reproduction or mastery of
concepts (see Radford, 2008b).

10.3 Categorising graphicacy

The findings from the studies presented in this dissertation seem to suggest a
refocus of attention to the characterisation of interaction with graphical
artefacts. Indeed this is a major contribution of this dissertation: the intention
being to encourage a categorisation that promotes a Critical-analytical
approach to graphicacy. ‘Categories’, as used in relation to Identification and
Critical-analytical approach to graphicacy, take on both a mundane as well as a
theoretical meaning. Thus, the categories can be regarded as ‘classes’ of
interaction with graphical artefacts in other words, as a way of talking about
graphicacy and also to be regarded as part of semiotics as shown in section 6.2.

In general, and as mentioned earlier, results from students’ interaction with
graphical artefacts might be characterized as reflecting ‘difficult’ as well as
‘easy’ tasks. While such characterisation might appear simplistic, a more
nuanced description is found in linking this to potential tools and forms of
expressions attached to these tasks. The idea being that it is the access and use
of appropriate tools that is perceived as a major factor influencing the
observed outcomes from interaction with graphical artefacts in a test or
assessment situation. While a perspective based on how difficult the task is
with respect to tools and forms of expression might stand out as rather
intuitive for characterising test results, the dominant perspective on interaction
with graphical artefacts does not seem explicitly embrace this, subsuming it in
general terms. In the study conducted by Curcio (1987) recognition is
appropriately given to the role of prior experience and its influence on the
nature of the outcome of interaction with graphical artefact. This fits into the
theoretical importance of collateral experience as understood and applied in
the present dissertation. However, Curcio analyses her work from schemata
theoretical perspective which basically suggests that the ability to interact with
graphical artefact is a function of schemata. Thus, the outcome of the
interaction based on this interpretation may be perceived as reflecting the
‘content or composition’ of this schemata. This study has shown that students
did not readily use the topical strand tools and forms of expression that they
had been taught [see section 5.3] and which possibly constituted aspects of
‘relevant schemata’. Instead, they sought and used means that they were familiar with to explain their solutions. Also, the relatively high score on some items of a multiple choice format, as mentioned earlier, seem to highlight a scaffolding effect on task format thus lessening the ease of access to tools and forms of expression.

The categorisation that is proposed in this dissertation shares some similarity with aspects of SOLO taxonomy in the sense that it can be used to analyse students’ written solutions [see chapter 9 also Shaughnessy, 2007]. However, in the I/CA model no strict claim on learning outcome and thus on hierarchy of students reasoning is suggested; this stance is justifiable since for example, as observed by Bakker (2004b) the students may not always utilise the [statistical] tools that they have learnt; a typical instance is illustrated by pre-service teachers who did not in fact use the methods that they had encountered during the lectures. The models that have been influenced by the SOLO taxonomy also seem to place order of superiority in the hierarchy such that those approaches at the lower level are perceived as less superior to those at the higher levels. In the I/CA model, while admitting that a grasp of subject specific tools and forms of expression may improve the quality and enhance the production of reasonable solutions, it does not perceive of the Identification approach as necessarily less superior to the Critical-analytical approach: the approaches are mutually constitutive (Roth & Bowen, 2001). Thus, rather than revealing the content of students’ ‘schemas’, the proposed model in this dissertation serves in some respect, to indicate students fluency in coming to terms with and utilising cultural forms of knowing. Since learning is not only a process of knowing but also of becoming (Radford, 2013a; see also Shaughnessy, Garfield, & Geer, 1996), it is noteworthy that the categories suggested in this dissertation seek to promote being Critical-analytical.

In as much as the present research does, not as such, focus on assessment practice, the particulars of the I/CA framework can be perceived as appropriate both during the construction of mathematics tasks and in the analysis of students solution strategies. The I/CA framework, as already mentioned, calls for vigilance as well as some level of mathematics [tools and forms of expression] awareness or consciousness regarding both students and educators. In this way it is possible to conveniently meet the pertinent
concerns of *didaktik* viz., the *what, why* and *how* in working with graphical artefacts in the mathematics classroom (cf. Pettersson et al., 2010).

### 10.4 Curriculum documents and graphicacy

The curriculum documents and syllabuses play a vital role in providing a *didaktik pathway* in institutionalised learning. With respect to graphicacy it is expected that these documents contribute to the fostering of reflective and Critical-analytical members of society within institutionalised settings. Indeed the syllabus for mathematics explicitly set out that students, alongside having a secure grasp of mathematical tools or concepts, should be able to carry out and use logical reasoning, draw conclusions and generalise providing arguments for their thinking (Skolverket, 2009). One way of achieving this would be to expose students to such items and environments that promote Critical-analytical approaches since these, often require a reflective stance as well as the expression of opinions and justification of methods used in problem solving (Gal, 1998). These learning environments should not only focus on interpretational interaction with graphical artefacts but also the production aspects of graphicacy. As mentioned earlier, activities that produce graphical artefacts also have the potential of influencing their interpretation. As much as the studies outlined in this dissertation did not diverge into graph production, it seems that not much scope is given to the design and production of graphical artefacts in the curriculum documents. Of course it is the ambition of these documents that students are able to compile information in table form and diagrams. However, these seem to be limited to the ‘common’ graphical artefacts such as bar graphs, pie-charts and line graphs (see Skolverket, 2009). It is probably the case that graphical artefacts of the type of Napoleon’s march to Russia [Fig. 1.3] are a rarity rather than the rule in mathematical or statistical classrooms. However, even to engage with some of the common graphical artefacts some knowledge of design is necessary. In the collaborative observational study, it was observed that although the students reflected a general understanding of graph production some of them seemed to totally ignore the elementary principles of graph construction, such as aligning the scale with the origin. The issue of design and production of graphical artefacts becomes necessary with increasing access to raw data and with the possibility for the consumer to design graphical artefacts from the data that best illustrate a phenomenon of interest.
10.5 Implications for practice

The general approach of the present study was a micro analysis of students’ interaction with graphical artefacts where focus was on subject specific tool use and forms of expression. This approach may help educators identify and deal with the students use or otherwise of subject tools and forms of expression. The analysis on students’ solution given in chapter nine may also be perceived as a form of assessment where focus is on the distribution of tool use within a classroom and in the process it might be possible to judge the social accessibility and possibly acceptance of the given tools. For example in the case of the pre-service teachers, one might apart from concluding that the students had no grasp of the boxplot method as a way of solving the task, claim that the result might also indicate an element of inadequate enthusiasm on the part of the students. Importantly, the realisation that focus on subject specific tool use and forms of expression accentuates the notion of students being able to identify and select appropriate ‘external’ resources for effective interaction with graphical artefacts. This observation underscores the need for developing teaching and learning activities that nurture students’ Critical-analytical disposition. The importance of fostering Critical-analytical competences is perceived as providing students with a chance to develop a deeper understanding of graphical artefacts and a Critical-analytical attitude to claims presented in sometimes complex or even misused diagrams. A recognition of the two categories [levels] of discussing graphicacy ought to be taken as calling on teachers’ to be vigilant about the type of tasks students encounter in teaching and learning situations and to be sensitive to these classes in the design and use of tasks containing graphical artefact.

Focusing and being vigilant about the tools and forms of expressions that students bring with them into the interaction with graphical artefacts not only highlight contextual factors but can be seen as removing the burden of searching for students’ misconceptions in their interaction with graphical artefact (Roth, 2012; cf. Hadjidemetriou & Williams, 2002). The chance should be availed of for observing the spectrum of tools and forms of expression which students use in interaction with graphical artefacts. These can either be strengthened in the case of ingenious tool application or attention can be led to the desirable subject specific tools and forms of expression in a quest to promote learning (see Shaughnessy, 2007). In the linear models of graphicacy
provided by Curcio (1987) for example, there is a risk of focusing on the unattained levels and misconceptions when interacting with graphical artefacts. This may limit focus on the tools and forms of expression which students’ bring with them in graphicy activities thus missing out on the potential of scaffolding them to desirable use of subject specific tools and forms of expression. In section 7.3.1, it was observed that the students’ discussion veered into personal-emotional concerns; focus on this aspect of the solution argument would have overlooked important statistical reasoning that eventually served to legitimise their conclusion. Similarly in section 8.3.2 a student’s online contribution using metaphorical explanations not only provided an intuitive aspect of variability but had the potential of introducing the concept of unalikeability which in turn had the potential to greatly enrich the learning process. The significance of these illustrations is that the observation that a Critical-analytical ‘leap’ might not necessary be that removed from the so called idiosyncratic and informal forms of interaction with graphical artefacts (cf. Aoyama, 2007; Watson & Kelly, 2008).

In terms of general interaction with graphical artefacts the present research seeks to bring clarity to the different ways that this has been characterised. In the three tier models, mentioned in chapter five, there appears to be divergent views as to what constitutes ‘reading between a graph’. Similar issues can also be observed in the SOLO inspired categories, for example Aoyama (2007) citing Watson and Callingham (2003), perceives the idiosyncratic level of interaction with graphical artefacts as including the use of basic terminology, mathematical skills associated with one-to-one counting and reading cell values in tables. Watson and Kelly (2008) on the other hand seem to associate idiosyncratic level with tautological responses that do not show any appreciation of meaning [it is noteworthy that Aoyama (2007) seems to take this view in the analysis of students’ solutions]. Thus, there appears to be some level of ambiguity, which in a way, is eliminated in the I/CA model by virtue of its reliance on subject [topic strand] oriented research to determine what can be considered as problematic tools (see section 5.5). The I/CA model as mentioned earlier brings clarity by simplifying the many levels inherent in some of the models, while basically still referring to the same thing. It is also perceived that the terminologies associated with I/CA resonates well with the
ambitions of teaching and learning viewed in a wider perspective, as well as with the desired disposition for effective citizenry.
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Students’ narratives from graphical artefact


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APPENDICES

Appendix I: Jeffersonian Transcription System

Below are the transcription codes that were used in the course of conducting this study. A more comprehensive transcription code and other information on transcription can be found at

http://homepages.lboro.ac.uk/~ssjap/transcription/transcription.htm

[ ] Square brackets mark the start and end of overlapping speech. They are aligned to mark the precise position of overlap as in the example below.

↑↓ Vertical arrows precede marked pitch movement, over and above normal rhythms of speech. They are used for notable changes in pitch beyond those represented by stops, commas and question marks.

→ Side arrows are used to draw attention to features of talk that are relevant to the current analysis.

Underlining indicates emphasis; the extent of underlining within individual words locates emphasis and also indicates how heavy it is.

CAPITALS mark speech that is hearably louder than surrounding speech. This is beyond the increase in volume that comes as a by product of emphasis.

“↑↓ I know it,” ‘degree’ signs enclose hearably quieter speech.

that’s right. Asterisks precede a ‘squeaky’ vocal delivery.

(0.4) Numbers in round brackets measure pauses in seconds (in this case, 4 tenths of a second). If they are not part of a particular speaker’s talk they should be on a new line. If in doubt use a new line.

(.) A micropause, hearable but too short to measure.
((sto:ccato)) Additional comments from the transcriber, e.g. about features of context or delivery.

**she wa::nted** Colons show degrees of elongation of the prior sound; the more colons, the more elongation.

**hhh** Aspiration (out-breaths); proportionally as for colons.

**.hhh** Inspiration (in-breaths); proportionally as for colons.

**Yeh,** ‘Continuation’ marker, speaker has not finished; marked by fall-rise or weak rising intonation, as when delivering a list.

**y’know?** Question marks signal stronger, ‘questioning’ intonation, irrespective of grammar.

**Yeh.** Full stops mark falling, stopping intonation (‘final contour’), irrespective of grammar, and not necessarily followed by a pause.

**bu-u-** Hyphens mark a cut-off of the preceding sound.

**>he said<** ‘greater than’ and ‘lesser than’ signs enclose speeded-up talk. Occasionally they are used the other way round for slower talk.

**solid.= =We had** ‘Equals’ signs mark the immediate ‘latching’ of successive talk, whether of one or more speakers, with no interval.

**heh heh** Voiced laughter. Can have other symbols added, such as underlinings, pitch movement, extra aspiration, etc.

**sto(h)p i(h)t** Laughter within speech is signalled by h’s in round brackets.
Appendix II: Graphical artefacts related to Nationellaprovet

In this appendix is given some of the items used in the study involving the Swedish National Test [Nationellaprovet]: Given the length of the oral examinations these that is, Task No. 1 and Task No. 4 are however not attached here but can be easily accessed through the link given below.

Task No. 1 & Task No 4.

Available in the website below as section A


Task No. 2

The diagram shows the weight and price for three packages of candy.

a) Which package costs least?

b) Put a new point in the diagram which shows a package of candy that weighs less than B, but has the same price per kg as B.

Answer:_________ (10)

© Skolverket, 2007

Task no. 2 basically testing graph ‘reading’
**Task No. 3:** A test perceived as testing multireferencing and graphing skills ‘reading’

In Auckland in New Zealand there is a building called Sky Tower. The tower is 328 m high and it is the highest building in the southern hemisphere.

a) An observation deck “Main observation level” is located at a height of 186 meters. There are 1 029 steps up to this level. Every year there is a contest to climb these steps as fast as possible. The standing record is 5 min 7 s. How many steps did the record winner take every second?

b) According to a tourist brochure you can see as far as 82 km from the highest look-out point on the Sky Tower (the tower is marked with an X). Would it be possible to see the island called Great Barrier Island?

The map is drawn in the scale 1 : 1 500 000
c) It takes 40 s to ride the elevator up to the deck at 186 m. The top speed of the elevator is 30 km/h. Which of the graphs A, B, C or D shows the relationship between the time and the position of the elevator? Explain your reasoning and or calculations.
**Task No. 5:** This task basically test what is perceived as graph ‘reading’ that is, demanding Identification approach.

The diagram shows how the temperature for water falls as the water cools down.

How many minutes does it take for the temperature to fall from 60 °C to 40 °C?  
Answer: _______ min  

© Skolverket, 2008
Task No. 6: This item is perceived as testing basic statistical concepts of mean and median.

The pupils in a school investigated how many siblings (brothers and sisters) they had. The result of the survey is shown in the diagram.

a) How many students took part in the survey? Explain your answer. (20)

b) Determine the mean and the median for the number of siblings. (12)

© Skolverket, 2008
Task No. 7: An item perceived as testing basic statistical concepts of mean and median

![Graph: Height and weight for class 9B](image)

a) Philip goes in class 9B and weighs 65 kg. How tall is he?  
Answer: ______ m  
(10)

b) What is the median height for the class?  
Answer: ______ m  
(0/1)

© Skolverket, 2009
Task No. 8: This task tests appreciation of linear relationships and models.

How old can a cat be?

A cat does not live as long as a human being. So you might say that a cat ages faster than a human. To compare a cat’s age (number of cat years) with those of a human (number of years) one might use different models.

Model A: Each year corresponds to 7 cat years.

Model B: The first year corresponds to 15 cat years.
The second year corresponds to 10 cat years.
Each additional year corresponds to 4 cat years.

a) Three years ago Maria got a newly born kitten. How many cat years old is her cat today according to Model A and Model B respectively?

b) Copy and fill in the table.

<table>
<thead>
<tr>
<th>Year</th>
<th>Cat’s age</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model A cat years</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>
c) How long does it take until the two models give the same age for a cat? Determine this as exactly as you can.

d) Cats can get rather old. It is not unusual that they live more than 20 years. Compare the two models for the life span of cats (number of cat years). Which of the two models is most reasonable? Explain and defend your conclusions.

In assessing your work the teacher will take into consideration
- what mathematical knowledge you have shown
- how well you have defended your conclusions
- how well you have presented the solution.

© Skolverket, 2009

**Task No. 9:** This test multiple competence including visual perception and mathematical skills

The diagrams show how the medals were distributed for some different countries at the Olympics in Peking in 2008. Determine whether each of the claims made below is true or false. Explain your reasoning.

- Australia won 46 medals
- Spain won 18 medals
- Great Britain won 47 medals

<table>
<thead>
<tr>
<th>Australia</th>
<th>Spain</th>
<th>Great Britain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
<td>Gold</td>
<td>Gold</td>
</tr>
<tr>
<td>Silver</td>
<td>Silver</td>
<td>Silver</td>
</tr>
<tr>
<td>Bronze</td>
<td>Bronze</td>
<td>Bronze</td>
</tr>
</tbody>
</table>

a) Great Britain won more gold medals than Australia.  

b) Spain won more silver medals than Australia.

© Skolverket, 2009
Appendix III: Graphical artefacts related to the PISA survey test

1. Below are some of the items used in the PISA survey test and which have been released into the public domain. We also illustrate how the proposed taxonomy has been applied. The bottom line is identifying items that require the application of simple visual inspection, elementary tools and forms of expression.

**M150Q01- 3**

![Graphical Artefact](image)

**Question 1: GROWING UP**

Since 1980 the average height of 20-year-old females has increased by 2.3 cm, to 170.6 cm. What was the average height of a 20-year-old female in 1980?

Answer: ..................................................cm

**Question 2: GROWING UP**

According to this graph, on average, during which period in their life are females taller than males of the same age?

..........................................................................................................................

**Question 3: GROWING UP**

Explain how the graph shows that on average the growth rate for girls slows down after 12 years of age.

..........................................................................................................................
- **M150Q01**: this item is coded as Identification (I) since the solution can be derived by visual inspection, picking the solution from the sentence and also by reading the values.
- **M150Q02**: this item is coded as Identification (I) since the solution can be derived by visual inspection, that is to say, by pointing out values from the graphical artefact.
- **M150Q03**: whereas aspects of this item are accessed visually, students seem to have difficulties explaining or articulating their solution strategies, thus it is classified as Critical-analytical (CA).

**M179Q01**

Question 1: ROBBERIES

A TV reporter showed this graph and said:

“The graph shows that there is a huge increase in the number of robberies from 1998 to 1999.”

Do you consider the reporter’s statement to be a reasonable interpretation of the graph?

Give an explanation to support your answer.
**M179Q01:** this item is classified as CA since it requires insight into the fact that it might be difficult to make adequate comparisons with limited data; the student is called upon to realize the important of aspects not presented in the graphical representation, it also requires the justification of solution strategies; an observation that might prove to be challenging to most students.

**M438Q01-2**

**EXPORTS**

The graphics below show information about exports from Zedland, a country that uses zeds as its currency.

**Question 1: EXPORTS**

What was the total value (in millions of zeds) of exports from Zedland in 1998?

**Answer:** ..................................................

**Question 2: EXPORTS**

What was the value of fruit juice exported from Zedland in 2000?

A 1.8 million zeds.

B 2.3 million zeds.

C 2.4 million zeds.

D 3.4 million zeds.

E 3.8 million zeds.
- M438Q01: this item is coded as I since the solution can be derived by visual inspection; picking the solution from the bar graphs.
- M438Q02: this item is coded as CA since the solution requires knowledge of proportions and calculation with percentage.

M467Q01

Question 1: COLOURED CANDIES

Robert’s mother let him pick one candy from a bag. He can’t see the candies. The number of candies of each colour in the bag is shown in the following graph.

What is the probability that Robert will pick a red candy?

A 10%
B 20%
C 25%
D 50%

- M467Q01: this item is coded as CA; it is a task with multiple solution strategies however, the general demand requires knowledge or appreciation of proportional calculation and percentages [and probably some elementary knowledge of the concept of probability]
Question 1: TEST SCORES

The diagram below shows the results on a Science test for two groups labelled as Group A and Group B.

The mean score for Group A is 62.0 and the mean for Group B is 64.5. Students pass this test when their score is 50 or above.

Looking at the diagram, the teacher claims that Group B did better than Group A in this test.

The students in Group A don’t agree with their teacher. They try to convince the teacher that Group B may not necessarily have done better.

Give one mathematical argument, using the graph, that the students in Group A could use.

- **M513Q01**: this item is classified as CA since it requires insight into the statistical concept of variability [a concept that, according to research, is challenging to most students]. The task also requires the justification of solution strategies; an undertaking that might prove challenging to most students.
Question 1: WATER TANK

A water tank has shape and dimensions as shown in the diagram.

At the beginning the tank is empty. Then it is filled with water at the rate of one litre per second.

Which of the following graphs shows how the height of the water surface changes over time?

A

B

C

D

E
- **M465Q01**: this item is classified as CA since it requires some element of multi-referencing and mental rotations. This item was used instead of M192Q01 which by the time of conducting the study had not yet been released to the public domain.

2. A portion of the data from the PISA database shown in table IV the actual success rate for the selected items and countries used in the present study.

**Table III: Data for items containing graphical items obtained from PISA database**


<table>
<thead>
<tr>
<th>Item</th>
<th>Credit</th>
<th>DNK</th>
<th>FIN</th>
<th>ISL</th>
<th>NOR</th>
<th>SWE</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Full</td>
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<td>98</td>
<td>94</td>
<td>94</td>
<td>97</td>
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</tr>
<tr>
<td>M302Q01</td>
<td>Full</td>
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<td>86</td>
<td>86</td>
<td>85</td>
<td>85</td>
<td>85</td>
<td>86</td>
</tr>
<tr>
<td>M438Q01</td>
<td>Full</td>
<td>85</td>
<td>88</td>
<td>76</td>
<td>74</td>
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</tr>
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<td>Full/Partial</td>
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<td>80</td>
<td>75</td>
<td>73</td>
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<td>80</td>
</tr>
<tr>
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<td>Full</td>
<td>68</td>
<td>67</td>
<td>76</td>
<td>71</td>
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<tr>
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<td>Full</td>
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<td>67</td>
<td>59</td>
<td>55</td>
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<td>M155Q04</td>
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<tr>
<td>M467Q01</td>
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<tr>
<td>M150Q03</td>
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<tr>
<td>M828Q01</td>
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<td>M828Q03</td>
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<tr>
<td>M302Q03</td>
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<tr>
<td>M179Q01</td>
<td>Full/Partial</td>
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<td>28</td>
<td>42</td>
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<td>M513Q01</td>
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<td>30</td>
<td>43</td>
</tr>
<tr>
<td>M155Q03</td>
<td>Full/Partial</td>
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<td>23</td>
<td>23</td>
<td>12</td>
<td>18</td>
<td>12</td>
<td>23</td>
</tr>
</tbody>
</table>
3. Fig. IIIa is a dendrogram showing selected items that received full credit: In this case the items that have been removed are M150QO2, M155Q02, and M155Q03 AND M179Q01. That the two clustered are maintained shows the stability of the group of items characterized as Identification and Critical-analytical approach items.

![Dendrogram](image)

Fig. IIIa: Dendrogram of the clusters exclusively awarded full points. Thus, pointing to the stability of the clusters formed.
4. The dendrogram presented in Fig. IIIb shows the formation of the clusters including the approximately unbiased (AU) p-values as well as bootstrap (BP) values computed using multistage bootstrap resampling.

Fig. III b: Cluster dendrogram with AU/BP values showing aspects of internal cohesion of the two clusters of items containing graphical artefacts.
Appendix IV: Graphical artefacts from observational studies

1. Illustration of a typical way of dealing with misleading diagrams in a teaching and learning situation
Students’ narratives from graphical artefact

Fig. IV a: A BBC website illustration of misleading diagram mirroring item M179Q01

2. Below are the students’ inscriptions and solutions from the observation study for the students groups.

Group I

[1]. Karolin here it is 507 and up here it is about 516 (pointing at the bars), that is a lot of robberies …in a year, it is quite a lot
[2]. Helen yeah…It is difficult, …however,
[3]. Karolin …(mutter)…it is quite a lot
[4]. Helen it depends on…
[5]. Karolin [while addressing the observer] shall we begin the discussion now …or…
[6]. Linn (interjects) … speak up?
[7]. Observer yes…I mean you can discuss
[8]. Karolin if you compare this one (pointing at the apex of the two bars with the pointing fingers) it is kind of huge
[9]. Helen it is kind of… (mutter)
[10]. Karolin yes…kind of …it is kind of a lot in a year
[11]. Linn mutters while pointing at the values
[12]. Karolin  but it is about eight …eight
[13]. Helen  but even this bit here …(aligning the apex of the bars with axis values using a pencil) this one is to here
[14]. Linn  no…!
[15]. Helen  yes! it is this …that there is up to here… (showing the units alignment on the y-axis) as if demonstrating how the graph should be read
[16]. Karolin  yes … eight, sixteen
[17]. Linn  …and it is at five hundred five hundred
[18]. Helen  …and here we have two, five, eight (counting the parts thought not visible on the scale) or…
[19]. Karolin  yes approximately [pause]
[20]. Karolin  but it is actually much in a year
[21]. Observer  so, what have you arrived at?
[22]. Helen  we don’t know really
[23]. Karolin  but if one reflects on this (pointing at the axle) it is quite a lot in a year that is five hundred robberies, it can be big as well as small
[24]. Helen  I think the increment is about eight or nine in a year [pause]
[25]. Karolin  yes…but then it depends on the kind of robbery. It doesn’t have to be such bank robbery, but it can be like a person is robed or something.
[26]. Helen  I agree with you it can be both big and small
[27]. Karolin  robbing a small person or something
[28]. Linn  Yes
[29]. Karolin  Yes
[30]. Observer  so what do we say or what have you agreed upon
[31]. Helen  these are values from 1998 and 1999
[32]. Karolin  yes it is quite a lot!
[33]. Observer  Okay
[34]. Helen  the question is if it was the same …it was probably the same then
[35]. Karolin  I don’t know how much it was before…it was probably less
[36]. Helen  no … I don’t think that… this is what is normal. I find it hard to think that it was less than 1999
[37]. Karolin  it seems like …..(makes some in audible comment)
[38]. Helen  Yes
[39]. Karolin  yes [as if to signal that they are through with discussion]
[40]. Observer  what is your conclusion
[41]. Karolin  yeah…
[42]. Helen  it is quite a lot
[43]. Karolin  Yes
Students’ narratives from graphical artefact

[44]. Helen …at the same time it is also difficult to ascertain
[45]. Observer why is it difficult to ascertain?
[46]. Helen …of course it can be… at the same time one gives in since it has to be huge
[47]. Karolin (interjects) yes eight robberies
[48]. Helen I mean it is hard to ascertain…it is just a year, it is possible
[49]. Karolin …(mutter)… small communities with their small robberies, it doesn’t have to be big robberies it can be like small …[Unclear statement]
[50]. Helen I think it may almost be true
[51]. Karolin what do you say Linn?
[52]. Linn I say the same thing
[53]. Helen I say it can be true
[54]. Linn it is our conclusion
[55]. Karolin Yes
[56]. Observer yes?
[57]. All yes!

Group II

[1]. Amis (reading the title of the task) Robberies
[2]. Johan (almost spontaneously) This is not a reasonable…reasonable interpretation of the diagram, that there is a huge increase
[3]. Amis mmh (Amis & Alex mutter)
[4]. Johan watch this! (aligning the apex of the bar with the axis using a pencil to highlight the gap between them) I think this is say 15 this is 7; there are 7 more robberies in a year [pause]
[5]. Johan it is this kind of…It is a classical case of misleading diagram
[6]. Alex yes it is true! …ah
[7]. Johan it is not really true
[8]. Alex yes, the diagram is correct…
[9]. Amis wait! (a moment!)
[10]. Alex [completing previous statement from turn 8]…but there is something about it
[11]. Johan Yes
[12]. Johan answer: No!…answer dot, dot (writes full colon) no,
in that case we say No then (mutter)

because the diagram just shows that approximately 500; one can guess or something like that (while writing the solution) the diagram shows that … or correctly put, that 515 is meant to be twice as much as 507 [pause]

what did you say it shows? What do you mean?

it shows that this 515 is twice as much as 507 or 508 or whatever it is

aha! 515 is twice as much as much (chuckles)

…15 is meant to be twice …as… much…as robberies for…year 1999, twice as much as about 507 robberies in 1998 although it is…

are you writing? [addressing Alex]

of course I am writing…

[continuation from turn 18]… just about eight robberies per year!
Fråga 1: RÄN

En TV-reporter visade detta diagram och sade:
"Diagrammet visar att det är en enorm ökning av antalet rån från 1998 till 1999."

Anser du att reporterens uttalande är en rimlig tolkning av diagrammet? Ge förklaring
som stöder ditt svar.

nej, därför diagrammet visar att 515

* Ta vara dubbelt så mycket som
från år 1999 - 507 1998

No...because the diagram shows that 515 should be twice as much as from the year 1999

Fig. IV b: Amis’ solution notes (Group II)
Fråga 1: RÄN

En TV-reporter visade detta diagram och sade:
"Diagrammet visar att det är en enorm ökning av antalet rån från 1998 till 1999."

![Diagram][1]

Answer: No

because the diagram shows that 515 robberies from the year 1999 should be twice as much as 507 robberies for the year 1998 although it is just an increase of 8 robbers/year

Fig. IV c: Johan’s solution notes (Group II)
Fråga 1: RÄN

En TV-reporter visade detta diagram och sade:
"Diagrammet visar att det är en enorm ökning av antalet rån från 1998 till 1999."

Antal rån per år

År 1998

År 1999

Anser du att reporterens uttalande är en rimlig tolkning av diagrammet? Ge förklaring som stöder ditt svar.

Nej, rånens ökning endast med 7-8 varje år men utformningen gör så att man tro att det är dubbelt så mycket rån 1999.

No, the robberies increase by just 7-8 every year but the presentation makes some to think that there is twice as much robbery in 1999

Fig. IV d: Alex's solution notes (Group II)
Group III

[1]. Susanne we take the first one…that is, this one (shuffling the papers)
[2]. which one?
  Observer
[3]. Susanne Robberies
[4]. Susanne I think this is not …reasonable, that is the axis is broken, actually if it started at zero (making marks on the bars as if extending then to the x-axis) they’d be the same, so it is not a huge increase at all as they claim. Here it appears as if there is a double increment and that is not the case. [pause]
[5]. Susanne that is what I’d say is correct
[6]. Susanne [while addressing Maria] …do you share the same opinion?
[7]. Maria yes I think it is true [pause]
[8]. Observer
[9]. Maria yeah…that it is not certain that…they …I mean if they begun from below …if they begun from zero may be it would be completely something else (laughs)
[10]. at least the same … they appear a little odd… (referring to the bars)
  Susanne
[11]. Maria Yes
[12]. Susanne maybe not the big picture (in audible) either [probably referring to the highlighted parts]


Fråga 1: RÅN

En TV-reporter visade detta diagram och sade:
"Diagrammet visar att det är en enorm ökning av antalet rån från 1998 till 1999."

Anser du att reporterens uttalande är en rimlig tolkning av diagrammet? Ge förklaring som stöder ditt svar.

- Misleading…
  It appears as though it is twice as much but actually it is not.

Fig. IV e: Petra’s solution notes (Group III)
Fråga 1: RÅN

En TV-reporter visade detta diagram och sade:

"Diagrammet visar att det är en enorm ökning av antalet rån från 1998 till 1999."

![Diagram of råns per year from 1998 to 1999]

Anser du att reporterns uttalande är en rimlig tolkning av diagrammet? Ge förklaring som stöder ditt svar.

Väldigt onormalt om man ritar diagrammet från 0 märker man att ökningen inte alls är så enorm.

Quite unreasonable if one draws the diagram from 0 one observes that the increment is not that huge

Fig. IV f: Susanne’s solution notes (Group III)
Fråga 1: RÅN

En TV-reporter visade detta diagram och sade:

"Diagrammet visar att det är en enorm ökning av antalet rån från 1998 till 1999."

Anser du att reporterns uttalande är en rimlig tolkning av diagrammet? Ge förklaring som stöder ditt svar.

Unreasonable that one draws the diagram from point 0

Fig. IVg: Maria’s solution notes (Group III)
Group IV

[1] Tim: you can start by discussing one
[2] John: Yes::: Precisely (0.4) OKay (;)
[3] Tim: >mhm< (0.3)
[4] John: "hmm (;) = (Reads) The graph shows that there is a huge increase in the
the number of robberies from nineteen ninety eight (;) to nineteen ninety nine (0.5) It ea[-
[6] John: Number of robberies<
[7] [8] John: Yes (0.1) precisely (continues to reads) the number of robberies
(0.2) yea: r ah (;)
[10] five hundred and seven, five hundred and nine
[11] John: nineteen o eight (;) yes< nineteen ninety eight it was abo(0.1) ut
(0.2)yes=: seven (;)
[12] Tim:
[13] John: five hundred and seven, five hundred and eight something >
"hmm< (;) and nin- ninety- nine it was aroundfive hundred and sixteen
"something like that"
[14] (;)"hmm
[15] Tim: [ yes (0.1) it is
[16] John: >it is not that mu[ch< 1 thin-
[17] Tim: ][No <- (0.2) an] (0.2)d > "the state(;me(0.1)nt is"
there
[18] is a huge Increase <
[19] Peter: ]Yes Precisely! <
[21] Tim: It is written (;) that (;)a huge increase \(0(0.1) and it isn't
[22] John: [yes (;) precisely (0.1)
\(No::<(0.3)
[23] Peter: so: (0.2) it-(0.1) yes:::(0.3)
[24] Tim: therefore;i[t is ]not(;) reasonable(0.1)
interpretation
[25] Peter: ]Here did you see something?\(;<(; (the express(is missing) (0.5)
Students’ narratives from graphical artefact

[26] John: [No]
[27] Tim: [>mmh < (0.5) if it was like (0.2)>yes<
[28] John: >it was in the range for (-.) fif- twenty, thirty, forty robberies< then it would have
[29] been different (-) but I don’t think it is a huge increase.
[31] Peter: But it is:: a rath::er big: increase
[32] John: Yes, yes but it is becau-[

[33] Peter: >[but(.)and: since it lies above five hundred (0.1) one has to count: (0.2) yes
[34] John: [Yes precisely
[35] Peter: one has to count the [numb:::er (0.2) in tótal
[36] John: No (.)
[37] Peter: Shall we write it here (0.2)
[38] Tim: Shall we?
[39] Peter: mh::mh (0.3)
[40] John: >yes<
[41] Peter: Shall we write down the answer or::?
[42] Tim: It is not such a huge increase if (.) one compares wit-
[43] John: It is a question of an increase maybe seven, eight robberies(-) in a

[44] year(0.1) of a-and it approximately five hundred (0.2) a-(-0.1) in reality (0.1) as
[45] such we perceive that (0.1) seven, eight robberies is not a huge increase
[46] Tim: >No<
[47] R: But(0.1) Ehms:: Okay (.4) is it just that, that you believe ehm(.) is the
[48] Explanation why there are:: (0.1) or are there other explanation ?

225
Peter: mhm(·) Yes: (0.5)
Tim:  
mhm:
Peter: What do you say? (0.5)
John: I do not know what one can say directly
Tim: No (0.4)
R: Okay
Tim: It appears of course-(0.2) This bar appear larger but then it is not from
the beginning of::
John: yes=Prec-(·) That is () it appe-it appear larger because they have here
drawn a little diagram () If it had been a (0.3) larger diagram () then this
would (0.2) if one had several numbers at the side so: (0.1) number of
robberies (0.1) If one had several to choose from then this distance would be
less and then it would also appear smaller (0.3)
R: ah: okay
John: but it is not really large actually(0.1) and now there is not much(0.1) to choose
from or see(0.2) or:: how [can one put it.
Tim: yes
[Precisely.
Appendix V: online logs for pre-service teachers discussions

1. Discussion group A

[A1] 18/11 19:01 Hello - Tutor
As mentioned earlier [from a previous common post] you see it is now possible to begin discussing the tasks. In case you have difficulties with anything just inform me. If you start early I might get the time to participate in the discussion.

Good luck.

NOTE: you decided amongst yourselves how to solve the tasks say, discuss one task at a time or discussing all the tasks simultaneously; just remember to arrive at a common conclusion.

[A2] 20/11 17:31 the tasks - Eva

…

I think in the variation table [meaning the graphs] the range is the same but the dispersion or variance [the Swedish word ‘spridningen’ can generally be translated as spread and may used statistically in relation to centre] is larger for class 2

…

What do you think?? 😊

/Eva

[A3] 21/11 11:36 Re: the tasks - Tutor
Of course it sounds good … would like you to explain a little why you think that diagram 2 has a larger variation (dispersion).

[A4] 21/11 11:54 Re(2): the tasks - Eva
Hello yes, I thought that in diagram 1 there were many in the middle of the range while those in diagram 2 were more evenly spread, but I think the range is the same for both [diagrams].

[A5] 23/11 17:03 The tasks - Helny
Hello! Eva and all members of this group!
... The task on range.

I interpret the task: the range extends from having 50 correct up to and including 95 correct and which is apparent for both graphs...

that is the range is of the same size. Alternative c

However, if one considers the number of students: “cognitive difference” then it is class two which has the largest variation. Based on [the observation] that every column [the tabular frequencies] contains an increasing number of students. Class 2 students have many 55 and 65 correct and 85/95 correct which implies that there is a large number of students at different cognitive level.

(hence I agree with Eva)

...

Who are [the members of] in this group?

Does someone have another suggestion?

Helny

hello helny ☺ it appears as if we have the same view on all the tasks. It is moa, karin, jarla, inga and lotta who are also members of this group. I believe that jarla had a reflection on the cartons [referring to one of the other tasks] but then she can talk about it by herself ;)

[A7] 23/11 18:45 Hello - Tutor
I think your discussion is quite fine…concerning the task on variation I wonder how it would look like if you used boxplot?

Regards

Tutor

[A8] 23/11 19:50 Re: Hello - Eva
if one would construct a boxplot then the variation on diagram 1 would just be around 75 correct but in diagram 2 would be around 65-85 correct and thus of course diagram 2 would have a larger variation...or I am wrong?

[A9] 23/11 22:10 The variation task and lottery - Karin
Variation task

Concerning the task on variation I perceive that alternative b) is the correct answer. Just like Eva and Helny said, here the distribution of the results has been more dispersed in Class II. Since most of the students are at different levels. In Class I the probability is higher that if a student asks a classmate what their grades is, then it will be 75. The interesting thing with both diagrams is that the number of grades still is very evenly distributed. Like in Class I that there are exactly as many with 55 and 95, then 65 and 85 and the rest at 75. The same applies to Class II, that the numbers are exactly evenly distributed from middle score. Interesting...Then it would be alternative c) because the variation is then actually the same in both classes, since they are “evenly” distributed. Or? Shall I still point out that I am sticking to alternative b), since there is a slightly larger distribution anyway.

...
interpretation…yeah yeah, kind of confusing. What I want to highlight is that it is a question of [personal] interpretation as regard our perspective on variation. End of story.

…


Well, the idea is that we should submit a solution that has the backing of all group members! What is your opinion about doing a boxplot for the variation task? I think that Inga’s summary on the article was representative of our views, or what do you think? The task with the carton we have calculated the same way…

What do you think? Today is the last day to get done with this!

/Eva


Hello group members!

I can but only agree with what you’re saying about the article and the task on variation.

…

See you tomorrow

[A16] 25/11 14:47 RE - Eva

…

[A17] 25/11 17:43 IN AGREEMENT - Hilary

… [mentions that the group appears to have reach some consensus on the tasks]

Can someone send in our group solution to Tutor?


…

I’ll say just like Eva, that we have had similar point of view around the task on variation that it, our answer choice is alternative b). In which case we perceive that the variation is in how the grades are distributed. Not so much the overall, rather how evenly distributed they are on the different [grades]
…

What do you think? Great that we have a consensus : p

Then this is our conclusion!

Sounds great Karin ☺

[A20] 26/11 17:59 concerning the tasks – Lotta
… [apologizes for group for not being active until now]

The agreed upon solution will certainly be good, I bet that I'll arrive at
the same conclusion like most of you.

My energy and commitment is not high just now, *sorry for that!* [words in
italics posted in the English language]

Lotta

Well…sat and reflected for a while on this [task], but I have to say that
alternative b seems appealing.

Exactly as Karin is saying, there are 50% with grade 75 I class 1. The
other 50% of the students consequently share the rest of the grades and
that dispersion is divided to 4 different grades

In Class 2 are the variations larger since more students have one and the
same grade but in total the students have different grade levels.
However, it appears as if the classes are the same…range…*sigh*

2. Discussion group B

[B1] 18/11 19:01 Hello – Tutor
[the group received the same instruction as for group A i.e. as contained
in A1]


[B3] 21/11 18:38 Re: graph of variation – Tutor
Interesting… we'll see what the others think …

Hello!

I had a slightly different thought to yours, the way I see it, there is a larger variation in class II.

It became easier when I thought of it as measuring different colors on students’ clothes in two classes. Then it is clear that there is a greater variation in Class II.

Regards Petra

[B5] 22/11 22:00 Re (2): graph of variation posted for Petra – James

Hi there!

(My computer problems have just been fixed and I have had some time to look at these graphs).

I think it is a pretty tough question; however, I am inclined to respond that option B is probably correct. Thus, I tend to agree with Petra on this issue, especially after reading her concretization.

Great, Petra!

- James

[B6] 24/11 06:44 Re (3): graph of variation posted for Petra – Tutor
Oh well... maybe it’s difficult; I do not expect that you will work with say standard deviation [a concept] that we have not covered. Try the boxplot; it is of course an approach that highlights the dispersion in the data set. I want to emphasize that the purpose of these tasks is that you should learn how to turn the task upside down to [be able to] say produce the measures of central tendency.

...

Good luck

Hello all!

Sorry that I delayed with posting my reflections on these tasks. I did not think the deadline would be up too soon and I haven’t really grasped the tasks due to the fact that I missed the lesson when you went through it. Anyway I have now gone through the tasks and as what you have written in your comments are in harmony with what I believe.

…

Diagrams:

As regards the next task on the results of the two classes, I think even in this case it ought to be very logical that it is option B, that class II has a larger variation than class 1. This I explain by that the majority in class I have obtained an average score [it is not clear if he refers to the mean] while class II has a more spread and even score. There are more in class II with a higher and lower score which also entails there being greater variation.

Now, I do not know if you have time to reflect on what I have posted since probably the online discussion group winds up soon. But if you can and have time to read through it and want to comment on and discuss, please feel free to send your views to my inbox here at FC.

See you tomorrow in the student text lesson.

With kind regards

Erik


That said I think you’re coming up just fine [with the discussion]...

…

Erik brings up another way of looking at the data which is really the same reasoning as the one Petra propagates, However, Erik uses the concept mean. This is also another way of working with the task alongside boxplot.
That is, from the diagrams calculate [figure out the mean] and then compute how the other “bars” deviate from the bar containing the mean and then determine the mean of these “differences”

Kind regards

[B12] 27/11 22:26 Now FINALLY I have embarked on this … anyone able to read? –
Charlotte

Hello!

Sorry that I am so late with this work. *I am ashamed* I am in the processes of sequentially completing my assignments (I still have some piled up...) and now it is time for this [the online discussion].

Since I have been away from mathematics lessons, I received assistance from my (quite humorous) husband.

Perhaps I have been too explicit in my explanation - but that is because it is too hard to just use text when one has to discuss these kinds of “problems”

The attached document “The spread in the results” is about the task with the graphs. In the document I explain how I settled for alternative B as the most correct.

I hope that you are able to read this one now [referring to her post]… I am aware that I have tendency to be verbose…

Have a good time - now I’ll have to do some abs

~*~ Charlotte ~*~

NOTE: the following explanation was given as a separate attachment

The spread in the results:

One way to discuss this is to perceive the results as pellet marks from a shotgun [aimed at] on a shooting target, where every student is assigned a pellet that hits within an area on the target. Then the tables could appear as follows:
Fig. 8.2. Charlotte’s characterization of the variability of scores for the two classes
Something that people talk about in connection with shotgun shots is the spread. This indicates how the pellets are scattered from the barrel. A narrow barrel means that one gets a little pellet spread and thus more pellets end up in the middle of the target. If one would liken the classes with a shot from a shotgun on a target then, it can be perceived that class II has a larger spread as more hits are further away from the centre of the target. If a similarity is made between variation and spread [of the pellets on the target board] then class II will have more variation than class I.

[B13] 27/11 23:54 Re: Now FINALLY I have embarked on this ... anyone able to read? - Tutor

Hehe of course this one has to read... thank you... I am not sure your colleagues will have the time to read [the post]. 'would like to see their reactions on your views

Kind regards

Tutor

After allowing access to all groups one member of group A commented on group B’s discussion posts

[B14] 16/12 10:32 Thanks - Lotta [a post from a member of discussion group A]

…very interesting contribution Charlotte. Cheers to Jacob and his shotgun

:D

[B15] 16/12 14:16 Re: Thanks - Charlotte:

It is I who should bow in gratitude that someone found time read through my solutions!