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This is a published version of a paper published in *Nordic Pulp & Paper Research Journal*.

Citation for the published paper:

Uesaka, T., Juntunen, J. (2012)

"Time-dependent, stochastic failure of paper and box"

Nordic Pulp & Paper Research Journal, 28(2): 370-374

URL: <http://dx.doi.org/10.3183/NPPRJ-2012-27-02-p370-374>

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Time-dependent, stochastic failure of paper and box

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KEYWORDS: Creep, paper, Board, Corrugated box, Failure, Stochastic

SUMMARY: Time-dependent, stochastic failure is most common in practical situations where paper and board are used, such as web breaks and box failure. A theoretical treatment has been presented, starting from Coleman's stochastic failure model to the models with structural hierarchy. Creep failure data of box and board published in the literature were analysed from the light of the proposed theoretical model. The results showed that the distribution of life time (not only mean life time) of corrugated boxes changes very sensitively with loading and environmental conditions. Generally the coefficient of variation varied from 6% to 90%. Another important parameter, the load sensitivity of damage growth also varied from 1.7 to 27. Such enormous variation of the key parameters is not a mere fluctuation of experimental condition, but represents the very nature of time-dependent, stochastic failure. The stochastic failure model presented provides a framework for analyzing experimental data as well as for predicting life time distributions under general fluctuating loading conditions other than creep.

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Failure phenomena are probably one of the most frequently-investigated subjects for many materials including paper and board. In these investigations, it is customary to *presume* that the failure occurs when stress σ exceeds a critical value (strength) σ_c , in the case of uni-axial state, or a combination of the stress components meets a certain criterion ($F(\sigma_{ij})=0$) in a multi-axial case. However, looking at failure phenomena in real world, we find that these stress-based failure criteria may not necessarily be applied. For example, when corrugated boxes are subjected to *nominally* constant load in compression (creep conditions), it is known that the box fails at different times. That is, the failure is time-dependent. Secondly we find that the time at which failure occurs (life time) always shows enormous scatters (stochastic nature). Fatigue strength is another notorious example that exhibits these features. Although such time-dependent, stochastic failure may be less evident in relatively fast stress-strain tests in the laboratory, it is more common in realistic situations (e.g., box failure and web breaks). The reason is that failure is a "process", and the failure process is governed by the evolution of damages created in ordered/disordered material structures. Considering the fact that the damage evolution is influenced by the boundary conditions, this also

implies that material strength is inherently size-dependent.

Although the time-dependency, stochastic nature, and size dependency have been recognized in the area, they are often treated as either annoyances or exceptions in many studies, because of the complexity of the problem and the lack of appropriate approaches to deal with the problem. Therefore, most studies typically focused on *mean* strength and *mean* life time, for example. This study concerns the first attempt to formulate time-dependent, stochastic, size-dependent failure of paper and board on a macroscopic scale. We first set up mathematical apparatus for dealing with the problem, and then to review and analyse the data in the literature in details from the light of the new theoretical framework.

Theoretical Framework

Paper and board have a hierarchical structure, as many materials do, starting from highly disordered fibre network structures, and then going down in the scales, e.g., fibres, fibrils, much smaller elementally fibrils, cellulose chains, and so on. Failure and damage might occur at different levels of this structure. For the macroscopic description, it is most convenient to have a general but still tractable model that contains the representation of the damage evolution within these multi-scale structures. We start from the seminal work by Coleman by defining an elementary process of time-dependent, stochastic failure. We tentatively call this structural element "fibre", but it doesn't have to be identified as a real pulp fibre. When a force history $f(t)$ is applied to this fibre, damages are developed. Accordingly, the probability of fibre failure also increases as the damages increase with time. We will first define the damage evolution law (or breaking rule) in the following:

$$\frac{d\Omega}{dt} = \phi\left(f, \frac{df}{dt}, \frac{d^2f}{dt^2}, \dots, \Omega, m, \frac{dm}{dt}, \frac{d^2m}{dt^2}, \dots, T, \frac{dT}{dt}, \frac{d^2T}{dt^2}, \dots\right) \quad [1]$$

where Ω is a damage parameter, whose rate of change is a function of load $f(t)$, its time derivatives (e.g., df/dt , d^2f/dt^2), the value of Ω , moisture content m and temperature T , and their time derivatives (e.g., dm/dt , d^2m/dt^2 , dT/dt , d^2T/dt^2). Obviously, the dependency on the moisture content and its time derivatives is included with the consideration of its well-known effect on accelerated creep and creep failure. The above expression, called Coleman's breaking rule of Class C, could include not only the damage evolution but also damage *healing* effects, and is a very convenient form for including various damage evolution rules. The dependency on load $f(t)$ has been investigated intensively in the past, for example from the point of reaction kinetics (Coleman 1956), but in this study we will tentatively take the most frequently-used form for the discussion purpose:

$$\frac{d\Omega}{dt} = \phi(t) = af(t)^\rho \quad [2]$$

where a and ρ are material constants. (Note that the dependencies on m and T are suppressed for the moment.) The exponent ρ represents the sensitivity of damage evolution to load $f(t)$: the higher the ρ , the damage grows faster. Integrating the above equation gives:

$$\Omega(t) = \int_0^t \phi(s) ds \quad [3]$$

This indicates that damage at time t is a function of the entire histories of load. Given the damage at time t , such as Eq (3), we can obtain the probability that the “fibre” fails between 0 and time t_B , by precisely following the extreme statistics argument (Coleman 1958):

$$F(t_B) = 1 - \exp\{-b\Omega(t_B)^\beta\} = 1 - \exp\left\{-c \left[\int_0^{t_B} f(t)^\rho dt\right]^\beta\right\} \quad [4]$$

where $F(t_B)$ is the probability that the failure occurs at time less than t_B , b and c are material constants, and the exponent β is related to the distribution (or scatter) of life time. It should be noted that this equation is applied for varying loading histories, such as cyclic fatigue tests, typical stress-strain tests, and random loading (Coleman 1958). Here we recover some of them for the discussions in the next section. Under creep condition ($f(t)=f_0$ for $t>0$), Eq (4) becomes

$$F(t_B) = 1 - \exp\{-cf_0^\rho t_B^\beta\} \quad [5]$$

That is, life time t_B follows Weibull distribution in the case of *fibre*. The mean life time and coefficient of variation in this case are given by (Coleman 1958)

$$Ex\{t_B\} = \Gamma(1 + \frac{1}{\beta}) [cf_0^\rho]^{-1/\beta} \quad \text{and} \quad [6]$$

$$\frac{\sigma_B}{Ex\{t_B\}} = \left[\frac{\Gamma(1 + \frac{2}{\beta})}{\Gamma^2(1 + \frac{1}{\beta})} - 1 \right]^{1/2} \quad [7]$$

where $\Gamma(\cdot)$ is the Gamma function. As seen in Eq (6), the average life time of fibre decreases with applied load very sensitively, depending on the exponent ρ . Second, the coefficient of variation is a unique function of another exponent β . To the authors' knowledge, there have been no experimental work that systematically measured life time on the level of fibre or paper/board, but we will leave these equations as the background information for interpreting existing data in the literature.

The above model typically describes the case, like Fig 1a, where the fibre fails when the element of the shortest life time fails. However, fibrous materials, such as paper and board, have hierarchical structures, actual failure mechanisms are much more complex than the case of Fig 1a. This situation is true for many other materials. For most of the non-brittle materials, the failure of one element doesn't lead to the entire failure, but the released load by the failed element is normally redistributed to other neighbouring fibres, and the propagation of the element failures highly correlated, but still influenced by

Fig 1. Hierarchical structure model.

stochastic structures. There have been no general treatments of this dynamic stochastic failure. One interesting structural hierarchy is “fibre bundle”, as shown in Fig 1b. In spite of its very simple appearance, this structure possesses rich statistical mechanics properties, and can present general time-dependent, stochastic failure phenomena that were shown with more complex network structures (Alava et al. 2006). In this model, as fibre fails, the load released by the failed fibre is carried by the neighbouring fibre(s) according to a load sharing rule (local load sharing). The system failure occurs when all fibres in the bundle fail. Life time of this system depends on the system size (the number of fibres N), and load, as well as the parameters that describe the failure characteristics of the element fibre (β and ρ).

Fig 2 shows the distribution functions of life time for the fibre bundles of different size N for $\beta=10$ and $\rho=3$ (Juntunen and Uesaka 2011). The distribution functions are plotted in the Weibull form so that if the distribution follows Weibull distribution, the plot should appear as a straight line. Clearly, as the system size increases, the distribution curves deviate from Weibull distribution. One interesting feature is that when plotting the curves by vertically shifting them by $\log(N)$, the curves tend to collapse into one single curve with increasing N . For $\beta=1$, Newman and Phoenix (Newman and Phoenix 2001) showed that the curves indeed collapsed into one as N increases. They also showed that as ρ increases, the curves showed much faster collapse. This special scaling behaviour presents weakest-link scaling, such as shown below:

$$1 - F_N(t_B) = (1 - F_0(t_B))^N \approx \exp\{-N \cdot F_0(t_B)\} \quad [8]$$

for $N > N_c$. The characteristic function $F_0(t_B)$, representing the collapsed curve, is the distribution function that is achieved asymptotically with increasing N . If there is another level of hierarchy, such as shown in Fig 1c, called the chain of bundle, the distribution function of this structure is given by

$$1 - F_{MN}(t_B) = (1 - F_0(t_B))^{MN} \approx \exp\{-MN \cdot F_0(t_B)\} \quad [9]$$

where M is the number of chains in the chain of bundle structure of Fig 1c. Essentially, Eqs (8) and (9) (or Figs

Fig 2. Weibull plot of distribution functions of life time for the fibre bundles with different system sizes. $\beta=1$ corresponding to highly disordered fibre, and $\rho=2$ to relatively low load sensitivity of damage growth.

Fig 4 Creep life time vs. applied load for A-flute corrugated container box. The data was reconstructed from (Koning and Stern 1977).

I_b and I_c have the same structure. Therefore, once we determine the functional form of $F_0(t_B)$, either experimentally or analytically, we can determine the distribution functions for various hierarchical systems, such as Figs 1b and 1c (Uesaka 2007, Uesaka 2011). It should be noted that although Eq (8) or (9) takes the form of weakest-link scaling, the distribution is *not* Weibull distribution, as seen in Figs 2 and 3. Generally the distribution curves show a convex shape in the Weibull plot. As is also evident in Eq (8) or (9), life time distribution of the larger systems is controlled by the low tail of the characteristic function $F_0(t_B)$. At this moment, there is no concrete expression for $F_0(t_B)$ for a general case of β and ρ , but Newman and Phoenix obtained approximate expressions for $\beta=1$ (Newman and Phoenix 2001). In our separate study we have found some small deviation from the weakest-link scaling for a certain combination of β and ρ . Details will be reported elsewhere (Juntunen and Uesaka 2011).

Fig 3. Weibull plot of distribution functions of life time for the fibre bundles with different system sizes. $\beta=10$ corresponding to more uniform fibre, and $\rho=5$ to medium load sensitivity of damage growth.

Time-dependent, stochastic failure of corrugated board and box

In this section, we will look at some of the published data of time-dependent, stochastic failure of corrugated board and box from the light of the theoretical framework discussed above. The literature abounds in the creep deformation and the relation between creep rate (particularly secondary creep rate) and creep life time (for example, Coffin 2011). However, it is surprisingly rare to see the studies on the stochastic aspects of failure, such as the distribution of life time and how it is affected by various loading conditions. It is partly because the creep measurement itself is already extremely time-consuming, and collecting a large number of data under the same and controlled conditions is also very difficult. Nevertheless, we will here try to extract some useful information from earlier work.

The easiest method to extract information on the parameters ρ and β is creep test. If the system follows the statistical distribution for the fibre level (i.e., Eq (5)), then the mean life time $Ex\{t_B\}$ and its coefficient of variation $\sigma_B/Ex\{t_B\}$ are given by Eqs (6) and (7), respectively. By plotting the logarithm of both sides of Eq (6), we can determine the parameter ρ (the force sensitivity to damage evolution) from the slope, and by obtaining the coefficient of variation, we can also determine β (the distribution of life time). Although the creep failure data in the literature is extensive, very few systematic data are, unfortunately, available for component board or paper. We will take Koning's and Stern's data (Koning and Stern 1977) for creep life time of corrugated box. It should be noted that boxes and corrugated panels possess higher level hierarchical structures which may not be described in Fig 1-b or Fig 1-c, and therefore, the discussions here are still of general nature.

Fig 4 shows the creep life time vs. load relation. Because of the scatter of the data, which is inherent to this type of test, it is difficult to judge linearity, but the

Fig 5 Weibull plots of life time distribution function for C-flute corrugated boxes (type 1). The data were reconstructed from (Nyman 2004).

Fig 7 Weibull plots of life time distribution function for C-flute corrugated boxes (type 2). Applied load=500 N. The data were reconstructed from (Nyman 2004).

slope showed an extremely high value of $\rho=14$, i.e. extreme sensitivity of box life time to applied load.

Analysing other data obtained by Morgan (Morgan 2005), Stott (Stott 1959), and Moody (Moody 1966) also yielded ρ values of 13 to 27 for corrugated boxes, and 3-7 for corrugated panels. At this moment, the factors influencing the load sensitivity parameter of damage evolution (ρ) are not known, but it would be interesting to launch systematic experimental studies for this property. In addition these high values of ρ also implies that, if there is any misalignment in box stacking, the loading non-uniformity created in such a way will immediately affect its life time and distribution (Coffin 2011).

The measurements of the corresponding coefficient of variation, COV (or β) also suffered the limited number of samples (e.g., typically 5 samples at a given load). The data in the above mentioned literature showed the extremely high values of COV (large scatter) in the range of 0.710 to 0.913. It should be noted that these data were obtained under still *nominally* the same environmental conditions of temperature and humidity.

Recently, a much larger number of samples were tested in creep for C-flute box by Nyman (Nyman 2004). About 30 boxes were subjected to a given load, and creep life time was measured under varying environmental conditions that are closely monitored. Fig 5 shows Weibull plots of life time distributions at different applied loads. The temperature and humidity histories were nominally the same for the samples loaded at 500 N and 1000 N. As seen in the figure, it is difficult to judge

Fig 6 Weibull plots of life time distribution function for C-flute corrugated boxes (type 2). The data were reconstructed from (Nyman 2004).

Fig 8 Mean life time against applied load. The data were reconstructed from (Nyman 2004).

whether the data follows Weibull distribution (straight line in Weibull plot), and in the case of structured systems like boxes, Weibull distribution is not expected, as discussed earlier in Figs 2 and 3. As load increases, the life time distribution curves shifts toward shorter times, as expected, but the shapes of the distributions also seem to change with load level.

Such change in the distribution shape is more significant for another type of C-flute corrugated box (see Fig 6). In this case, the width of the distribution is much wider than the previous type (type 1) in Fig 5. For example, the mean life time at 500 N load was 21.4 days, whereas the shortest life time in this population was only 2.5 days. In both cases, it is obvious that the shape of the distribution changes with applied load. The effect of load on life time distribution is analytically predicted in the case of Eq (5) (Weibull distribution for fibre) and the case of $\beta=1$ for the fibre bundles. Both predict that load scales with time, so that life time distribution shifts in the horizontal axis in the Weibull plot as the load is changed. The above results clearly showed otherwise, suggesting the macro-structure effects.

The effects of environment were clearly demonstrated in Fig 7, where the type 2 boxes were subjected to 500 N load, and the test was repeated at a different time. The difference in the environment mainly came from temperature changes (around 0 degree for rectangular data points and 20 degrees for circles) and associated relative humidity changes. As seen in the figure, the

mean life time varied from 11.4 days to 21.4 days at the *same* applied load.

Fig 8 shows the plot of mean life time against applied load in a logarithmic scale for the type 1 and type 2 boxes. Each point represents the average of about 30 measurements. There is a large difference in the slope, that is, the load sensitivity factor $\rho=1.7$ vs. 6.6. Interestingly, the coefficient of variation (COV) varied from only 6.3% up to 91% for the whole data set (Nyman 2004). (Note that the COV values are obtained for about 30 samples.)

Concluding Remarks

As is clearly seen in the data analysed based on the literature, the stochastic aspect of life time is extremely important. If life time varies with the coefficient of variation of 90% under nominally the same loading and environmental conditions, we must seriously question designing box and board based on the mean value. In practice it is the low tail of the distribution that is perceived as the failure of the boxes. Secondly, it is also interesting to note that some boxes showed very little variation (COV) of life time, only 6%. Another interesting aspect is the load sensitivity factor ρ , which also varied greatly from 1.7 to 27 within the literature data analysed. It would be interesting to collect, systematically, the data for ρ and COV (or β) to identify furnish, papermaking, converting and design factors that control these parameters. The time-dependent, stochastic failure model presented in this paper provides a theoretical framework for the analyses of experimental data, and also can be used to predict life time and life time distribution under *varying* load.

Acknowledgements

The authors would like to thank financial supports from Bo Rydin Foundation. This study was inspired by various discussions of one of the authors (T. Uesaka). He would like to thank Christer Söremark for his introduction of time-varying load effects on life time, people at SCA R&D centre in Sundsvall, Sweden for their demonstration of this outstanding problem with overwhelming historical data, and people from Billerud both in Stockholm and Gruvön, Sweden for their introduction of the concept of CCT10. The authors benefitted immensely from these discussions.

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