Lateral light scattering in fibrous media

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Abstract: Lateral light scattering in fibrous media is investigated by computing the modulation transfer function (MTF) of 22 paper samples using a Monte Carlo model. The simulation tool uses phase functions from infinitely long homogenous cylinders and the directional inhomogeneity of paper is achieved by aligning the cylinders in the plane. The inverse frequency at half maximum of the MTF is compared to both measurements and previous simulations with isotropic and strongly forward single scattering phase functions. It is found that the conical scattering by cylinders enhances the lateral scattering and therefore predicts a larger extent of lateral light scattering than models using rotationally invariant single scattering phase functions. However, it does not fully reach the levels of lateral scattering observed in measurements. It is argued that the hollow lumen of a wood fiber or dependent scattering effects must be considered for a complete description of lateral light scattering in paper.

OCIS codes: (010.5620) Radiative transfer; (100.2810) Halftone image reproduction; (290.4210) Multiple scattering; (290.7050) Turbid media.

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#185971 - $15.00 USD Received 25 Feb 2013; revised 15 Mar 2013; accepted 16 Mar 2013; published 22 Mar 2013

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1. Introduction

The underlying physics of light scattering in turbid media is important in a wide variety of fields. Light scattering in paper has a fundamental role as the optical appearance is one of its main functions. Lateral light scattering has a key role in the optical dot gain in paper, the so called Yule-Nielsen effect [1], where printed dots due to light scattering appear larger. It has also been suggested that monitoring scattering in paper can potentially be used in measuring fiber formation [2].

Proper models for lateral light scattering in turbid media are requried to correctly relate to the optical properties in materials such as paper. It has been shown by for example Neuman et al. [3] that both single scattering anisotropy and the medium mean free path has a big impact on the point spread function. Modulation transfer function (MTF) analysis is a convienient way of evaluating lateral light scattering [4]. Arney et al. [5] compared measured values of the inverse frequency at full width half maximum of the MTF, k_p, with a model based on the Kubelka-Munk (KM) scattering coefficient S. Recently Coppel et al. [6] compared these results with Monte Carlo simulations based on the Henyey-Greenstein phase function with two different asymmetry factors. They found that neither isotropic scattering nor strong forward scattering correctly could predict the large lateral scattering observed in the measurements. Since paper is highly anisotropic, based upon layers of cellulose fibers, phase functions that depend on the absolute direction of light within the structure are called for. Applying such phase functions to address similar observations has been suggested for other turbid media. Anisotropic directional dependent scattering can for example be modelled by using phase functions derived from an analytical solution of electromagnetic scattering by an infinite cylinder [7]. This was first proposed by Kienle et al. [8, 9] who observed the anisotropic shapes of backscattered point spread functions (PSF) for different stochastic representations of the cylinder alignments. Similar Monte Carlo simulations have since then been used to model light scattering in anisotropic structures such as biological tissue [10, 11], softwood [12] and textile [13]. A detailed description of the implementation of such a Monte Carlo model can be found in Yun et al. [14]. One benefit of using analytical solutions is that the phase function is derived from material parameters like particle size and refractive index. This means that the asymmetry factor g is no longer required in the same way as in for example the Henyey-Greenstein phase function. Another benefit is that the scattering efficiency Q_s can be obtained, it can be used to define how scattering distance depend on the absolute direction of photons propagating in aligned fiber structures.

The purpose of the present work is to utilize the cylinder phase function through Monte Carlo simulations to test if it can be used to mimic scattering in paper. The structural anisotropy is considered by aligning the cylinders isotropically in the plane of the paper and with a small Gaussian distribution in the thickness direction. The simulated MTFs are then compared to both the measured [5] and simulated values [6] of k_p to test whether they can predict the amount of lateral scattering in the real paper samples.

#185971 - $15.00 USD Received 25 Feb 2013; revised 15 Mar 2013; accepted 16 Mar 2013; published 22 Mar 2013

2. Method

The Monte Carlo model in this work utilizes phase functions from an analytical solution of scattering by an infinite cylinder [7]. For perpendicular incidence ($\zeta = 90^\circ$) light scatters in a disc around the longitudinal axis of the cylinder and for oblique incidence light scatters in a cone around the longitudinal axis where the phase function specifies the distributed intensity. The half angle of the scattering cone equals the incident angle ζ governing the directional dependencies observed by for example Kienle et al. [8]. μ_s is related to the density of cylinders C_a, cylinder diameter d and scattering efficiency Q_s for any incident angle ζ as

$$\mu_s(\zeta) = C_a d Q_s(\zeta). \tag{1}$$

The cylinder density is defined as the total length of cylinders per volume unit. It can be valid to test the performance of the model in relation to C_a as it is the concentration of particles, especially since the cylinder phase function only is valid for plane electromagnetic waves.

The scattering parameters are determined by matching the reflectance and transmittance factors to those of the previously published measurements for each of a set of 22 samples. The shape of the simulated MTFs are then compared to the Monte Carlo simulations using the Henyey-Greenstain phase function with isotropic single scattering ($g = 0.0$) and forward single scattering ($g = 0.8$) and the simulated values of k_p are also compared to the measured values.

2.1. *Material parameters and parameter estimation*

Paper is essentially an entangled network of fibers, however the model used in this work assumes that the fibers are not in contact with one another. Surface effects are not considered as the scattering only take place at the air-cylinder interfaces in the model. The cylinders are aligned isotropically in the plane of the simulated microstructure and are assumed to be following a Gaussian distribution with standard deviation $\sigma_z = 5^\circ$ in the thickness direction. A more detailed description of the cylinder alignment has been reported in [2]. The refractive index of cellulose ($n = 1.55$) is used for the cylinders and the surrounding medium is air ($n = 1.0$). The cylinder diameter is chosen to be 20 μm as typical wood fiber dimension [15]. The wavelength of the light was chosen to be $\lambda = 510$ nm as the experiments in Arney et al. used a green filter. We optimized C_a iteratively using a Gauss-Newton method in the $0^\circ/d$ geometry with the Monte Carlo simulation tool to match the measured total transmission and reflection for each of the paper samples with their given thicknesses. The obtained cylinder density C_a, scattering coefficient in the thickness direction $\mu_s(\bar{\mathbf{z}})$ and absorption coefficient μ_a are shown in Table 1.

2.2. *Simulation of the edge response*

The Monte Carlo simulation tool (available at *http://fibermc.sourceforge.net/*) is used to compute the point spread function (PSF) for each of the 22 sets of paper sheets. Both for a single paper sheet and for an opaque pad of paper sheets. The resolution was chosen to 10 μm and the incident trajectories of the photons was tilted with a 20° angle toward the surface normal to match the measurements. A 2D convolution between the simulated point spread function and a intensity distribution $i(x,y)$ was used to derive the edge response

$$ESF(x,y) = PSF(x,y) * i(x,y). \tag{2}$$

The intensity distribution is uniformly distributed and cut off by a sharp knife edge, i.e. a matrix consisting of 1's on one side and 0's on the other. A similar convolution was used by Ukishima et al. [16] who considered the intensity distribution of pencil light together with the point spread function. This approach is different compared to Coppel et al. who simulated the edge response directly by distributing each of the photons over an area. Since the statistical response of a PSF

#185971 - $15.00 USD   Received 25 Feb 2013; revised 15 Mar 2013; accepted 16 Mar 2013; published 22 Mar 2013

Table 1. Sample thickness t, cylinder density C_a, scattering coefficient $\mu_s(\bar{\mathbf{z}})$, absorption coefficient μ_a and simulated values of k_p for a single sheet and an opaque pad (k_p^∞).

sample	t μm	C_a 10^9 m^{-2}	$\mu_s(\bar{\mathbf{z}})$ mm^{-1}	μ_a mm^{-1}	k_p mm	k_p^∞ mm
1	42	0.71	32.2	0.13	0.250	0.794
2	71	1.39	63.1	0.0	0.346	0.718
3	45.5	0.78	35.4	0.0	0.266	0.807
4	76.5	1.45	65.9	0.0	0.360	0.693
5	97	4.73	214.4	0.295	0.224	0.254
6	109.5	4.795	217.3	1.40	0.200	0.208
7	95	9.03	409.3	0.332	0.142	0.153
8	98	3.62	164.1	0.0	0.275	0.372
9	89	0.895	40.6	0.0	0.453	0.771
10	130	0.43	19.5	0.0	0.641	0.881
11	84	2.87	130.1	0.0	0.291	0.455
12	65	7.155	324.3	0.372	0.153	0.180
13	89	8.06	365.3	0.479	0.149	0.163
14	122	2.965	134.4	0.0	0.333	0.441
15	126	4.30	194.9	0.158	0.259	0.279
16	99	3.83	173.6	0.105	0.263	0.318
17	124	5.80	262.9	0.139	0.213	0.239
18	104	3.77	170.9	0.0	0.275	0.370
19	110	1.07	48.5	0.0	0.470	0.757
20	293	5.92	268.3	0.268	0.218	0.229
21	93	8.51	385.7	0.312	0.146	0.161
22	130	6.29	285.1	0.275	0.193	0.203

is more focused it greatly reduces the amount of simulated photons required to keep down the noise levels. It is therefore preferred to use the convolution as Monte Carlo simulations are very time consuming.

2.3. MTF

The line spread function (LSF) is obtained by taking the derivative of the edge spread function (ESF, see Fig. 1(a)). We then obtain the MTF by taking the Fourier transform of the LSF. The inverse frequency at full width half maximum of the MTF, k_p, is defined as

$$MTF(\frac{1}{k_p}) = 0.5. \tag{3}$$

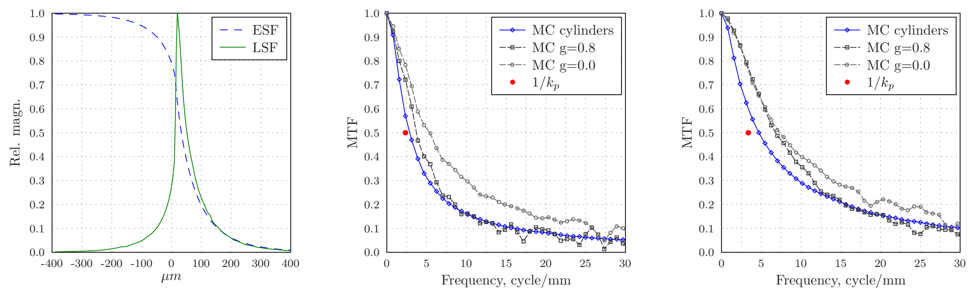
Only measured values of k_p are compared to the simulations since Arney et al. only reported k_p. The full MTF would have been preferred over the metric k_p as it only indicates the amount of lateral light scattering and lose a lot of the information held by the MTF [6].

3. Results

The simulated MTFs of samples 2 and 17 plotted for the three phase functions are shown in Fig 1(b-c). The shape of the simulated MTFs is narrower at low frequencies for the simulations with the cylinder phase function compared to the previous Monte Carlo simulations but seems to agree well with simulations with anisotropic scattering ($g = 0.8$) at higher frequencies. The

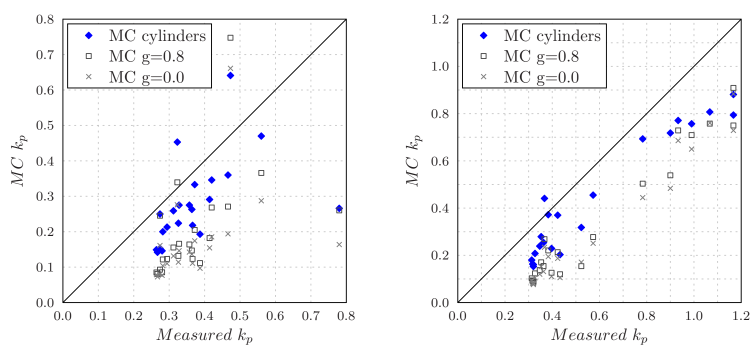
#185971 - $15.00 USD Received 25 Feb 2013; revised 15 Mar 2013; accepted 16 Mar 2013; published 22 Mar 2013

measured values of $1/k_p$ are presented by red dots in the figures. Figure 2 shows the simulated results for all the 22 samples for both single sheets and opaque pads of paper sheets in relation to the measurements and previous simulations. The general trend is that the simulated values of k_p are underestimated in comparison to measurements but they predict larger values than simulations with rotationally invariant phase functions.



(a) ESF and LSF of sample 2 (b) MTFs of sample 2 (c) MTFs of sample 17

Fig. 1. Simulated ESF and corresponding LSF in (a) and simulated MTFs for single paper sheet sample 2 in (b) and sample 17 in (c). The main difference between the samples is that sample 17 has a about four times larger scattering coefficient compared to sample 2. The red dots indicate the measured value of k_p presented by Arney et al.



(a) Single (b) Opaque

Fig. 2. Measured values of k_p versus simulated values for a single sheet (a) and an opaque pad of sheets (b).

Figure 3 shows the values of k_p in relation to C_a for a single sheet and an opaque pad of paper samples. At low concentrations, or low scattering coefficients, the cylinder model seems to agree better with the previous simulations than with the measurements. It appears that the best correlation to the measurements are found in the middle region where C_a lie between $3 \cdot 10^9$ and $5 \cdot 10^9$ m^{-2}. For larger values of C_a a decresing correlation can be observed.

4. Discussion and conclusions

Simulations and measurements of lateral light scattering in paper was compared to a Monte Carlo model employing scattering by infinitely long homogeneous cylinders. The cylinder

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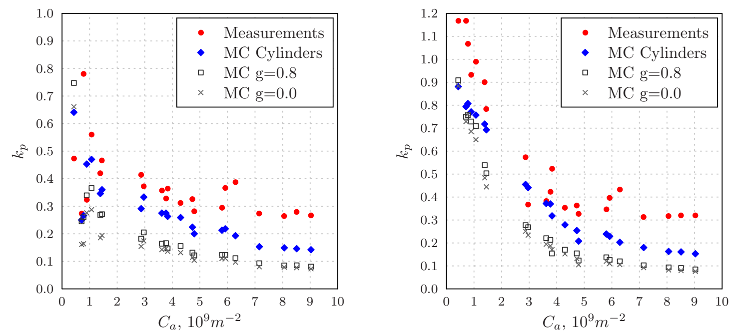

(a) Single (b) Opaque

Fig. 3. Measured and simulated values of k_p for a single paper sheet (a) and an opaque pad of paper sheets (b) plotted against the density of cylinders C_a.

model clearly predicted a higher value of k_p than models using rotationally invariant isotropic and strong forward scattering. However, the cylinder model still underestimates k_p compared to the measured values. This means that the model predicting less lateral scattering than observed.

The conical scattering by a cylinder has a tendency to sustain photon propagation in the plane of the paper thus increasing the lateral scattering. We originally thought that low correlation between simulations and measurements for the samples with low optical thickness (Fig. 3) were due to photons only scattering once before leaving the medium. Increased lateral scattering can only be observed if the photons manage to scatter so that they start propagating in the plane. However, the trend remains for the low concentrations in an opaque pad (Fig. 3(b)) and therefore contradicts this. The decreasing correlation at the larger values of the scattering coefficient, i.e. for very dense papers, is not suprising. The phase functions for scattering by cylinders is only valid for plane waves which means that the scatterers need to be in the far-field of one another. A possible explanation for the large values of the cylinder density is that the fibers in paper often are of banded or elliptical shapes. It shall be emphasized that it is a rough approximation to model a wood fiber as a homogenous cylinder. Additional simulations were made with adjusted values of the material parameters that affect the phase function, like e.g. the diameter of the scattering cylinders d and wavelength λ. We observed that this did not have any significant effect on the shape of the PSF and the resulting MTFs. This indicates that the model cannot predict more lateral light scattering for parameters within the natural parameter range of wood fibers. Arney suggested that the hollow lumen of a wood fiber could increase the lateral light scattering through a light-piping effect. This seem to be a reasonable explanation to why the model still predicts less lateral scattering than measured. Another reasonable explanation is that the concentration of particles is high, causing dependent scattering effects giving rise to interactions which are different from the interdependent scattering theory used in this work. Part of the lateral scattering can, however, be explained by the conical scattering by cylindrical objects. Compared to the Monte Carlo simulations using the Henyey-Greenstain phase function it roughly closes half the gap between earlier MC simulations and the experimental results. The cylinder phase function gives a good idea of how the lateral light scattering in fibrous materials is generated and has potential to better model for example optical dot gain.

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