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# Point spreading in turbid media with anisotropic single scattering

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**Abstract:** Point spreading is investigated using general radiative transfer theory. We find that the single scattering anisotropy plays a significant role for point spreading together with the medium mean free path, single scattering albedo and thickness. When forward scattering dominates, the light will on average undergo more scattering events to give a specific optical response in reflectance measurements. This will significantly increase point spreading if the medium is low absorbing with large mean free path. Any fundamental and generic model of point spreading must capture the dependence on all of these medium characteristics.

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**OCIS codes:** (030.5620) Radiative transfer; (100.2810) Halftone image reproduction; (170.3660) Light propagation in tissues; (290.2558) Forward scattering; (290.4210) Multiple scattering; (290.7050) Turbid media.

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## 1. Introduction

The point spread function (PSF) describes the response of a medium to a point light source. The lateral distribution  $D(x, y)$  of the reflected light is obtained by integrating the PSF and the illumination  $I$  over the reflecting medium, that is

$$D(x, y) = \int_{\Delta v} \int_{\Delta u} PSF(x, y; u, v) I(u, v) du dv \quad (1)$$

where  $\Delta u$  and  $\Delta v$  delimit the medium. In the present work we deal with laterally homogeneous turbid media with point source illumination. The PSF is then isotropic, i.e., it depends on radial distance only, and the reflected light is given by  $D(r) = PSF(r)I$  if the illumination is incident at radial coordinate  $r = 0$ .

The PSF is a medium characteristic and is related to the scattering and absorbing properties of the medium. Several attempts have been made to model the PSF or its Fourier transform, the modulation transfer function (MTF). Oittinen [1] proposed a Kubelka–Munk (KM) [2] based model of the PSF of paper substrates. Engeldrum and Pridham [3] found that Oittinen's model poorly described layered substrates and that it described plain papers such as newsprint well only at low spatial frequencies. Arney et al. [4] showed that the Oittinen model overestimated the effect of absorption on the PSF, and proposed a simple model based only on the KM scattering coefficient. Gustavson [5] and Emmel [6] used an exponential approximation of the PSF. Mourad [7] considered lateral fluxes in a KM framework leading to a much more complicated form of the KM based PSF that better fit experimental data from Arney et al. [4].

The PSF is closely connected to the phenomenon known as optical dot gain, when printed dots are perceived larger than their actual physical size due to lateral spreading of light. This is important in graphic arts where continuous tones are reproduced through halftoning using dots varying in size and spacing. Optical dot gain then makes the printed image appear darker than expected, which has to be compensated for. Point spreading is also important in computer rendering for a realistic appearance of rendered images [8], and in optical tomography where light scattering is used for medical diagnosis [9].

General radiative transfer (RT) theory can also be used to study the PSF and optical dot gain of a medium. While the KM model approximates the light intensity within a medium with two diffuse fluxes, general RT theory is angle resolved. A basic reference for RT is Chandrasekhar [10]. Considering light of a single wavelength, the familiar RT equation can be stated as

$$\frac{dI(x, y, z; \theta, \varphi)}{ds} = \sigma_e [-I(x, y, z; \theta, \varphi) + S]. \quad (2)$$

The symbols have their usual meaning;  $I(x, y, z, \theta, \varphi)$  is intensity at position  $(x, y, z)$  at polar angle  $\theta$  and azimuthal angle  $\varphi$ ,  $s = (x^2 + y^2 + z^2)^{1/2}$  is distance,  $\sigma_e$  is the extinction coefficient

and  $S$  is a source function. The extinction coefficient is the sum of the scattering and absorption coefficients  $\sigma_s$  and  $\sigma_a$  and also the inverse of the mean free path  $\ell_e$ . The source function accounts for light scattered to  $\theta, \varphi$  at position  $(x, y, z)$  from all other directions. It can be written

$$S = \frac{a}{4\pi} \int_{4\pi} p(\cos \Theta) I(x, y, z; \theta, \varphi) d\omega, \quad (3)$$

where  $a$  is the single scattering albedo defined as  $a = \sigma_s / (\sigma_s + \sigma_a)$ ,  $\omega$  is solid angle and  $p(\cos \Theta)$  is the phase function. Here  $\Theta$  is the angle between the directions of the incident and scattered light. The phase function describes the angular distribution of the single scattering process. A commonly used phase function is the Henyey–Greenstein (HG) phase function [11] which contains a single parameter, the asymmetry factor  $g$ . It ranges from  $-1$  to  $1$  with  $g = -1$  meaning complete back scattering,  $g = 0$  isotropic scattering and  $g = 1$  complete forward scattering. Also,  $g$  is the average of the cosine of the scattering angle and the first moment in an expansion of any phase function. For organic materials such as paper it has been shown that  $g$  has values in the interval 0.6–0.9 approximately [12–14].

We can characterize a turbid medium with the albedo  $a$ , the mean free path  $\ell_e$  and the asymmetry factor  $g$ . These medium characteristics are independent of position in a homogeneous medium but normally vary between wavelengths. Equation (2) can then be solved for each wavelength if the wavelengths are independent. This is not the case in fluorescing media, but in the present work we do not consider fluorescence.

Considering only one spatial coordinate Eq. (2) becomes a 3-dimensional problem and can be solved numerically using e.g. the method of discrete ordinates (DORT) and a Fourier expansion of the azimuthal dependence [15]. In the present work we are interested in laterally resolved (5-dimensional) radiative transfer and we choose to use Monte Carlo simulations since this problem has not been solved using DORT-methods. Coppel et al. [16] presented an open source Monte Carlo model called Open PaperOpt (available at <http://openpaperopt.sourceforge.net>), adapted to simulations of light scattering in paper and print that is suitable for the problem at hand.

Previous work on point spreading using general RT theory includes Gustavson [5] who developed an approximate Monte Carlo model of radiative transfer in turbid media to study point spreading, but considered isotropic single scattering only. Furthermore, Sormaz et al. [17] presented a method to speed up Monte Carlo simulations and used optical dot gain as an illustrating example. Chen et al. [18] compared measurements of transmittance through paper with the HG phase function, in this case representing the angle resolved transmittance. This is not correct if light can be multiply scattered, which is the case for paper media. When multiple scattering is considered the framework of general RT theory is necessary since the KM approach to the radiative transfer problem has been shown insufficient [19, 20].

The general purpose of the present work is to investigate point spreading using the framework of RT theory, which has not previously been done exhaustively. More specifically we want to determine how the point spreading is related to the medium parameters  $a$ ,  $\ell_e$  and  $g$ . In this way we can indicate if this dependence can be captured by a simplified model and give important guidelines when developing simplified models of point spreading.

## 2. Method

### 2.1. Material

Relevant numerical values for the medium parameters  $\ell_e$  and  $a$  are assessed using a set of paper samples. We choose two lightly dyed and two non-dyed 30 g/m<sup>2</sup> paper samples with and without fillers, giving four samples in total. Samples varying in filler content differ in mean free path  $\ell_e$  since the addition of fillers significantly increases scattering, thus decreasing the

mean free path. The addition of a blue dye increases absorption, thus decreasing the albedo, in the wavelength interval 550–700 nm approximately, and the effect of increased absorption on point spreading can be investigated by studying wavelengths in this interval. No samples contain fluorescent whitening agents. We denote the samples M1–M4, where M1 contains no dye or filler, M2 contains dye but no filler, M3 contains no dye but filler and M4 contains both dye and filler.

## 2.2. Estimation of medium parameters using DORT simulations

The paper industry uses standardized reflectance measurements to assess the reflectance factor in d/0 geometry (diffuse illumination and detection in the normal direction) [21]. We use these measurements to determine  $\ell_e$  and  $a$  by measuring the reflectance factor from a single sheet and an opaque pad of paper sheets. We then get a well-posed optimization problem that can be solved for  $\ell_e$  and  $a$ , e.g., by using the RT based DORT2002 model [15, 22] (freely available). This model can accurately describe the illumination and detection conditions of the d/0 instrument. The asymmetry factor  $g$  is varied from 0 to 0.8 in steps of 0.2, and the inverse RT problem is solved for each of these  $g$  values. We include  $g = 0$  since this is an assumption in the KM model. Each parameter setup  $[a, \ell_e, g]$  will then give the same optical response in the d/0 instrument for the particular medium studied, despite the variations in  $g$ , and the medium thus has the same scattering power irrespective of the  $g$  value.

## 2.3. Monte Carlo simulations of the PSF

When we have the parameter setup  $[a, \ell_e, g]$  we can estimate the PSF of a medium with a given thickness  $t$  through Monte Carlo simulations. We do simulations using both the thickness  $t$  corresponding to 30 g/m<sup>2</sup> paper and an opaque medium where  $t \rightarrow \infty$  in order to study the influence of transmittance on the PSF. The thickness of the paper samples is measured with a micrometer and found to be 65  $\mu\text{m}$ . We simulate illumination incident normally on a point and to minimize noise  $10^8$  wave packets are simulated in each run. We choose the wavelength of light that is most heavily absorbed by the blue dye (620 nm) to represent the four media M1–4.

The PSF is obtained by averaging the simulation data over azimuthal angle. We then introduce a single number metric  $\bar{r}$  to represent point spreading. This metric is the mean radial distance a wave packet travels before exiting the medium, i.e.,

$$\bar{r} = \left[ \sum_i PSF(r_i) \right]^{-1} \sum_i r_i PSF(r_i). \quad (4)$$

The metric we introduce here is in the distance domain, as opposed to the frequency domain metric  $k_p$  used by other authors and introduced by Arney [4], which is the frequency where  $MTF = 0.5$ . A distance domain metric is more useful when dealing with point spreading since the frequency resolution of the reflected light then is of minor interest.

Furthermore, we calculate using the Monte Carlo simulations the mean number of scattering events that a wave packet undergoes before leaving a medium. This allows for important conclusions to be drawn about the dependence of point spreading on the medium characteristics.

## 3. Results

### 3.1. Medium parameters

Table 1 shows the values of  $a$  and  $\ell_e$  obtained from d/0 measurements when  $g$  is varied. As expected M2 and M4 have the lowest albedos and M3 and M4 have the shortest mean free path. We can see that the mean free path decreases as  $g$  increases. This can be understood intuitively

since if the light is scattered more in the forward direction when it impinges on the medium surface, the medium must be highly scattering in order to reflect the measured amount of light towards the detector. In this way the scattering power is the same irrespective of the  $g$  value.

Table 1. Albedo ( $a$ ) and mean free path ( $\ell_e$ ) of the media at 620 nm obtained from d/0 measurements when the asymmetry factor  $g$  is varied. Parameters here are used in the Monte Carlo simulations

	M1		M2		M3		M4	
$g$	$a$	$\ell_e$ [ $\mu\text{m}$ ]						
0	0.9992	44.40	0.7717	42.52	0.9979	20.14	0.9038	19.53
0.2	0.9994	38.49	0.8204	34.33	0.9991	15.99	0.9253	15.63
0.4	0.9996	28.13	0.8670	25.90	0.9994	11.88	0.9454	11.72
0.6	0.9997	18.36	0.9120	17.41	0.9996	7.85	0.9644	7.83
0.8	0.9999	9.04	0.9562	8.81	0.9998	3.89	0.9825	3.92

### 3.2. Point spread simulations

The data in Table 1 is used in Monte Carlo simulations with the Open PaperOpt model. Fig. 1 shows how our PSF metric  $\bar{r}$  varies with  $g$  for the different media and for the two different thicknesses. It can be seen that  $\bar{r}$  increases with  $g$  for all media. The high albedo medium with large mean free path (M1) has the largest mean radial distance  $\bar{r}$ . The low albedo medium with short mean free path (M4) has the smallest  $\bar{r}$ . We can also see that  $\bar{r}$  is larger for the opaque media, with the most noticeable difference for M1. Hence, transmittance can obviously have a significant effect on point spreading. We can thus conclude that point spreading, as represented by  $\bar{r}$ , depends on asymmetry factor, albedo, mean free path and medium thickness.

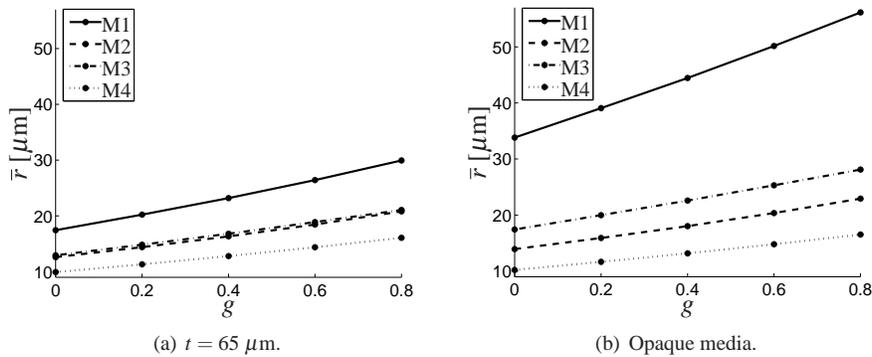


Fig. 1. Point spreading as represented by  $\bar{r}$  for two thicknesses of media M1–4 when  $g$  is varied. Point spreading increases with  $g$  for all media. M1 (high albedo and large mean free path) has the largest point spreading and the most noticeable difference when varying the thickness. M4 (low albedo and short mean free path) has the smallest point spreading.

Figure 2 shows the average number of scattering events that the wave packets undergo before exiting the medium. It can be seen that increasing  $g$  increases the number of scattering events. This holds for both thin and opaque media, but when the mean free path  $\ell_e$  is large (as for M1 and M2) the light is scattered fewer times in thin media. When the effect of transmittance is

eliminated (Fig. 2(b)) there is only a small difference in the number of scattering events when varying the mean free path (M1 vs. M3 and M2 vs. M4).

Based on these observations we can relate point spreading to the scattering properties of a medium. Media with high albedo (M1 and M3) have a similar contribution to the reflectance from different scattering orders. Media with low albedo (M2 and M4) are also similar in this respect. This means that the point spreading is determined by the distance that the multiply scattered light can travel. This distance is larger if the mean free path is large and absorption is low. A medium with high albedo and large mean free path will thus give the largest point spreading, and it will increase further if light is scattered more in the forward direction.

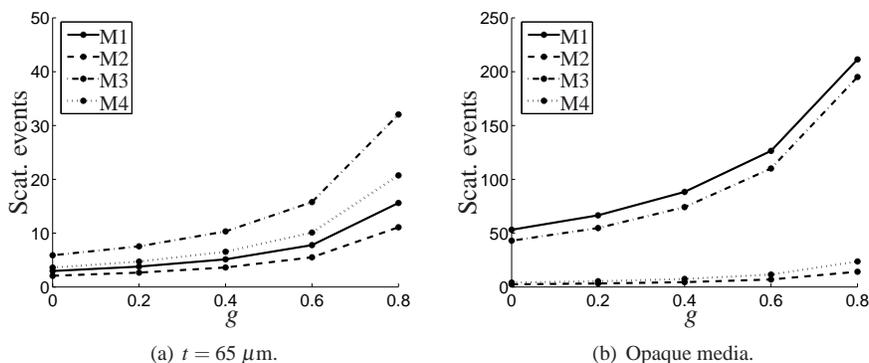


Fig. 2. The mean number of scattering events that the wave packets undergo before exiting the medium. We see that the asymmetry factor  $g$  affects the number of scattering events, and that increasing the albedo leads to more scattering events in opaque media.

#### 4. Discussion and conclusions

We have shown that the asymmetry factor  $g$  plays a significant role in point spreading together with the albedo, mean free path and thickness of a medium. Given an optical response from a medium, assumptions about the  $g$  value will greatly alter the point spreading. Any model for point spreading must take these medium parameters into account and describe their relative influence in order to be based on physics rather than ad hoc assumptions. Thus, models of point spreading cannot be single parameter models. A generic model of point spreading can be on the form

$$PSF(r) = \sum_i \alpha_i \exp(-\beta_i r) \quad (5)$$

where the constants  $\alpha_i$  and  $\beta_i$  can be determined from measurements or simulations and thereby related to  $a$ ,  $\ell_e$  and  $g$ . This could be a topic for future work.

This is of importance for example when predicting the optical dot gain to adjust halftone prints. By using a model based on fundamental knowledge it can be generic in the sense that the dot gain of any paper substrate can be estimated. Furthermore, a thorough understanding of lateral diffusion of light is important for fields such as image rendering and optical tomography.

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