Trycktekniskt Forskning

Performance and Application of the DORT2002 Light Scattering Simulation Model

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Abstract

Models for calculating the light intensity inside an illuminated turbid medium, e.g. paper, involve several numerical challenges, and are crucial for the paper and printing industries. DORT2002 is a fast and numerically stable solution procedure for this problem, which has been implemented in MATLAB. This report studies the performance and application of DORT2002, and is the documentation of an extensive test series.

All major steps that are taken to improve stability and speed of DORT2002 are discussed, and the resulting improvements are illustrated. Comparison of accuracy, applicability and speed between DORT2002 and three other models when applied to different sets of relevant test problems is covered.

The performance tests show that the steps that are taken to improve stability and speed of DORT2002 are very successful, together giving an unconditionally stable solution procedure to a problem previously considered numerically intractable, and together decreasing computation time with a factor 1 000-10 000 in typical cases and with a factor up to and beyond 10 000 000 in extreme cases. Further investigations and developments are suggested, that can have a large positive impact on computation time.

The application tests show very good agreement with three other model types. DORT2002 is shown to have better accuracy and much larger range of applicability than Kubelka-Munk, and to be much faster than Grace in relevant test cases. It is also shown that DORT2002 and Grace have good agreement, and that the agreement between the results from DORT2002 and DISORT – which is claimed to be "the finest radiative transfer software available" – is very good without exception, which gives strong support for the accuracy of DORT2002.

The conclusion is that Kubelka-Munk should be replaced with DORT2002 for most applications, and that a combination of Grace and DORT2002 should be used for accurate modeling of paper and print.

1. Introduction

Models for calculating the light intensity inside an illuminated turbid medium, e.g. paper, involve several numerical challenges, and are crucial for the paper and printing industries. The general problem, with several different applications, is known as the radiative transfer problem. The problem was for long considered intractable because of numerical difficulties. DORT2002 is a fast and numerically stable solution procedure for this problem, which has been implemented in MATLAB. This report studies the performance and application of DORT2002, and is the documentation of an extensive test series.

Chapter 1 gives a brief introduction and some notes and references. In chapter 2 the steps that are taken to improve stability and speed of DORT2002 are discussed, and the resulting improvements are illustrated. Chapter 3 covers comparison of accuracy, applicability and speed between DORT2002 and three other models when applied to different sets of relevant test problems. Suggestions for future work are given in chapter 4, and chapter 5 gives some concluding remarks. Some larger test cases are given in appendix.

1.1. Some Notes and References on Radiative Transfer

Radiative transfer theory describes the interaction of radiation with scattering and absorbing media. Solution methods for radiative transfer problems have been studied during the last century [1-10]. In the beginning most radiative transfer problems were considered intractable. Therefore coarse approximations were used, and methods developed slowly due to the lack of mathematical tools. Among the solution methods in use today are discrete ordinate methods.

Stamnes et al [11, 12] reported on a stable discrete ordinate algorithm, and later provided a complete software, DISORT. Thomas and Stamnes [13] also wrote a textbook on radiative transfer in the atmosphere.

1.2. Some Notes on DORT2002

DORT2002 is a numerical implementation of a discrete ordinate solution method for the radiative transfer problem in vertically inhomogeneous turbid media. The model is based on papers by Edström [14] and Edström and Lehto [15], which extensively treat a radiative transfer problem formulation and a solution method, including all necessary steps to get a fast and numerically stable solution procedure. The DORT2002 software is implemented in MATLAB, and is adapted to light scattering simulations in paper and print.

The mentioned papers on DORT2002 describe the underlying structures, e.g. intensity and phase function, and the relevant coupled systems of integro-differential equations that describe the intensities. The goal is to solve these equations under certain assumptions for the equation's kernel, i.e. the phase function. If the interaction described by the phase function depends on the scattering angle only, the complexity can be reduced. Fourier analysis on [-1,1] then gives a system of uncoupled equations, one for each Fourier component. The integral equations are then discretized using

numerical quadrature, which through the number of terms in the quadrature formula gives one more degree of freedom. This gives a system of first order linear differential equations. The natural solution procedure gives an eigenvalue problem, i.e. the initial problem has been transferred to a problem on eigenvalues of matrices. The eigenvalue problem is solved and the Fourier coefficients are found, which are then used to reconstruct the intensities. The situation with boundary and continuity conditions is also treated. Finally the problem of extending the computed intensity from the quadrature points to the entire interval [-1,1] is treated by creating interpolation formulas.

The main steps to get a numerically stable solution procedure include the continuous formulation with expansion in Legendre functions and Fourier cosine series, the evaluation of normalized associated Legendre functions, the choice of numerical quadrature, the matrix formulation of the discretization, the reduction of the eigenvalue problem, the preconditioning of the system of equations for the boundary and continuity conditions, and the avoidance of over- and underflow in the solution and interpolation formulas. Important are also the recognition of potential divide-by-zero situations, and reformulation of those.

Several measures are taken to make the code fast. This includes exploitation of the sparse structure of the system of equations for the boundary and continuity conditions. Several features allow high speed by maintaining accuracy at significantly lower number of terms in the quadrature formula than would otherwise be needed, or by automatically stopping calculations earlier when certain convergence criteria have been met.

2. Performance of DORT2002

This chapter discusses the steps that are taken to improve stability and speed of DORT2002. All major steps are covered in separate sections. The sections describe a problem to be overcome, discuss the actions taken, and illustrate the resulting improvements. Most actions are in more or less frequent use in other areas but seldom in this area, and so far not together.

2.1. Small Fundamentals for Stability

2.1.1. Evaluation of the Normalized Associated Legendre Functions

DORT2002 makes use of normalized associated Legendre functions, $\Lambda_l^m(u)$, which are related to the associated Legendre functions, $P_l^m(u)$. The normalized functions are preferred, since they remain bounded. The non-normalized functions can become large enough to cause overflow.

There are many ways to evaluate associated Legendre functions numerically, and a lot of them are bad. For example, explicit expressions involve cancellation between successive terms, which alternate in sign. For large *l*, the individual terms become larger than their sum, and all accuracy is lost.

The associated Legendre functions satisfy a number of recurrence relations on either or both of *l* and *m*. Most of the recurrences on *m* are unstable, and hence numerically unsuitable. DORT2002 uses a stable three-term recurrence on *l*.

It should be noted that the polynomial coefficients are not calculated, and the functions are not used explicitly. Instead the recursion is on the function values themselves. All together this constitutes a numerically stable way to compute the normalized associated Legendre functions.

2.1.2. Choice of Numerical Quadrature

One problem in radiative transfer is to approximate integrals on [-1, 1] with a finite sum, a numerical quadrature formula. Different choices of the weights and nodes give different quadrature formulas. If the nodes are taken linearly spaced from -1 to 1, there is a unique choice of weights that gives the quadrature an order of accuracy at least *m* - 1. This is known as a Newton-Cotes formula. It is simple and useful for small *m*, but for larger *m*, their weights have oscillating signs and amplitudes of order 2^m, which causes numerical instability.

Gauss showed that if not just the weights but also the nodes are chosen optimally, the result is a formula of order 2*m* - 1, which is the best possible. This is known as a Gaussian quadrature formula. The optimal nodes are the zeros of the Legendre polynomial $P_m(u)$. Furthermore, the weights are all positive, which makes the formula numerically stable even for large *m*.

There are closed expressions for the coefficients in the Legendre polynomials, but there is a risk of overflow for large *m*. A numerically stable method for finding the Legendre

polynomial coefficients is the Lanczos iteration, but it is still unstable to find zeros directly from polynomial coefficients. However, there is a closed expression for the Jacobi matrix used in the Lanczos iteration. By solving an eigenvalue problem for the Jacobi matrix, the optimal weights and nodes can be found without even forming the Legendre polynomials. The eigenvalues of the Jacobi matrix are the wanted nodes, and the weights are twice the square of the first component of the eigenvectors. Thus, this is a stable and fast method for finding the nodes and weights for a quadrature formula with optimal accuracy.

There is an advantage of using Gaussian quadrature of even order; the nodes come in pairs and the corresponding weights are equal. This symmetry is essential in the effective solution of the eigenvalue problems – it allows for reducing the eigenvalue problems with a factor 2 and thus the eigenvalue calculations with a factor 8.

Gaussian quadrature assumes that the integrand is a smooth function. It is known, however, that the intensity changes rapidly close to $u = 0$ near the boundaries. Furthermore, Gaussian quadrature has the nodes the least dense close to $u = 0$, where the intensity changes the most. In order to improve the situation, a modification to the Gaussian quadrature is used. Double Gauss approximates the integral over the two hemispheres separately, and the nodes and weights are chosen for the "half interval" $[0,1]$. For highest accuracy, the optimal Gaussian quadrature should be used on the new interval $0 \le \mu \le 1$ instead of the old $-1 \le u \le 1$.

2.1.3. Interpolation Formula for the Intensity

The general solution for the discrete problem gives the intensity at any depth, but only in the directions corresponding to the quadrature points. If the intensity in an arbitrary direction is wanted, an interpolation formula is needed.

It is always possible to fit a polynomial to a number of points. A polynomial of sufficiently high degree will be exact in all points to be fitted, but will normally perform badly in between. If a polynomial of lower degree is chosen, it will perform better between the points to be fitted, but on the other hand it will not be exact in those points, even though they are known. It is also possible to use cubical splines. They will be exact in all points to be fitted, but they will also perform badly in between if there are large changes in some point.

Another way is to make analytical interpolation formulas for the intensity expressed in the solutions of the eigenvalue problem for each layer. The interpolation formula for the intensity in DORT2002 is exact in the quadrature points. It also satisfies the boundary conditions for all μ, even though such conditions were imposed only at the quadrature points.

It is worth noting that the user supplied output polar and azimuthal angles are entirely decoupled from the quadrature points in the core calculations, and a high angular resolution does *not* require a large number of terms in the quadrature. In fact, it is one of the main features of DORT2002 to offer high accuracy and resolution at a small number of channels, thus giving a large decrease in computation time.

There is a risk for overflow in the general solution, as well as in the interpolation formula for the intensity, but by using the same scaling as with the boundary and

continuity conditions (see the section on preconditioning) the risk of overflow is avoided.

There is also a risk that the denominators in the interpolation formula can be close to zero. This risk can be entirely eliminated by identifying these situations and reformulating them analytically beforehand. If a denominator is close to zero, the corresponding term in the interpolation formula is simply substituted, which can be seen as an application of l'Hospital's rules.

2.2. Small Fundamentals for Speed

The DORT2002 code has been optimized in many ways, using known programming principles for MATLAB. This includes code vectorization to best exploit MATLAB's excellent matrix handling. The MATLAB Profiler has been extensively used to identify and reformulate functions and lines of code that allow for performance improvement.

Preallocation of large variables has been introduced, which prevents from reallocating variables dynamically as they grow. This saves memory and calculation time, and avoids memory defragmentation.

A great effort has been put into handling and exploiting the sparse structure of the systems of equations for the boundary and continuity conditions. Even moderately sized sparse systems of equations benefit from using sparse solvers instead of standard solvers that do not make use of the sparseness. Even if they are not so large that they do not fit into memory, the calculation time is drastically reduced. For large systems, the sparse storage is necessary to make the problem tractable, otherwise memory is exceeded and the solution time is too long.

2.2.1. Unresolved Problem

The system of equations for the boundary and continuity conditions has a sparse block diagonal structure. A system is generated and solved for each Fourier component. For the 0^{th} component the matrix has full diagonal blocks. For all other Fourier components even the diagonal blocks themselves are sparse.

Due to the internal representation of the sparse data structure in MATLAB, the generation of the sparse coefficient matrix is a bottleneck. The elements of the matrix arise blockwise in the solution procedure, one block from the eigenvalue problem for each layer. The nonzero elements that form the diagonal blocks and their row and column indices are known from the eigensolutions. This is used to make a mapping from the diagonal blocks to a list [i, j, s] of matrix elements, which is then used to create the sparse matrix (which is preallocated with `spalloc`) using `sparse(i,j,s)`.

Still, the pure administrative task of setting up the sparse matrix from known elements requires a factor of 10-100 more time than the actual solving of the sparse system of equations, and consumes around 75% of the total execution time, which is clearly unsatisfactory. This problem reduces the gain from exploiting the sparseness for small systems. Large systems, however, always reduce calculation time drastically when exploiting the sparseness. The implementation in DORT2002, as described above, has

been worked out in cooperation with technicians from Math Works, and is according to them the best that can be done in MATLAB today. Improvements in this direction have been set in sight in future versions of MATLAB.

2.3. Preconditioning

The multilayer solution contains $2N \times L$ constants to be determined. The boundary and continuity conditions constitute a $(2N \times L) \times (2N \times L)$ system of equations for the $2N \times L$ unknown coefficients. The coefficient matrix is sparse and block diagonal, with $6N$-1 diagonals. The blocks on the diagonal lead with a $3N \times 2N$ block, then follow L-2 blocks of size $4N \times 2N$, and the diagonal ends with a $3N \times 2N$ block.

These equations are ill conditioned due to exponentials with positive arguments. This is the reason for the fact that the method was considered as numerically intractable in the past, and consequently discarded. The ill conditioning can be removed with a scaling transformation as preconditioner. This makes all exponentials in the system of equations have negative arguments, the ill conditioning is avoided, and the problem of solving for the coefficients is unconditionally stable.

There is also a risk for overflow in the solution for each layer, but by using the same scaling in the solution and interpolation formulas as in solving the system of equations, the risk of overflow is avoided.

2.3.1. Condition Number

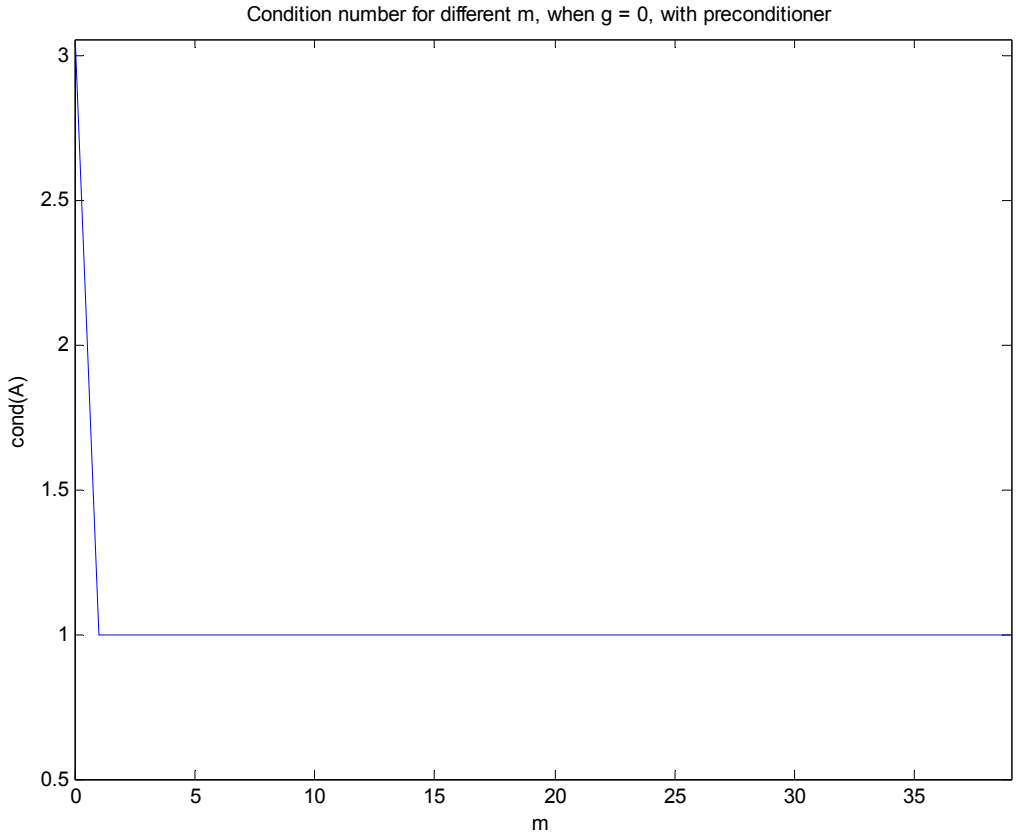
The following plots show the condition number of the coefficient matrix for the system of equations for boundary and continuity conditions with and without the preconditioner. The plots are generated with different L and m, i.e. with varying number of layers and Fourier component number, for three different asymmetry factors. The coefficient matrix is studied for the following parameter set, which is representative for any other set of parameters.

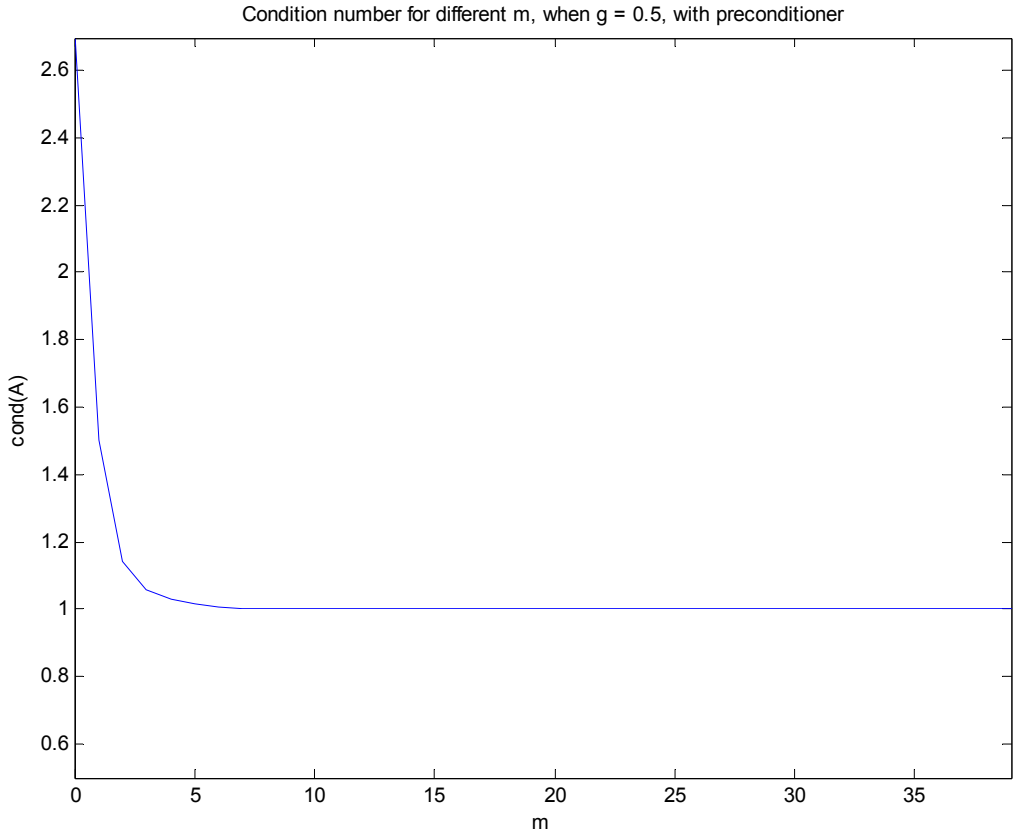
Diffuse intensity	0.3
Beam intensity	1.0
Beam polar angle cosine	0.5
Beam azimuthal angle	$\pi/2$
Depth at upper boundary	0
Underlying surface	Diffuse
Underlying surface reflectance	0.5
Number of channels	40
Maximum relative error	0
Number of layers	Varying from 1 to 10
Layer thickness (layer p)	$0.02/p$
Scattering coefficient	100
Absorption coefficient	10
Phase function	Henyey-Greenstein
Asymmetry factor	0, 0.5 or 0.9

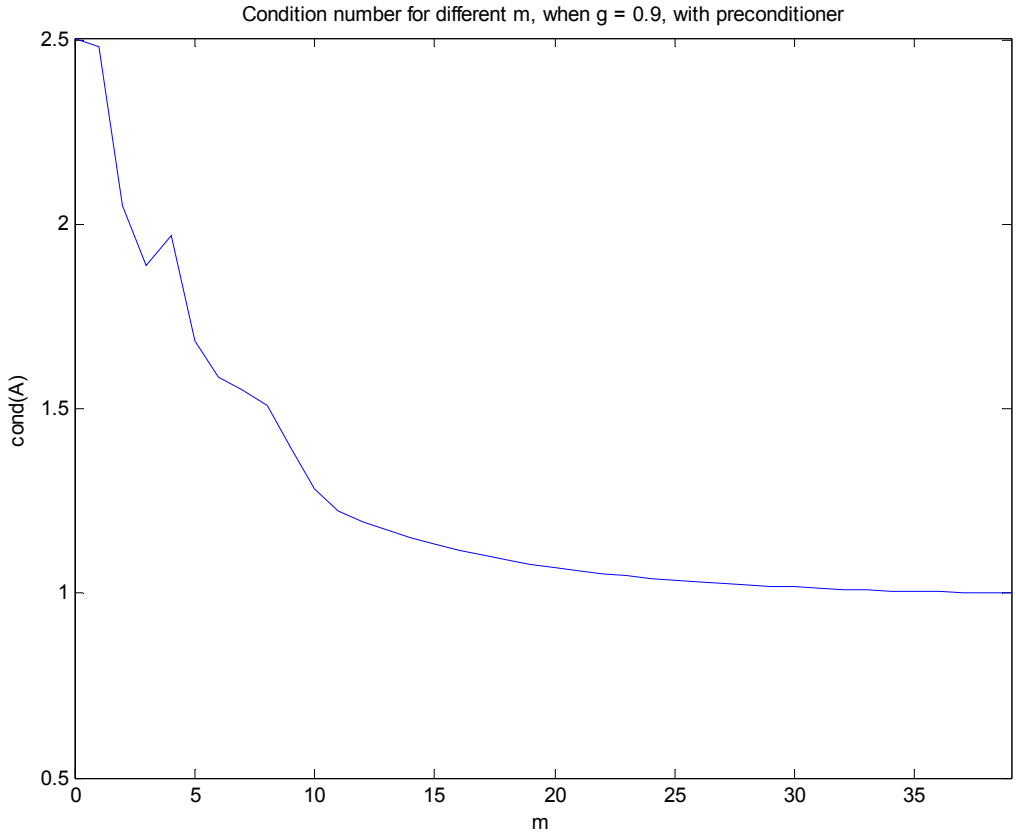
The plots show that the preconditioner works very well, giving condition number close to 1 in most cases, and around 30 in the worst case. They also show that the problem is

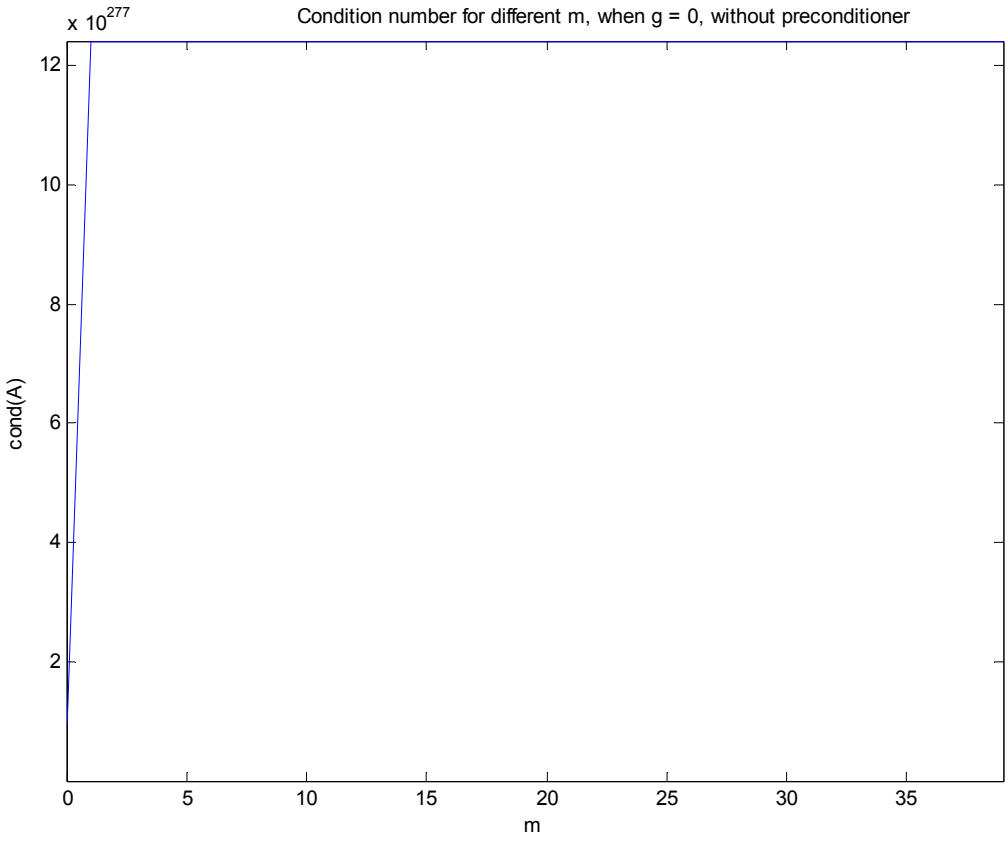
very ill conditioned without the preconditioner, having condition number near the largest positive floating-point number for the system (note the factor $10^{270} - 10^{300}$ on the axis scale in those plots).

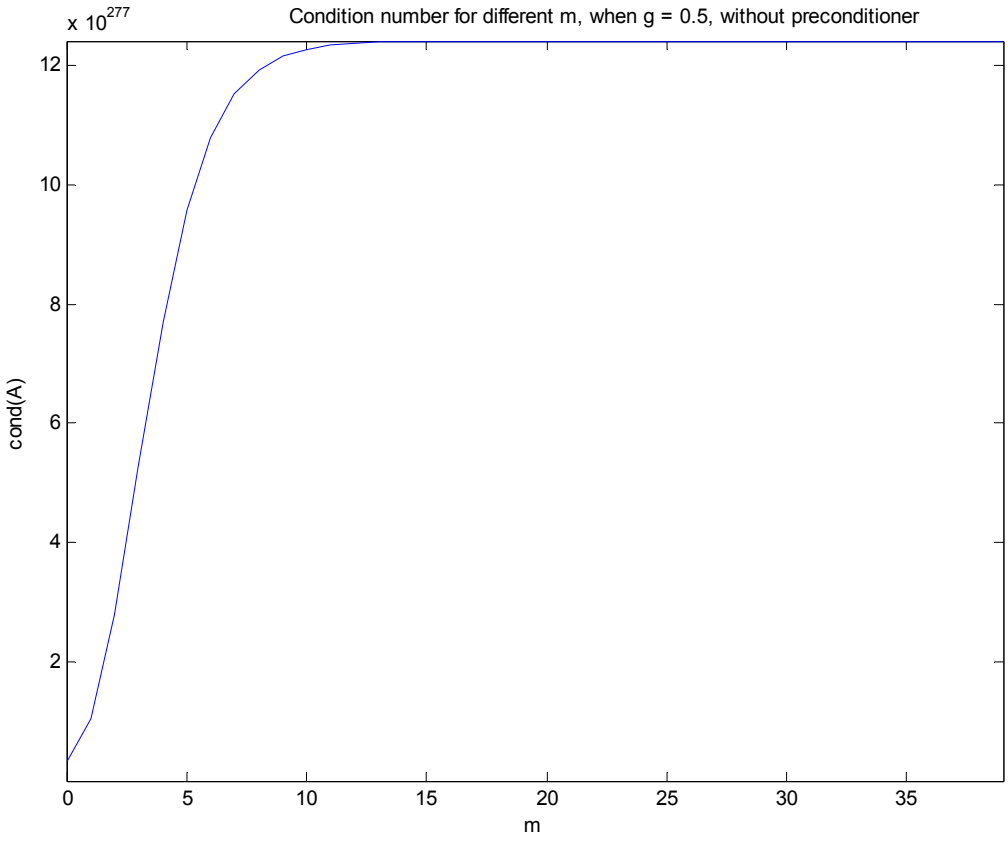
As can be seen, the 0^{th} Fourier component is the worst case, with condition number rapidly decreasing with increasing Fourier component number. It can also be seen that the condition number decreases slowly with increasing *g*.

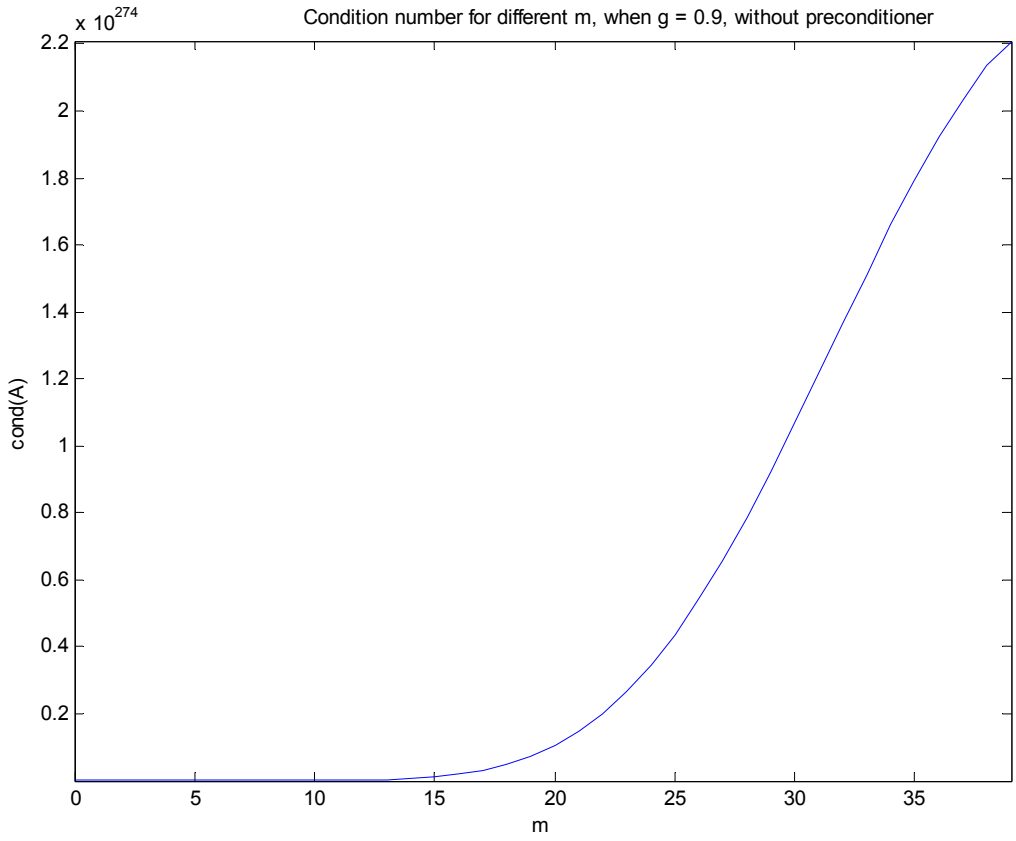


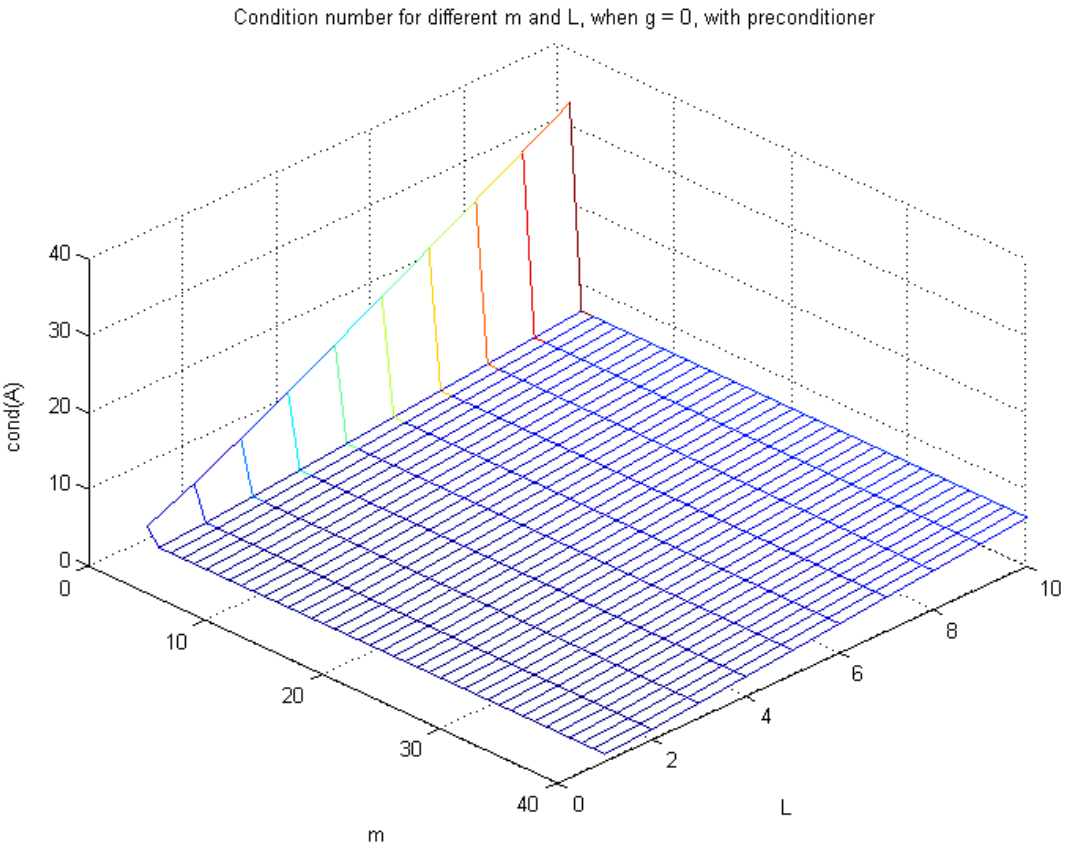

Condition number for different m, when g = 0.5, with preconditioner
cond(A)
m
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0
5
10
15
20
25
30
35

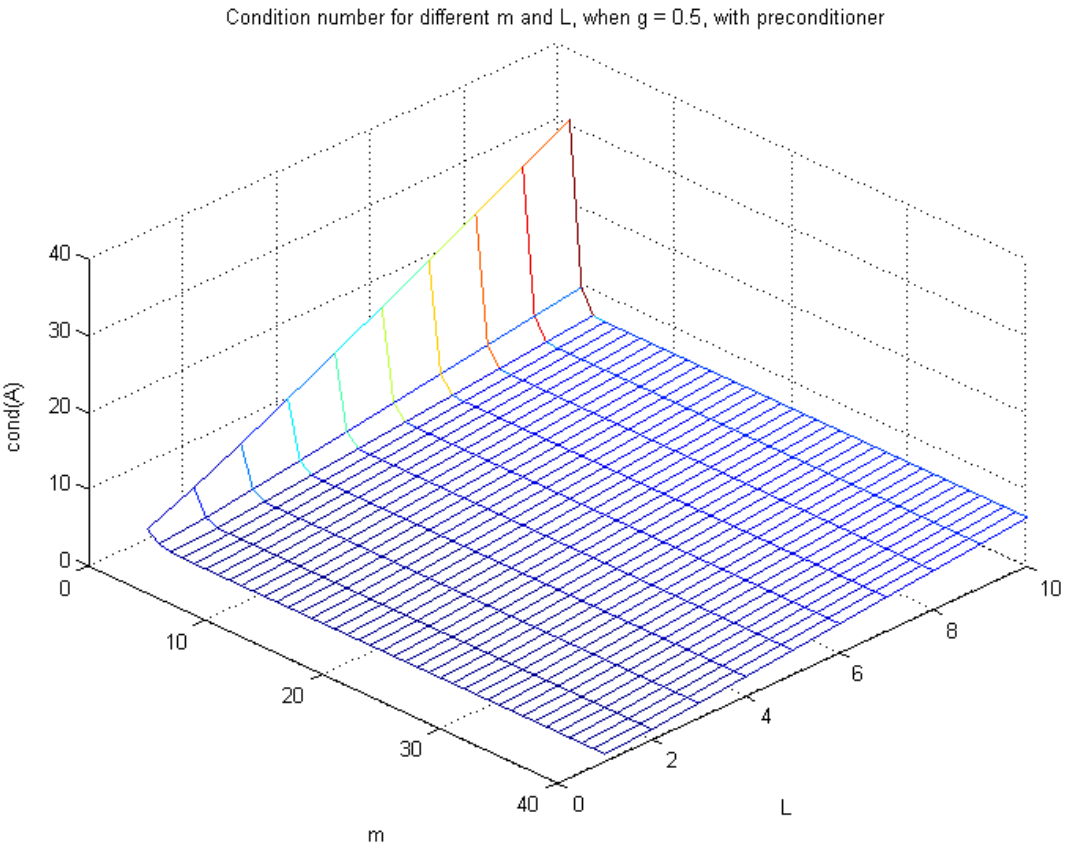

Condition number for different m, when g = 0.9, with preconditioner
cond(A)
m
2.5
2
1.5
1
0.5
0
5
10
15
20
25
30
35

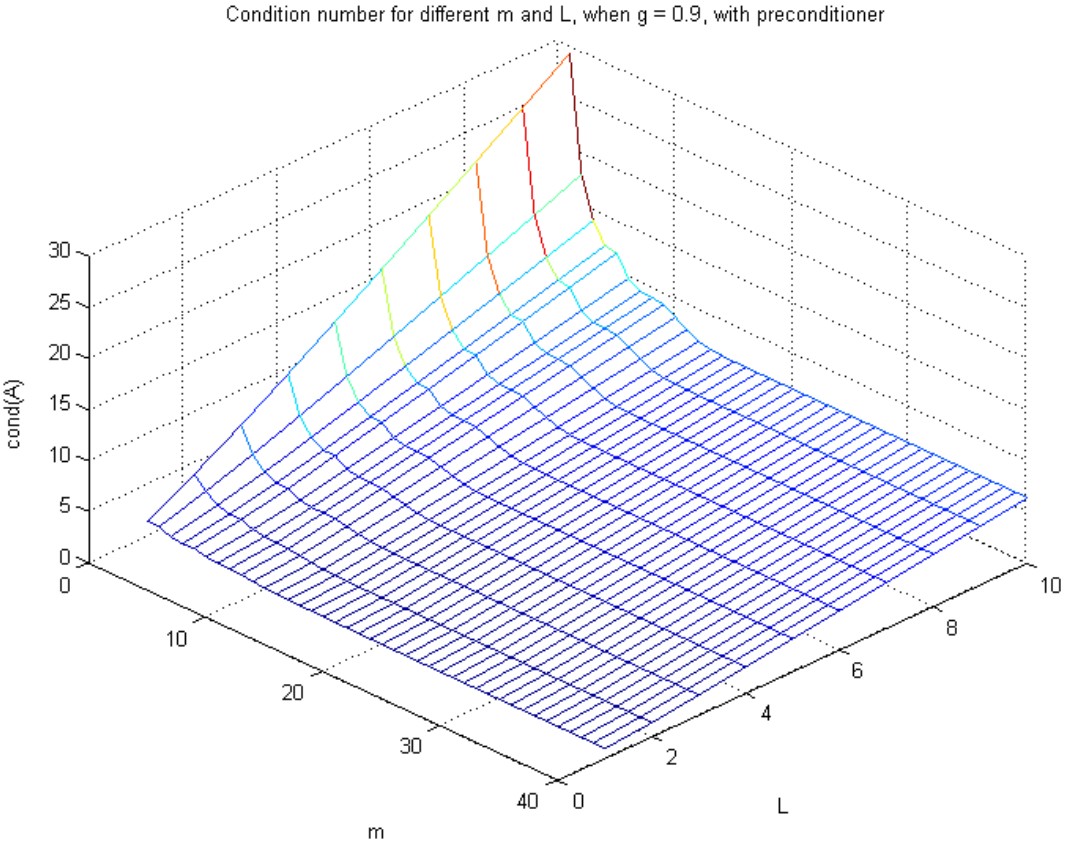

Condition number for different m, when g = 0, without preconditioner
x 10^277
cond(A)
m
0
5
10
15
20
25
30
35
2
4
6
8
10
12

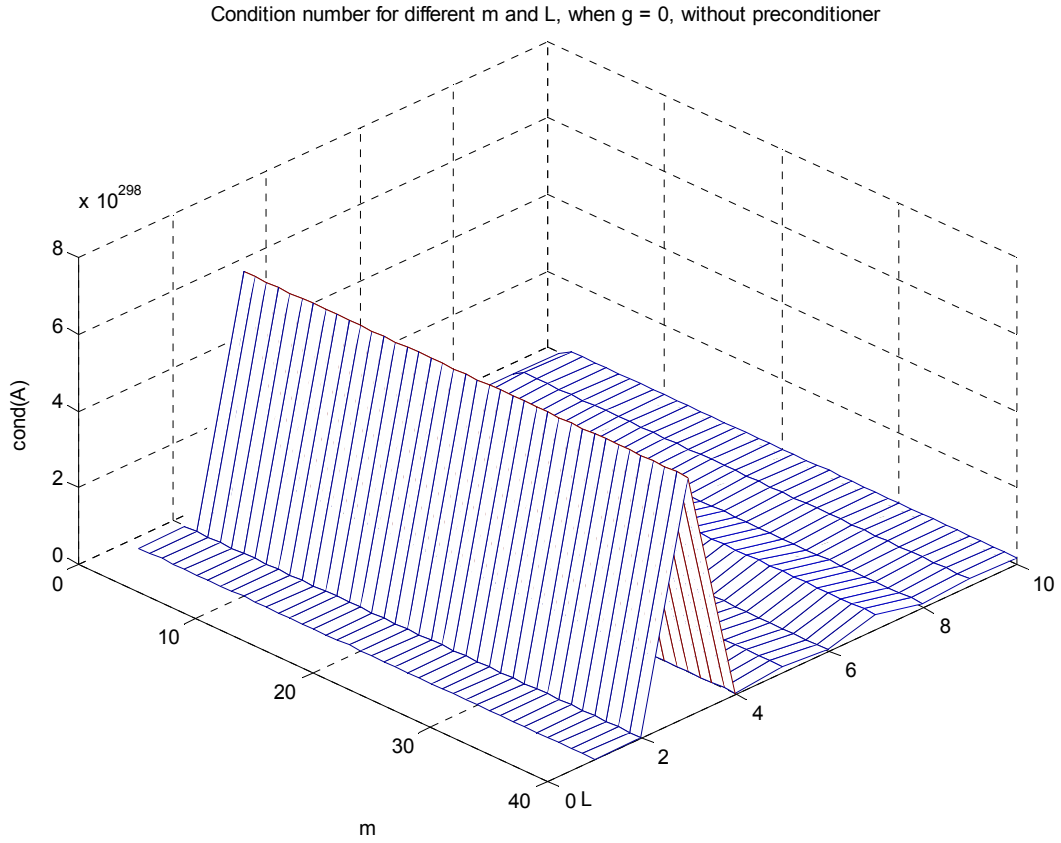

Condition number for different m, when g = 0.5, without preconditioner
x 10^277
cond(A)
m
0
5
10
15
20
25
30
35
2
4
6
8
10
12

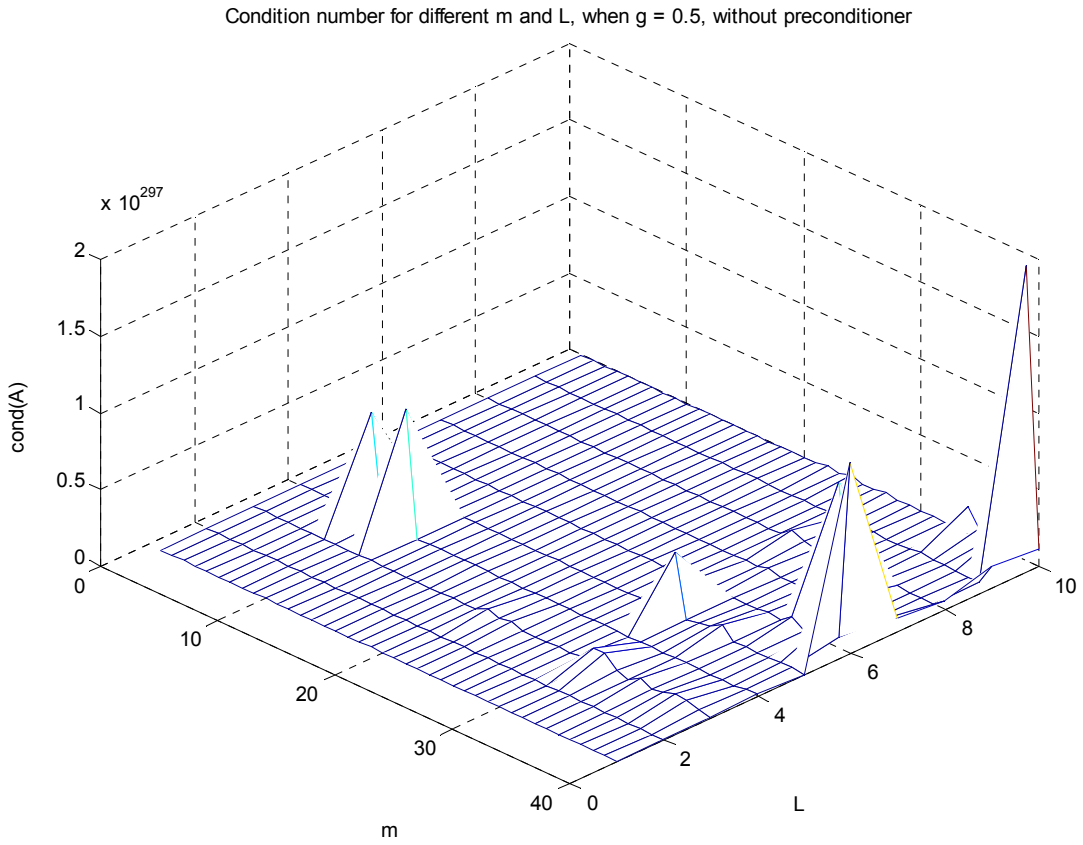

Condition number for different m, when g = 0.9, without preconditioner
x 10^274
cond(A)
m
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
5
10
15
20
25
30
35

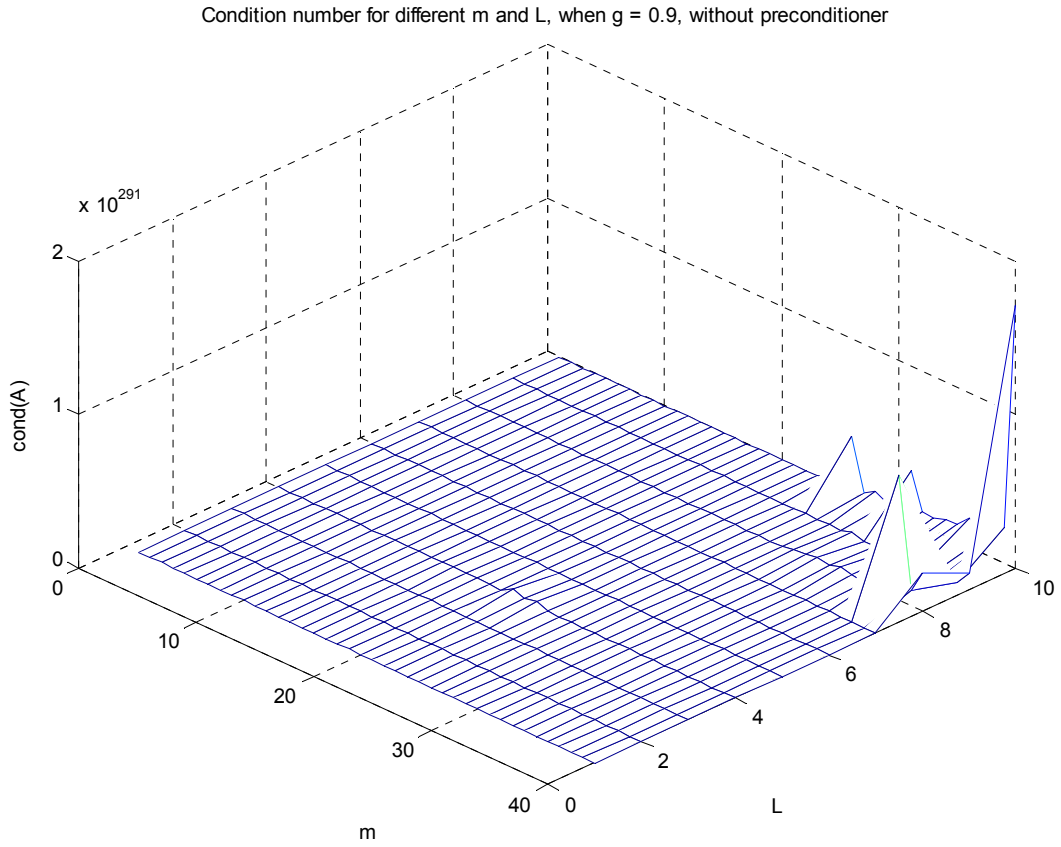

Condition number for different m and L, when g = 0, with preconditioner
cond(A)
40
30
20
10
0
0
10
20
30
40
m
0
2
4
6
8
10
L


Condition number for different m and L, when g = 0.5, with preconditioner
cond(A)
0
10
20
30
40
m
0
10
20
30
40
L
0
2
4
6
8
10


Condition number for different m and L, when g = 0.9, with preconditioner
cond(A)
30
25
20
15
10
5
0
0
10
20
30
40
m
0
2
4
6
8
10
L


Condition number for different m and L, when g = 0, without preconditioner
x 10^298
cond(A)
8
6
4
2
0
0
10
20
30
40
m
0
2
4
6
8
10
L


Condition number for different m and L, when g = 0.5, without preconditioner
x 10^297
2
1.5
1
0.5
0
cond(A)
0
10
20
30
40
m
0
2
4
6
8
10
L



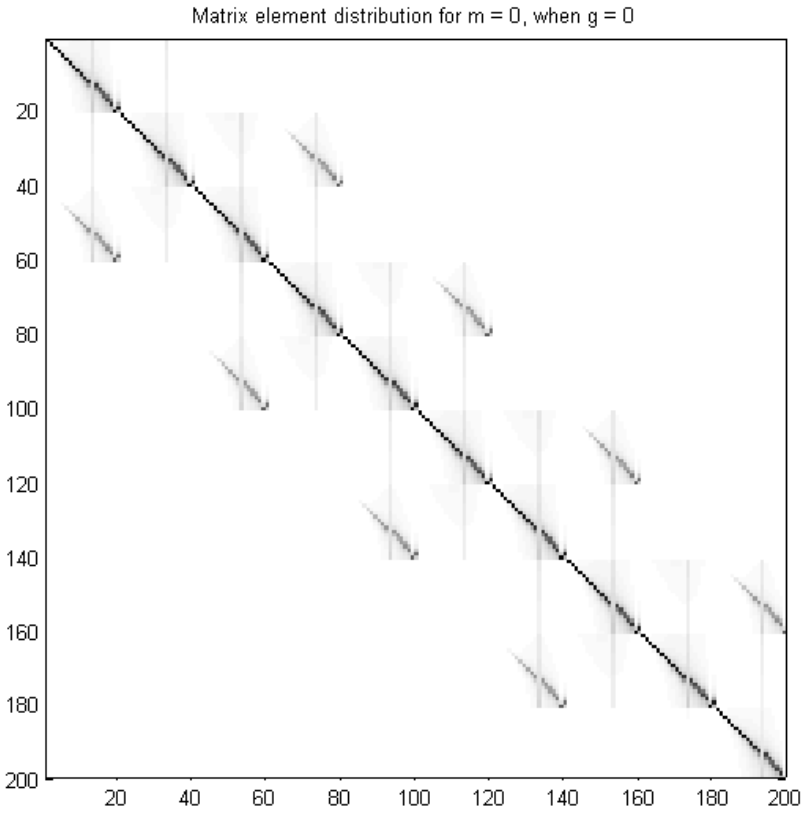
2.3.2. Element Size Distribution and Sparsity Pattern

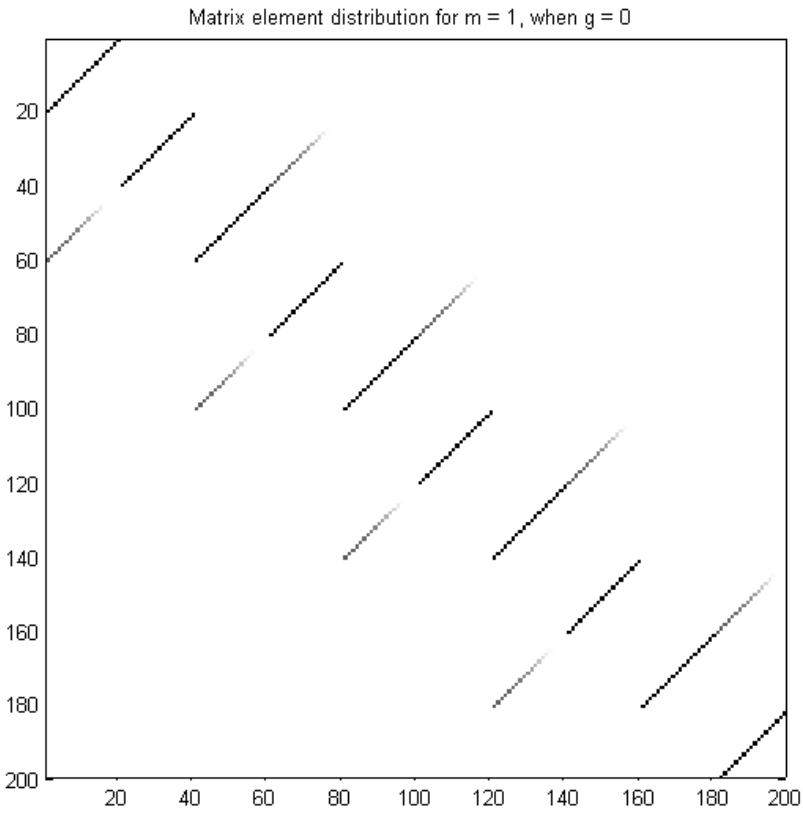
The following plots show the element size distribution and the sparsity pattern of the coefficient matrix for the system of equations for boundary and continuity conditions after applying the preconditioner. The plots are generated with $N = 20$ and $L = 5$, i.e. with 40 channels and 5 layers. This gives a 200×200 coefficient matrix. The single layer case can be obtained by extracting the top left 40×40 part.

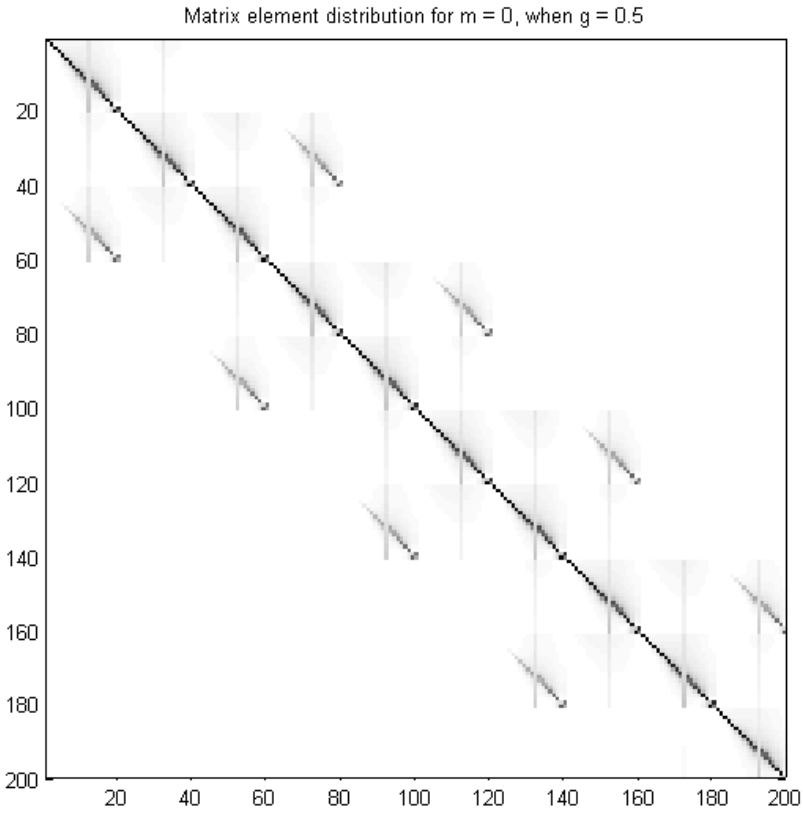
The coefficient matrix is studied for $g = 0$, which is a case with special structure since it is the perfectly diffuse case, and for $g = 0.5$, which is representative for all other g. It is also studied for $m = 0$, which is a case with special structure since it is the azimuthally averaged case, and for $m = 1$ and $m = 3$, which are representative for all other m.

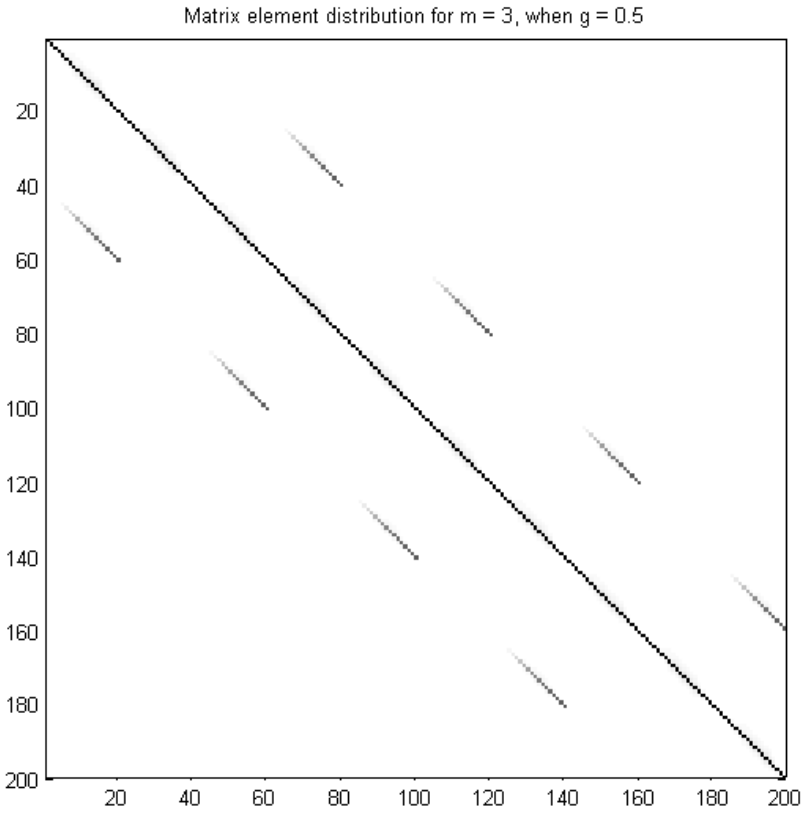
The plots show that the large elements are concentrated near the diagonal, and that all non-zero elements are inside the diagonal blocks. This indicates that the preconditioner works, and yields a well-conditioned system of equations.

As can be seen, the preconditioner preserves the sparsity pattern. The coefficient matrix is still block band diagonal, and the blocks are often sparse themselves. The sparsity is used to solve the system of equations efficiently.


Matrix element distribution for m = 0, when g = 0
20
40
60
80
100
120
140
160
180
200
20
40
60
80
100
120
140
160
180
200


Matrix element distribution for m = 1, when g = 0
20
40
60
80
100
120
140
160
180
200
20
40
60
80
100
120
140
160
180
200


Matrix element distribution for m = 0, when g = 0.5
20
40
60
80
100
120
140
160
180
200
20
40
60
80
100
120
140
160
180
200



2.4. Eigenvalue Problem

The eigenvalue problem is an important part of the core of DORT2002, and it is solved in the innermost loop. Any improvement in speed there will have a large effect on the overall performance. It is also important that the eigenvalue problem is well conditioned.

2.4.1. Reduction of Size

Two deliberate choices concerning the properties of the phase function and the numerical quadrature give the eigenvalue problem a symmetric structure that is possible to exploit. The structure of the *2N* x *2N* matrix comes from the choice that the phase function depends on the scattering angle only (so that the azimuthal dependence can be factored out), and the choice that the numerical quadrature has the symmetry properties $\mu_{-i} = -\mu_i$ and $\omega_{-i} = \omega_i$ for the nodes and weights.

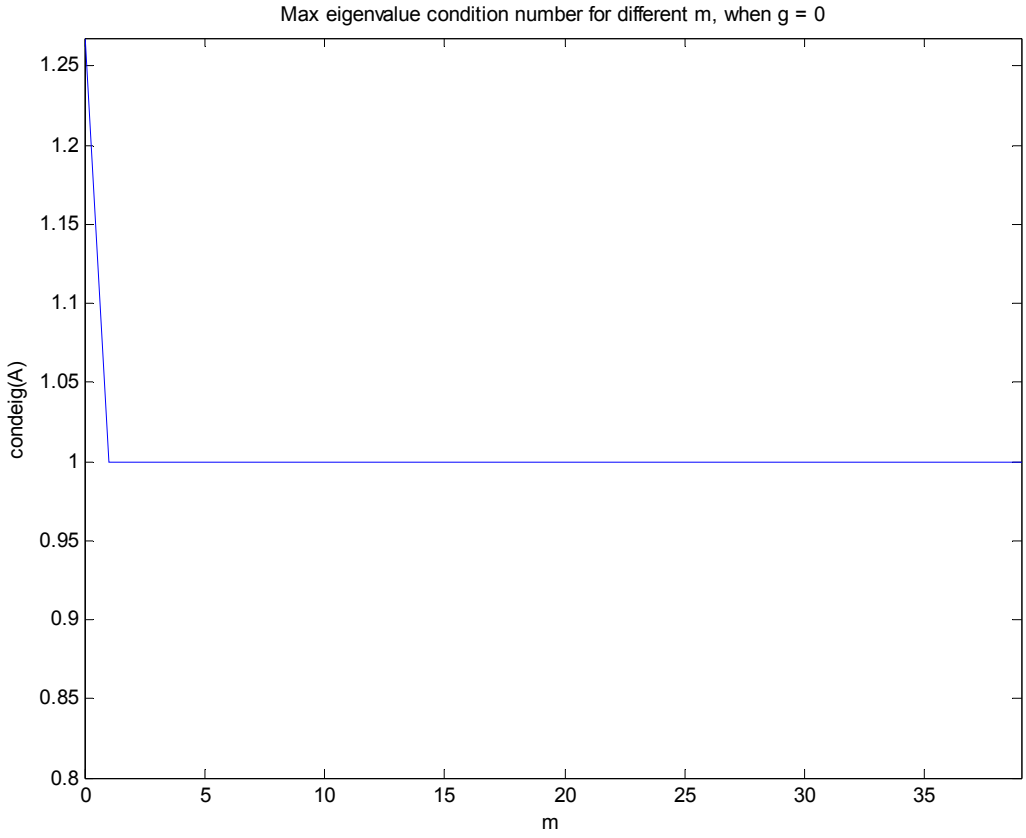
The structure of the matrix makes the eigenvalues come in positive/negative pairs. This allows reducing the size of the eigenvalue problem by a factor 2. Since the calculation time for an eigenvalue problem grows approximately as the third power of the size, the eigenvalue calculation time is thus reduced by a factor 8.

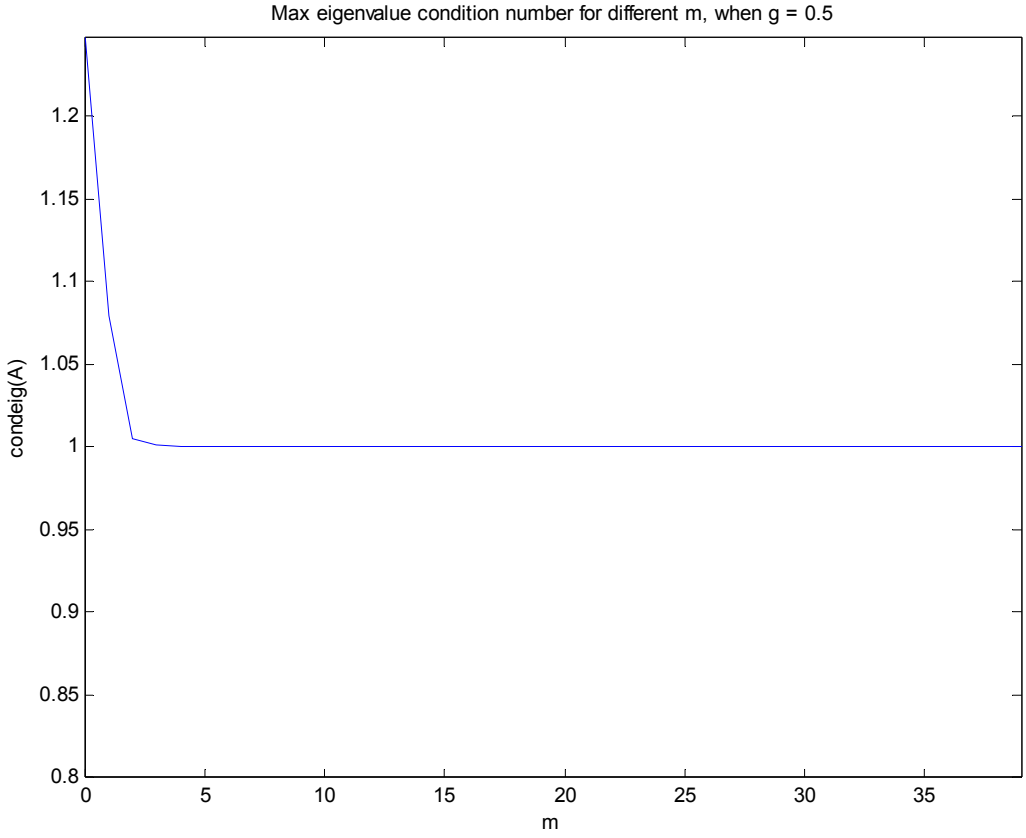
2.4.2. Condition Number

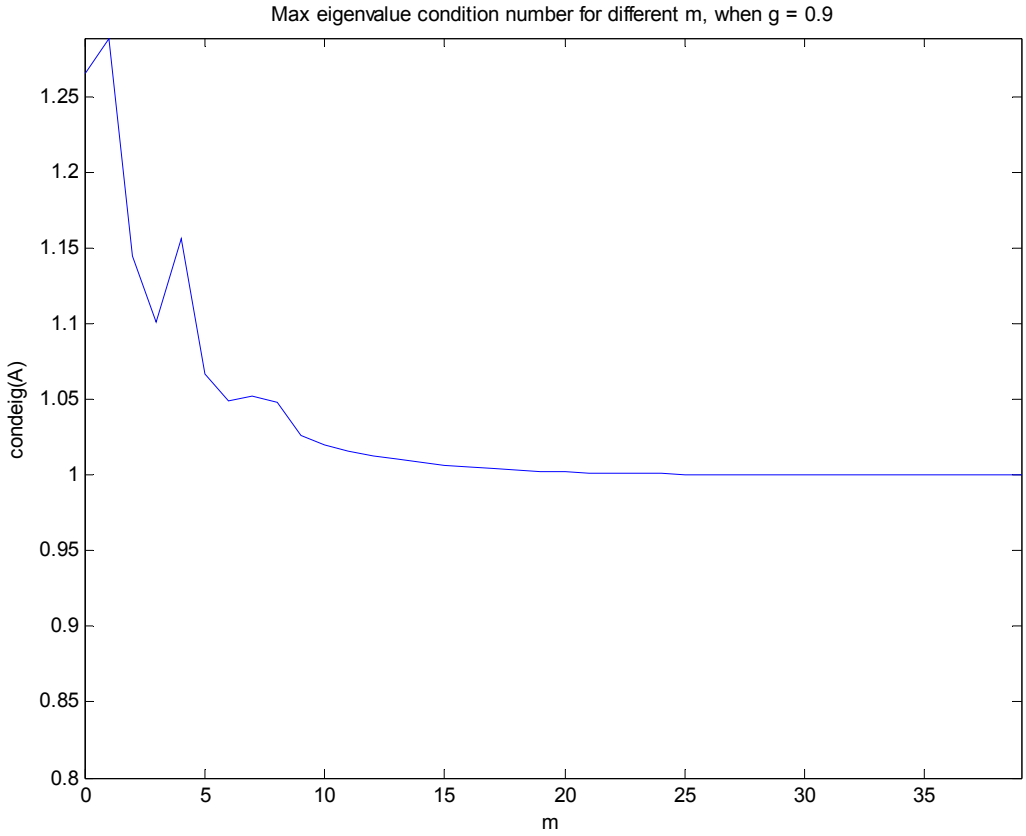
The following plots show the largest eigenvalue condition number after the size reduction. The plots are generated with different *m*, i.e. with varying Fourier component number, for three different asymmetry factors. The eigenvalue problem is independent of the number of layers, since it is solved for each layer separately. The eigenvalue problem is studied for the following parameter set, which is representative for any other set of parameters.

Diffuse intensity	0.3
Beam intensity	1.0
Beam polar angle cosine	0.5
Beam azimuthal angle	$\pi/2$
Depth at upper boundary	0
Underlying surface	Diffuse
Underlying surface reflectance	0.5
Number of channels	40
Maximum relative error	0
Layer thickness	0.02
Scattering coefficient	100
Absorption coefficient	10
Phase function	Henyey-Greenstein
Asymmetry factor	0, 0.5 or 0.9

The plots show that the reduced eigenvalue problem is very well conditioned, giving condition number close to 1 in all cases. As can be seen, the 0^{th} Fourier component is the worst case, with condition number rapidly decreasing with increasing Fourier component number. It can also be seen that the condition number is approximately independent of *g*.


Max eigenvalue condition number for different m, when g = 0
condeig(A)
m
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
0
5
10
15
20
25
30
35


Max eigenvalue condition number for different m, when g = 0.5
condeig(A)
m
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
0
5
10
15
20
25
30
35



2.5. Handling Strongly Forward Peaked Scattering

If the scattering is strongly forward peaked, an accurate expansion of the phase function needs a large number, up to several hundreds or thousands, of terms. To maintain the accuracy throughout the solution, a comparable number of terms are needed in the numerical quadrature used to approximate the integrals. This quickly gives very large eigenvalue problems and systems of equations, and since the computation time for these grows roughly as the third power of the size, the problem soon becomes intractable. The memory requirements also grow fast.

2.5.1. The δ - N Method

A transformation procedure, the *δ-N* method, has been implemented to allow handling of strongly forward peaked phase functions (*g* close to 1) with maintained accuracy without a tremendously increased computational burden. The *δ-N* method also gives maintained accuracy for all *g* for significantly lower *N* than otherwise needed. However, the closer *g* is to zero, the smaller *N* is needed anyway, so the savings in computation time diminishes with decreasing *g*. The overhead introduced by the method is insignificant compared to the core calculations.

The *δ-N* method does not change the mathematical form of the radiative transfer equation. It only changes the optical properties of the medium to make it appear less anisotropic.

2.5.2. Intensity Correction Procedures

The accuracy of the intensity computation is generally improved by the use of the δ-N method except near the direction of the forward peak, but the δ-N method also introduces minor errors in other directions. However, combining the δ-N method with exact computation of low orders of scattering can reduce the error considerably. The purpose of this is to achieve high accuracy with small N, to speed up calculations. The TMS and IMS methods serve to correct for single scattering and secondary and higher orders of scattering respectively.

The TMS method subtracts the erroneous single-scattered intensity from the δ-N method, and adds back the exactly calculated single-scattered intensity. The TMS method gives a substantial improvement for the computed intensity. Only in the direction of the forward peak an error remains. This can be corrected by accounting for secondary and higher orders of scattering, which is done in the IMS method.

The TMS and IMS intensity correction procedures further enhance the handling of strongly forward peaked phase functions beyond the capabilities of the δ-N method. These procedures also give maintained accuracy for all g for significantly lower N than otherwise needed. However, the closer g is to zero, the smaller N is needed anyway, so at some point the possible savings in computation time are smaller than the overhead introduced by the correction procedures. The intensity correction procedures are therefore automatically turned off when they are not needed, in order to save computation time.

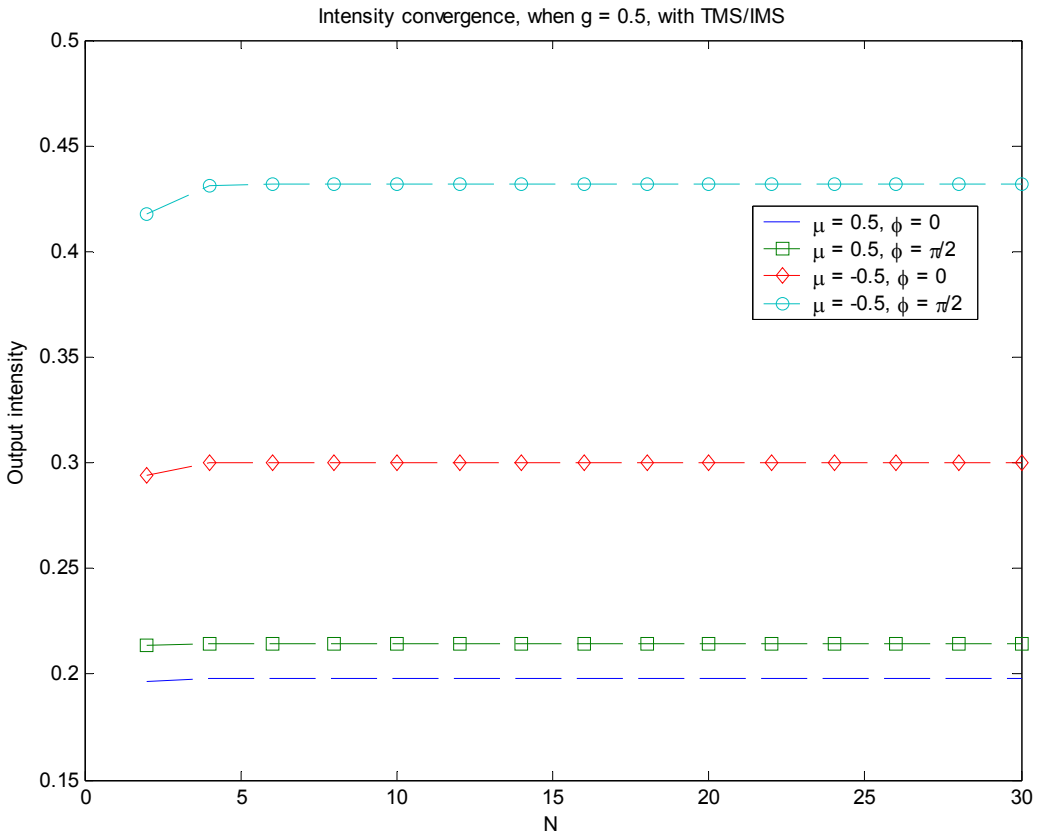
It should be pointed out that the intensity correction procedures take some extra time themselves, but the vast majority of the overhead introduced consists of evaluating Legendre functions Λ_l^m for the larger l and m that are used.

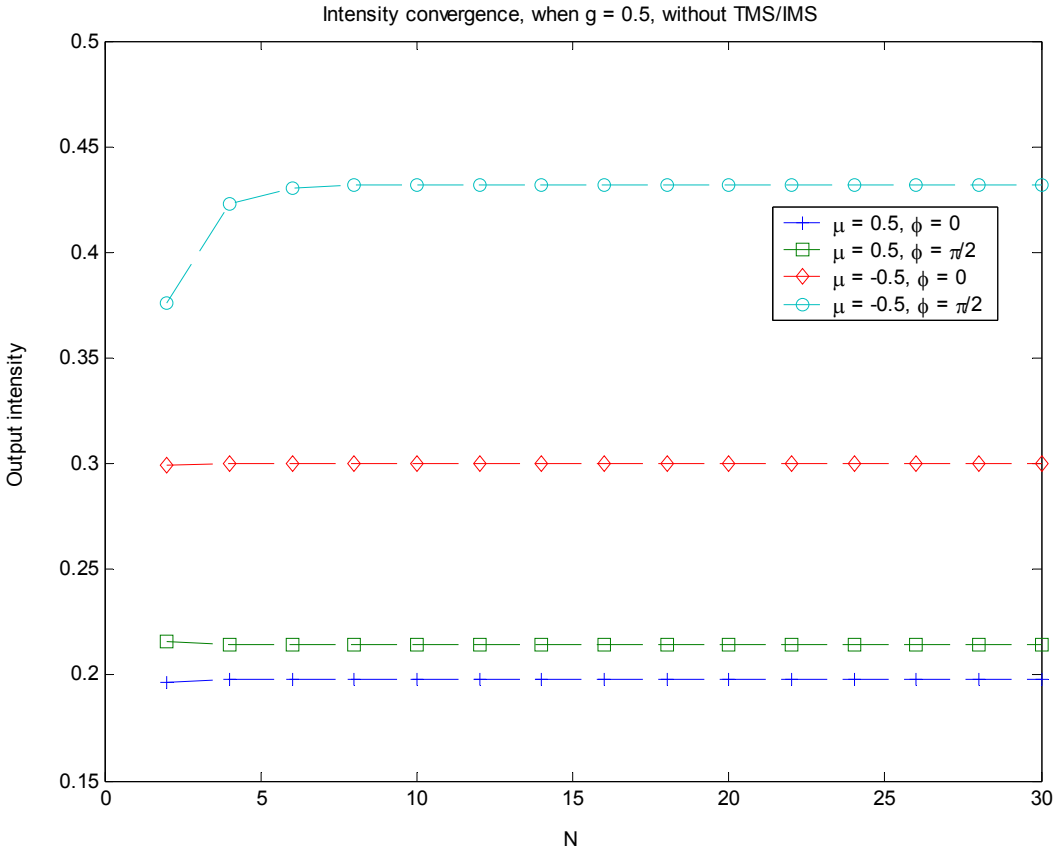
It should also be noted that if computation time is to be compared with and without the intensity correction procedures, it should be done at the same accuracy, not the same N. A typical improvement with these methods is that for a medium with strongly forward peaked scattering the needed N for a reasonable accuracy decreases from 100-1000 to 10-40. Since the overall computation time grows as $\sim N^2 - N^3$, this decreases the computation time with a factor ~1 000 – 10 000. What N is needed depends on g. For $g = 0$, $N = 1$-3 is sufficient, for $g = 0.9$, $N \approx 30$ is needed, depending on needed accuracy. For a multilayer medium with a mix of different g the needed N is harder to predict, but it is always true that the intensity correction procedures give a certain given accuracy at significantly lower N than otherwise needed.

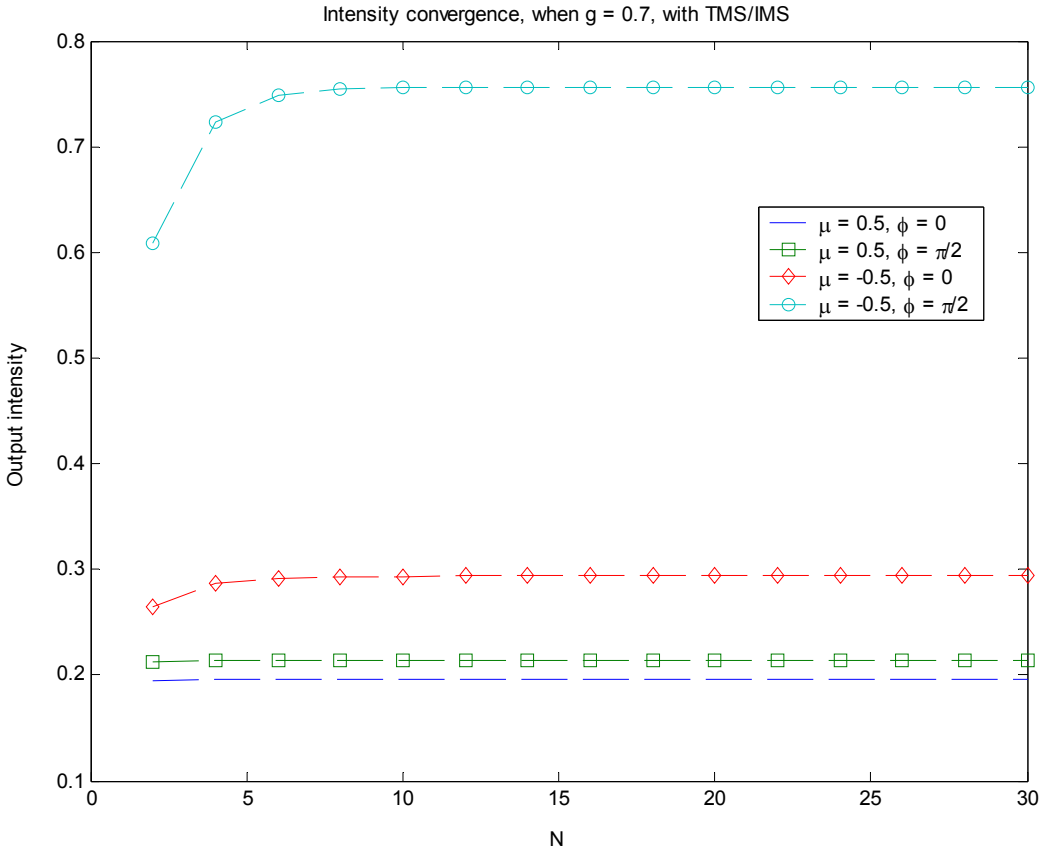
The following plots show the convergence behavior with and without the intensity correction procedures. The plots are generated with different N, i.e. with varying number of channels, for three different asymmetry factors. The resulting intensity is studied in the middle of the medium in four different directions (the incident direction, the direction of specular reflection, and two perpendicular directions). The “true” intensity was calculated with $N = 60$ using the intensity correction procedures. The convergence behavior is studied for the following parameter set, which is representative for any other set of parameters.

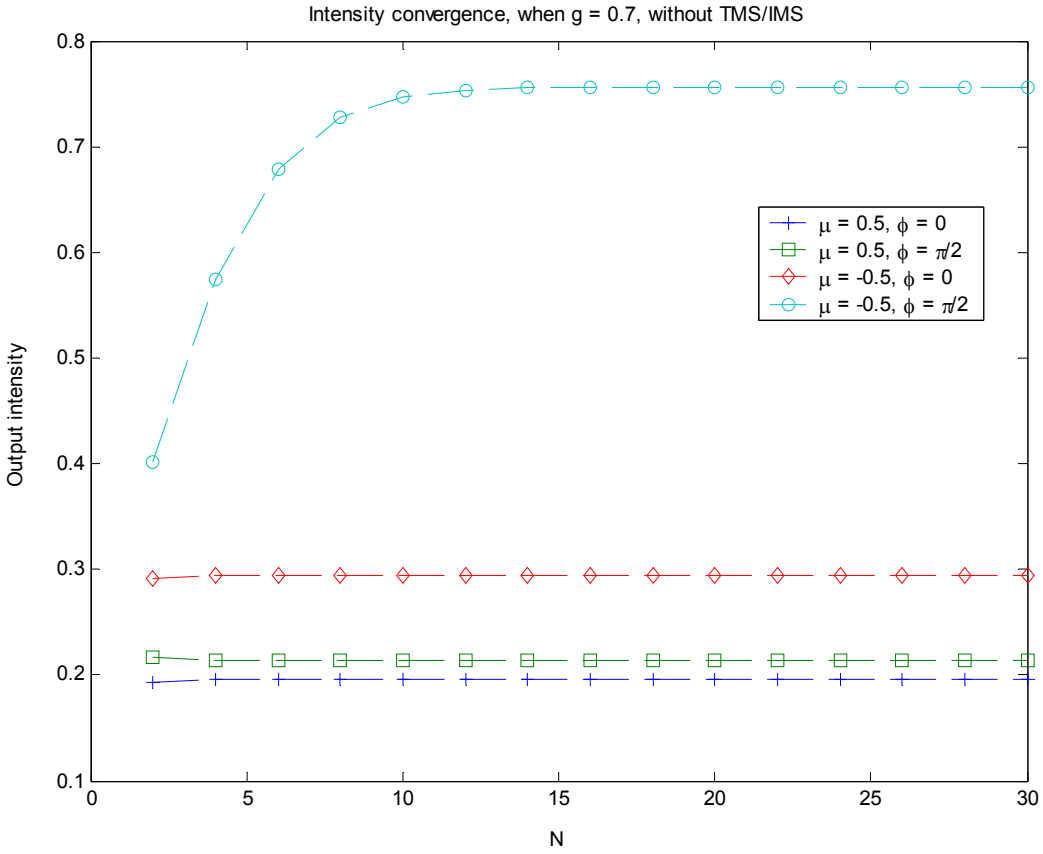
Diffuse intensity	0.3
Beam intensity	1.0
Beam polar angle cosine	0.5
Beam azimuthal angle	$\pi/2$
Depth at upper boundary	0
Underlying surface	Diffuse
Underlying surface reflectance	0.5
Number of channels	Varying
Number of layers	5
Layer thickness	0.01
Scattering coefficient	100
Absorption coefficient	10
Phase function	Henyey-Greenstein
Asymmetry factor	0.5, 0.7 or 0.9

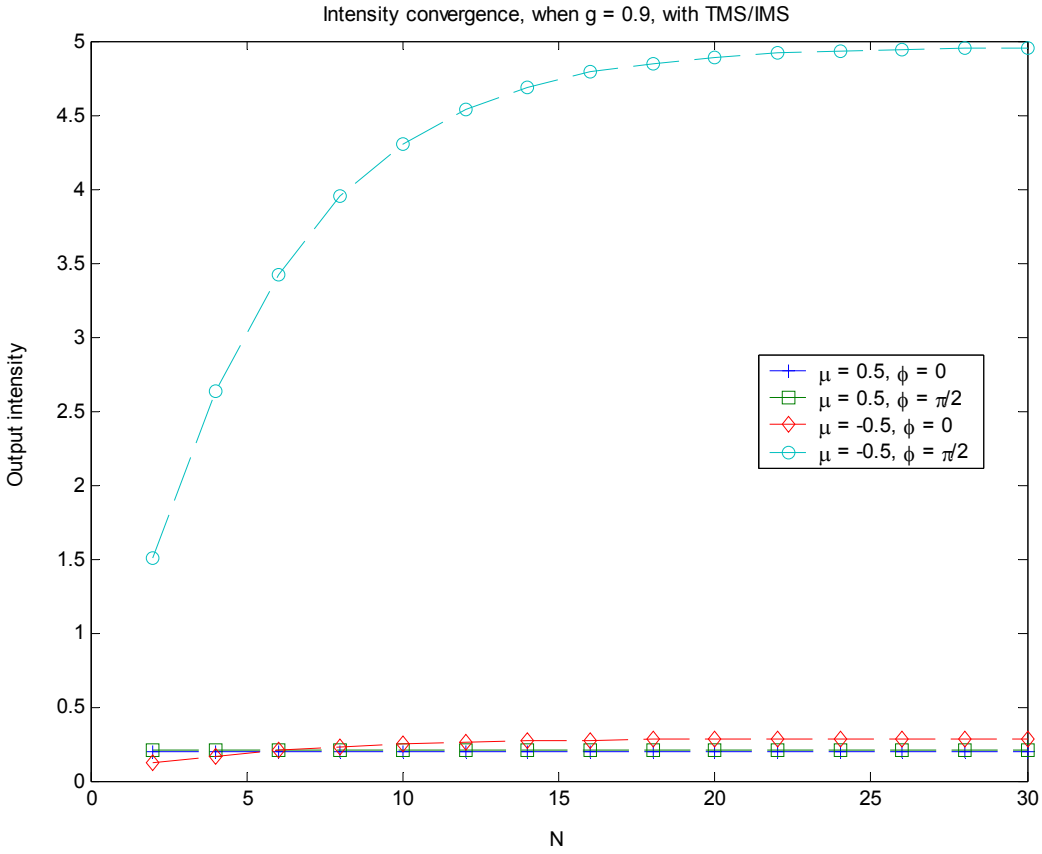
The plots show that DORT2002 always converges to the true value when *N* increases. As can be seen, a larger *N* is needed to maintain accuracy for larger *g*. It is also obvious from the plots that a larger *N* is needed to maintain accuracy when the intensity correcting procedures are not used. The plots show that it is always the incident direction (the o's) that converges the slowest and that is most sensitive to *g*, as should be expected for forward peaked scattering.

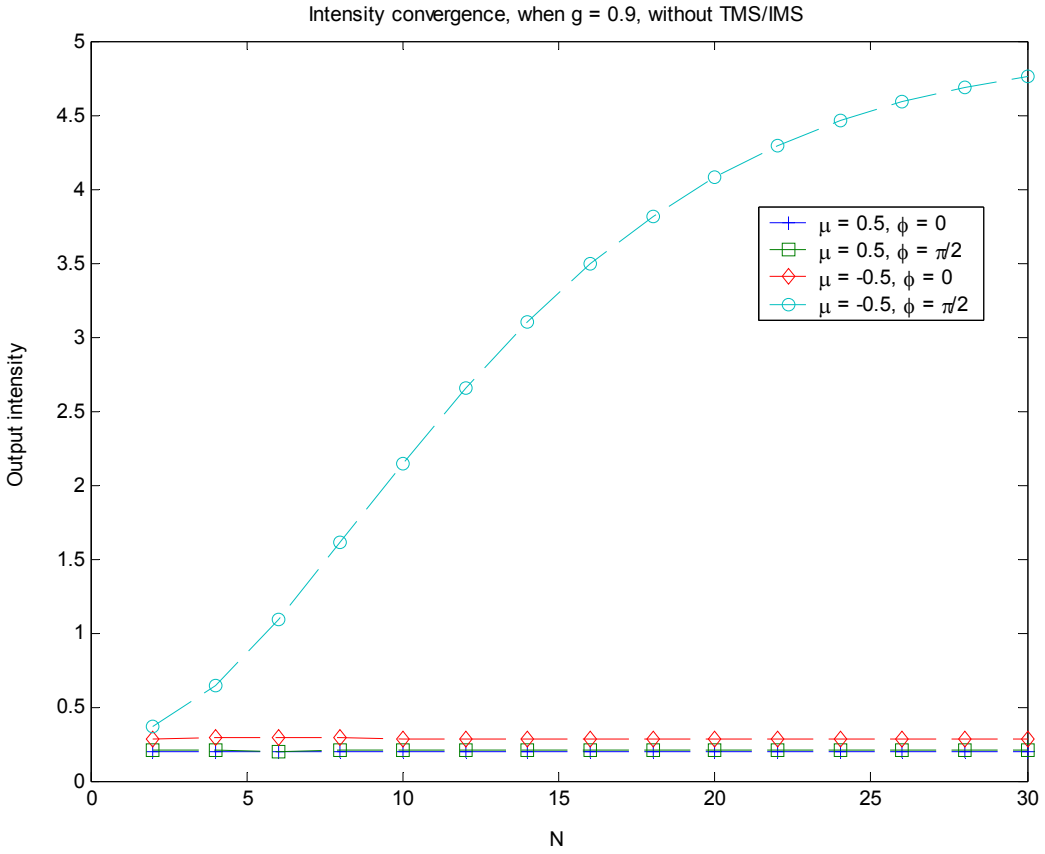



Intensity convergence, when g = 0.5, without TMS/IMS
Output intensity
N
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0
5
10
15
20
25
30
μ = 0.5, ϕ = 0
μ = 0.5, ϕ = π/2
μ = -0.5, ϕ = 0
μ = -0.5, ϕ = π/2


Intensity convergence, when g = 0.7, with TMS/IMS
Output intensity
N
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
5
10
15
20
25
30
μ = 0.5, ϕ = 0
μ = 0.5, ϕ = π/2
μ = -0.5, ϕ = 0
μ = -0.5, ϕ = π/2


Intensity convergence, when g = 0.7, without TMS/IMS
Output intensity
N
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
5
10
15
20
25
30
μ = 0.5, ϕ = 0
μ = 0.5, ϕ = π/2
μ = -0.5, ϕ = 0
μ = -0.5, ϕ = π/2


Intensity convergence, when g = 0.9, with TMS/IMS
Output intensity
N
μ = 0.5, ϕ = 0
μ = 0.5, ϕ = π/2
μ = -0.5, ϕ = 0
μ = -0.5, ϕ = π/2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
5
10
15
20
25
30



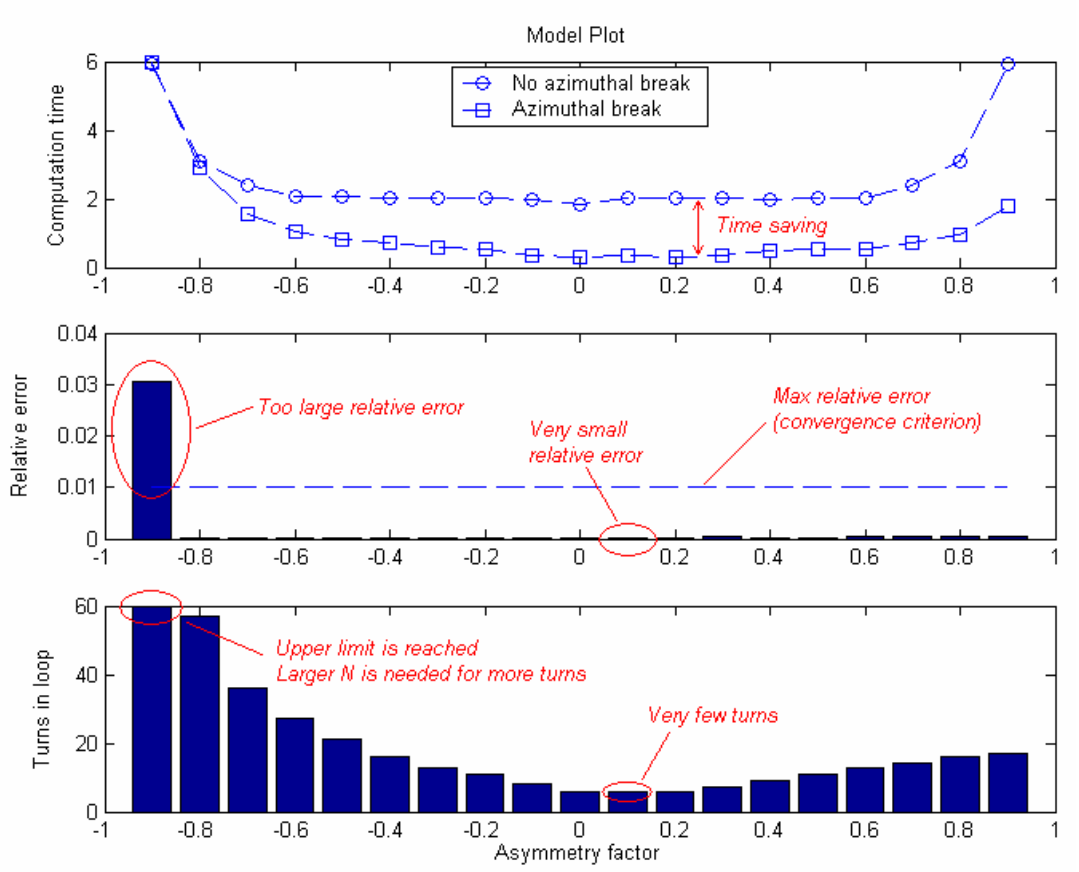
2.6. Breaking the Azimuthal Loop

In many cases the intensity converges well before the azimuthal loop has ended. Since it is the outermost loop, much is gained if it can be terminated earlier. A method has been implemented that breaks the azimuthal loop when a convergence criterion has been met. The objective is to stop the calculations well before $m = 2N - 1$, based on the assumed knowledge that the total contribution from the omitted terms is less than a user supplied limit. This saves a large amount of computation time in the vast majority of cases.

The method is an engineering method based on an ad hoc assumption that the Fourier components of the intensity are approximately exponentially decreasing. Experimental numerical studies of the Fourier components indicate that this assumption is valid in most cases. The algorithm has been carefully tested under a wide range of conditions and has proven to be valid for all tested single layer cases. Some multilayer cases give bad results. This is not surprising since the Fourier component distribution will in the multilayer case be a sum of different Fourier component distributions from the different layers. Therefore the first few terms, that are used to predict the needed number of turns in the azimuthal loop, do not necessarily show the assumed exponential behavior. It is possible to turn off this feature if there is any doubt that it yields bad results in a particular case.

2.6.1. Single Layer Case

The following plots show the performance of the loop-breaking algorithm for the single layer case. The plots are generated with different g, i.e. varying asymmetry factor, for three different depths (top, middle and bottom). The plots are divided into three parts. The first part shows the computation time with and without the loop-breaking algorithm, and thus the saving in computation time. The second part shows the max relative error used as convergence criterion as a dashed line together with the relative error for the different asymmetry factors, which makes it easy to see if the convergence criterion was met (The "true" intensity was calculated with $N = 60$, and the largest error over all directions was used). The third part shows the number of turns used in the azimuthal loop, where the goal was to make it as far below $2N = 60$ as possible. The first plot is a model plot.

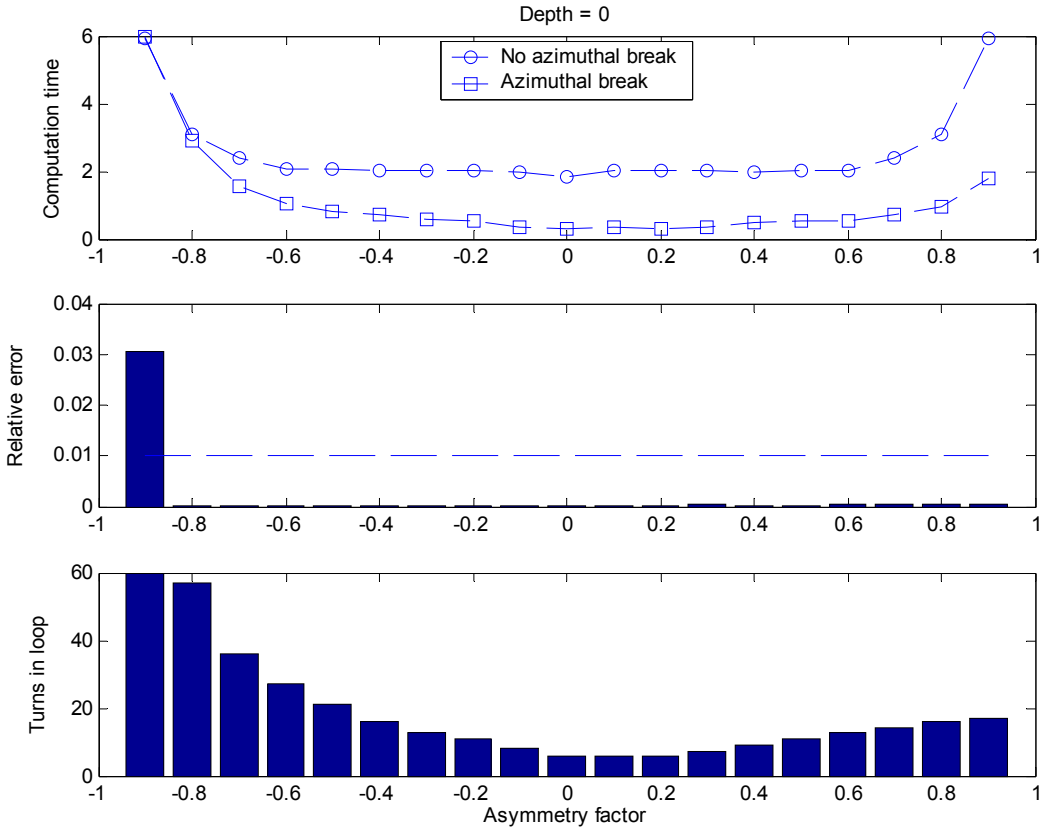


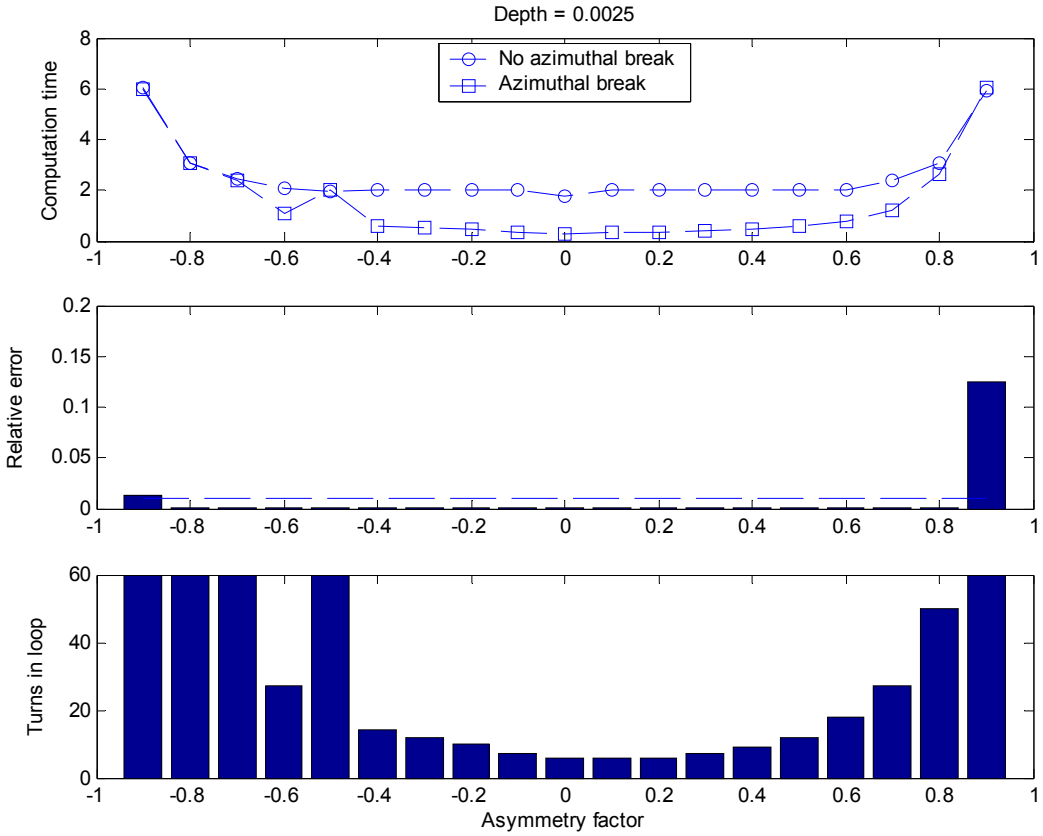
The performance of the loop-breaking algorithm for the single layer case is studied for the following basic parameter set. To study different interesting or extreme cases, one or more parameters were changed for some groups of plots, as indicated in the text below. Note that the scales on the y-axes vary between groups of plots.

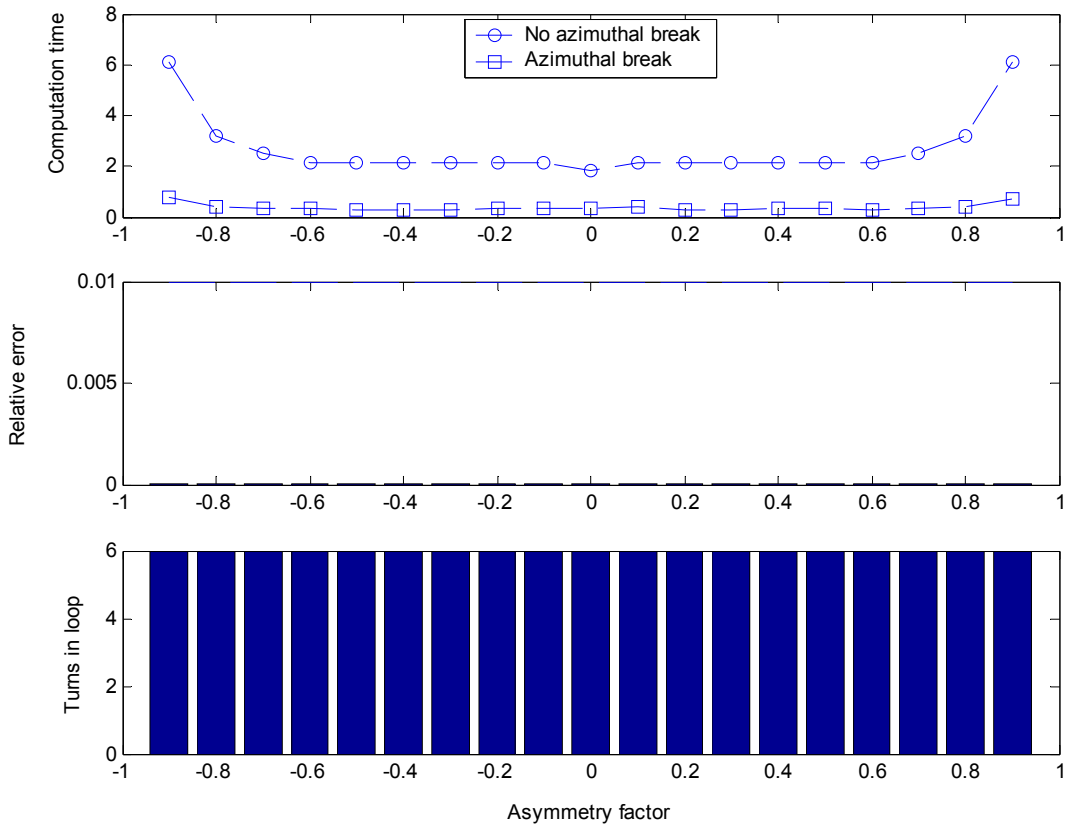
Diffuse intensity	0
Beam intensity	1.0
Beam polar angle cosine	0.5
Beam azimuthal angle	$\pi/2$
Depth at upper boundary	0
Underlying surface	Diffuse
Underlying surface reflectance	0.5
Number of channels	60
Number of layers	1
Layer thickness	0.005
Scattering coefficient	100
Absorption coefficient	10
Phase function	Henyey-Greenstein
Asymmetry factor	Varying

The plots show that the algorithm always works better for positive than negative g, that g closer to 0 gives better performance, and that the algorithm on the average gives 90% saving in computation time. The convergence criterion is not met only for some extreme cases with $|g|$ close to 1, and only because N was not sufficiently large. The algorithm can thus be said to work very well in the single layer case.

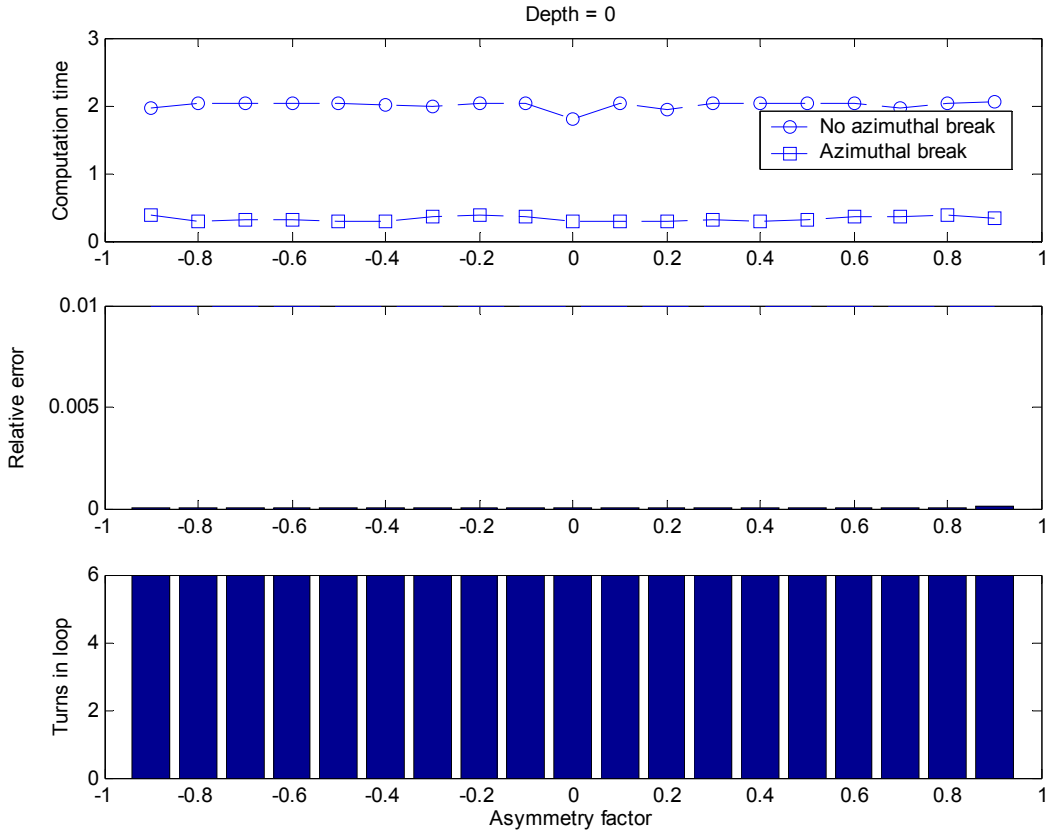
The first group of plots uses the basic parameter set. As can be seen below, at the top the relative error for $g = -0.9$ is too large, and the azimuthal loop was not broken. A larger N is needed to achieve the specified accuracy. For all other g the convergence criterion was met, the loop was broken well before its natural ending point, and the saving in computation time was around 90% on average. In the middle the relative error was too large for $g = 0.9$. The loop was not broken for some negative g, although it could be broken since the convergence criterion was met. This is because the behavior of the Fourier components is sometimes hard to predict for negative g, so a more pessimistic bound is used there. At the bottom, the performance was extremely good, which is always the case for a diffuse underlying surface. The loop was broken already after the minimum number of turns that are needed to investigate the initial behavior of the Fourier components.

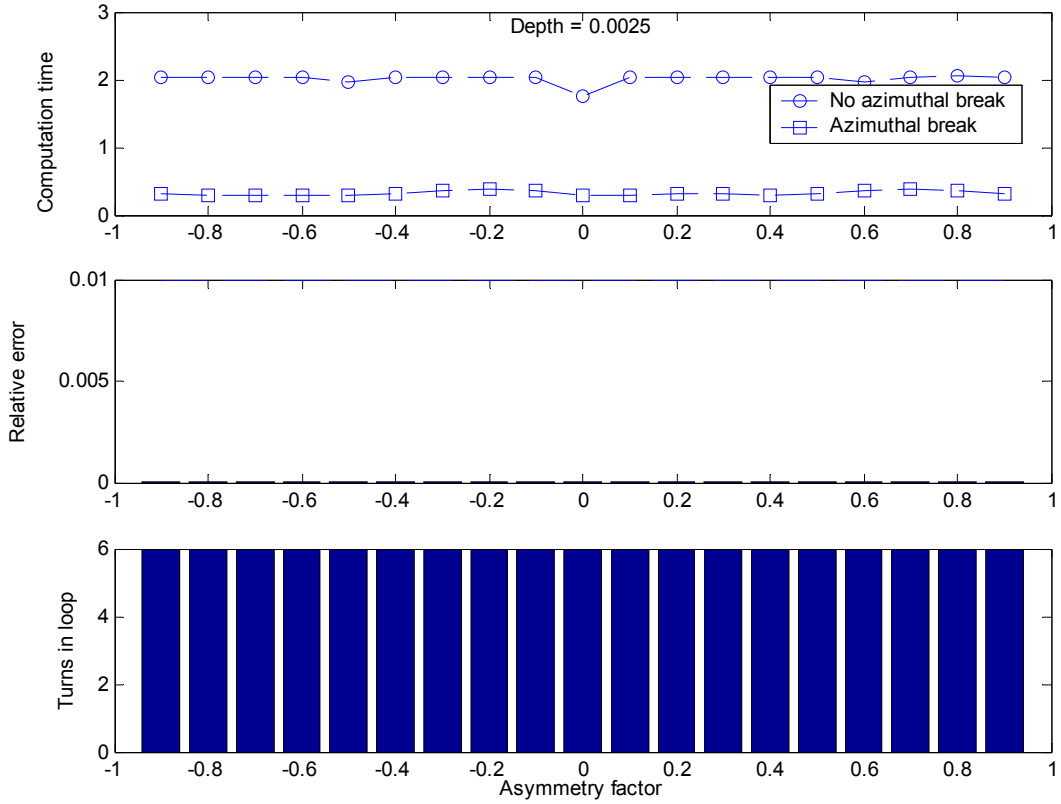

Depth = 0
No azimuthal break
Azimuthal break
Computation time
Relative error
Turns in loop
Asymmetry factor

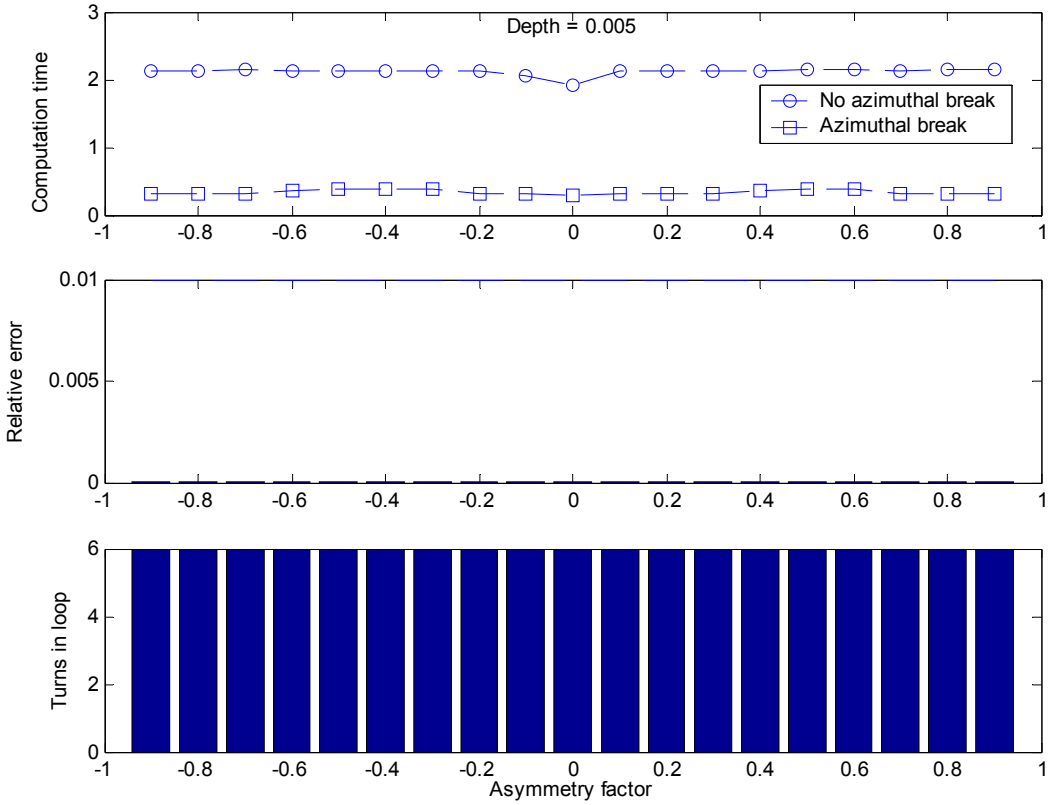

Depth = 0.0025
No azimuthal break
Azimuthal break
Computation time
Relative error
Turns in loop
Asymmetry factor



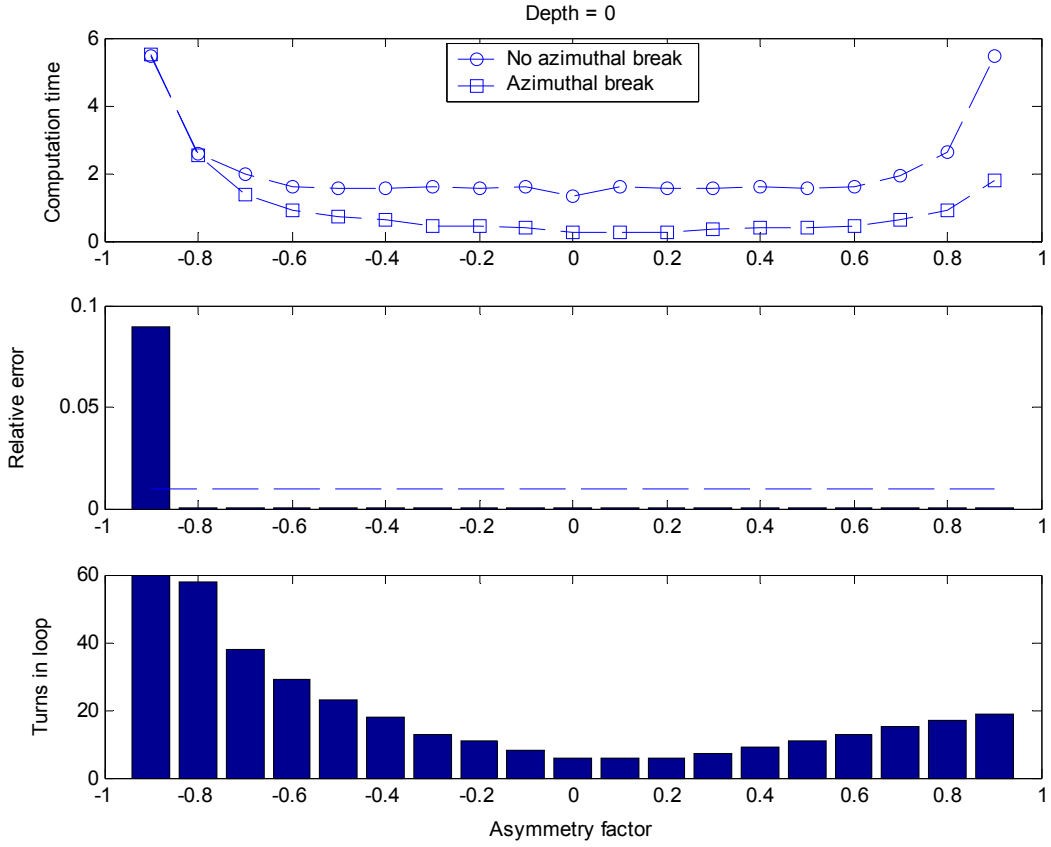
The second group of plots uses the basic parameter set, except for diffuse intensity = 0.3 and beam intensity = 0. As can be seen below, the diffuse illumination and the absence of a beam source give extremely good performance at all depths and for all *g*. The loop was in all cases broken already after the minimum number of turns that are needed to investigate the initial behavior of the Fourier components. The saving in computation time was at least 90% in all cases.

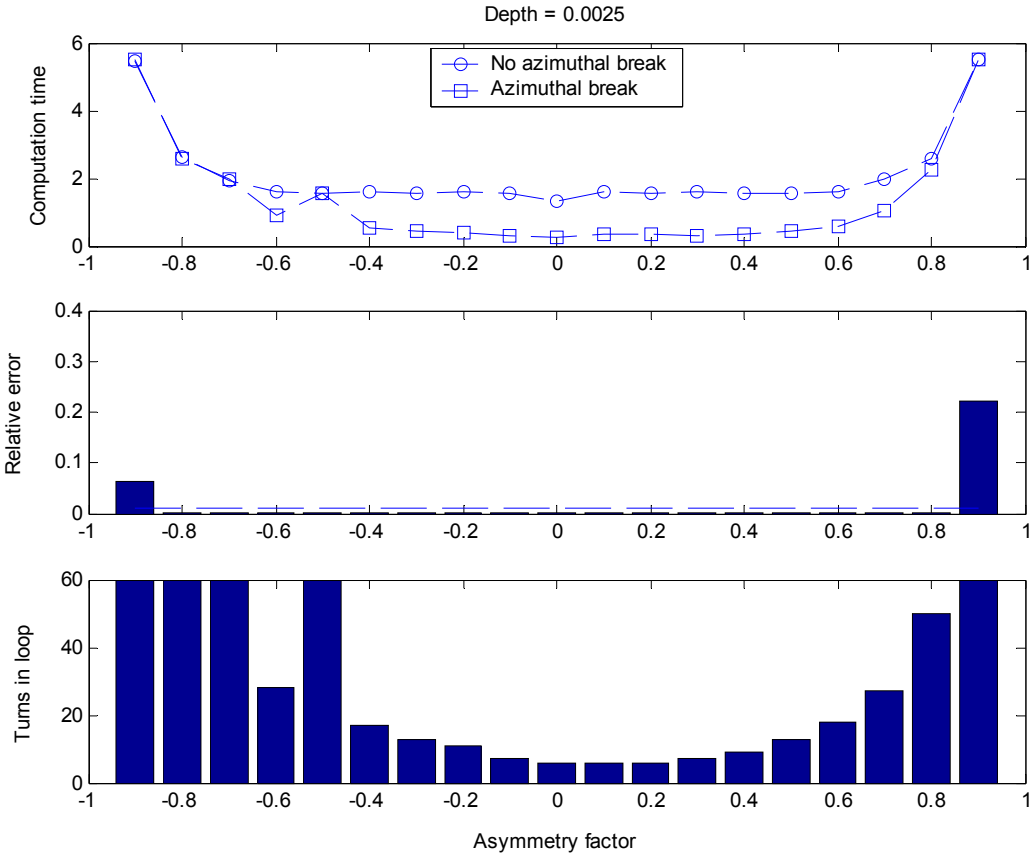

Depth = 0
Computation time
No azimuthal break
Azimuthal break
Relative error
Turns in loop
Asymmetry factor

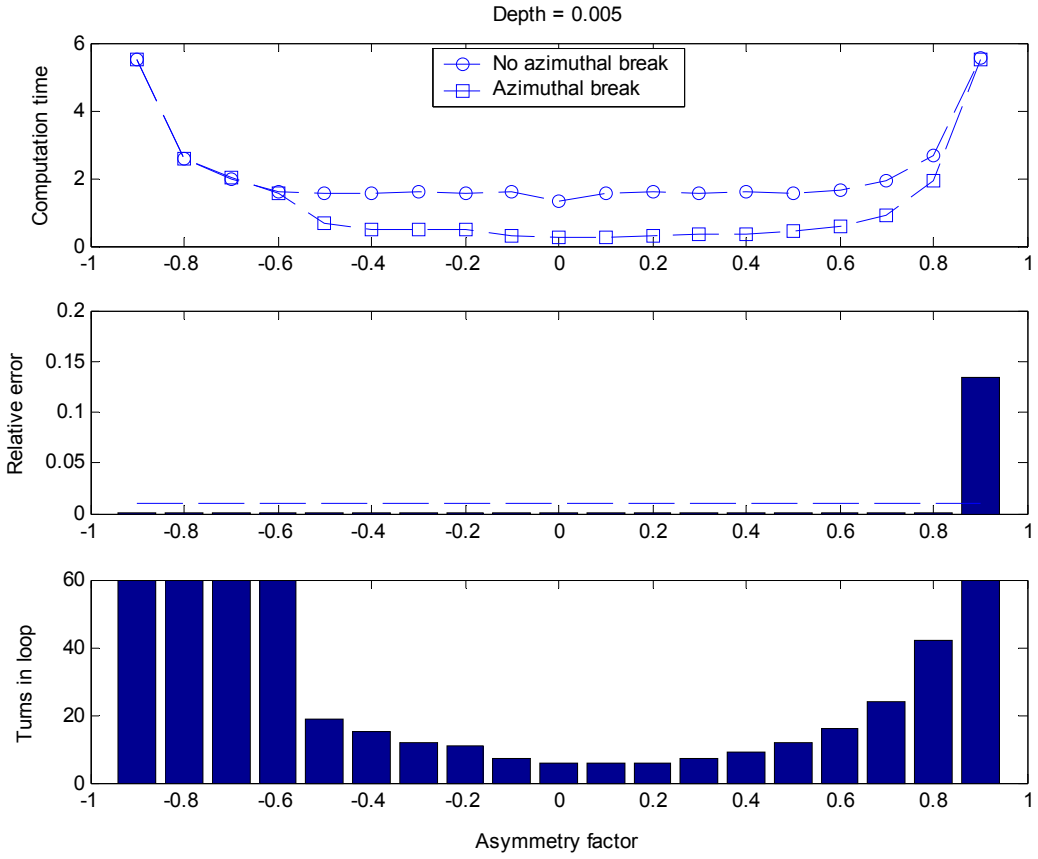

Depth = 0.0025
Computation time
No azimuthal break
Azimuthal break
Relative error
Turns in loop
Asymmetry factor



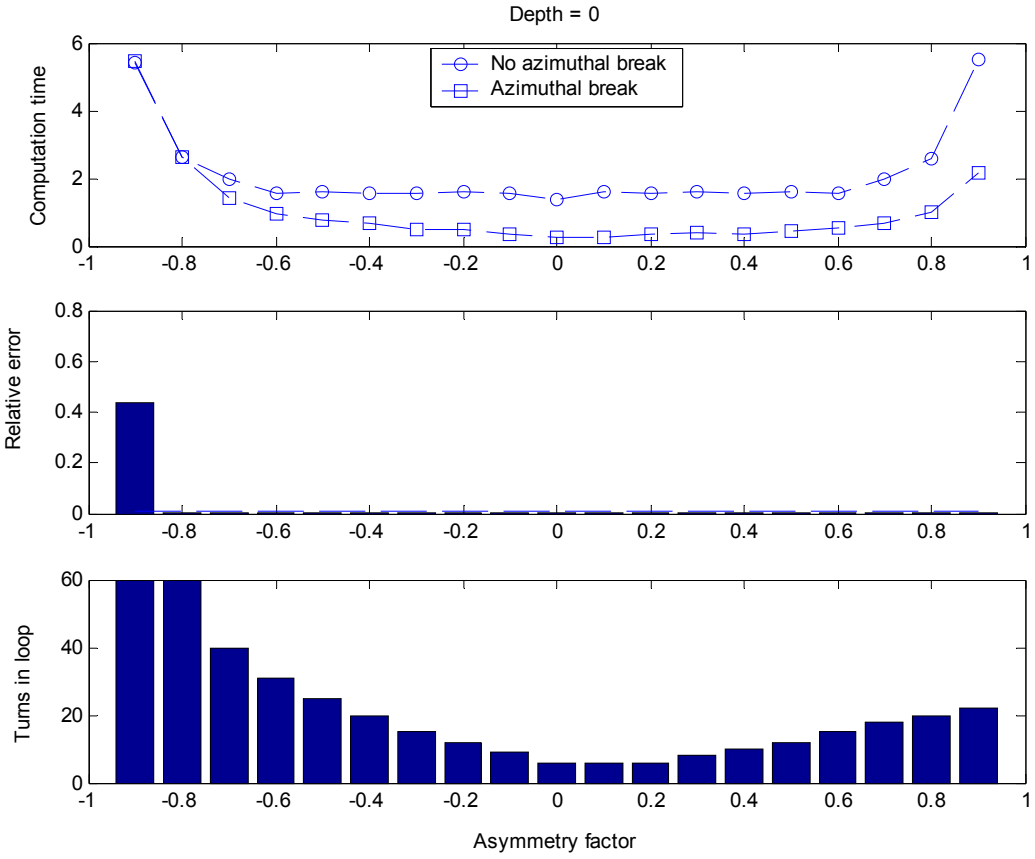
The third group of plots uses the basic parameter set, except that there is no underlying surface. The behavior is the same as in the first group of plots at the top and in the middle, which is not surprising since the circumstances are hardly changed there. The absence of a diffuse underlying surface makes the bottom behave as the middle, as can be expected.

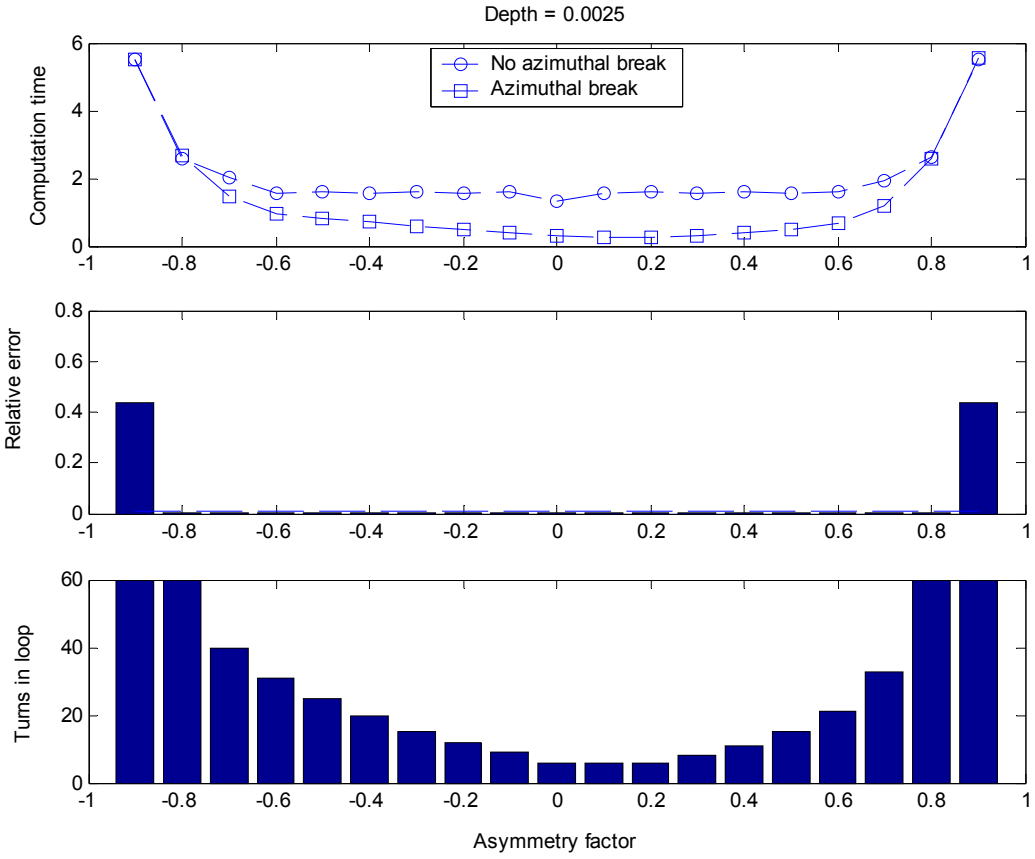

Depth = 0
No azimuthal break
Azimuthal break
Computation time
Relative error
Turns in loop
Asymmetry factor

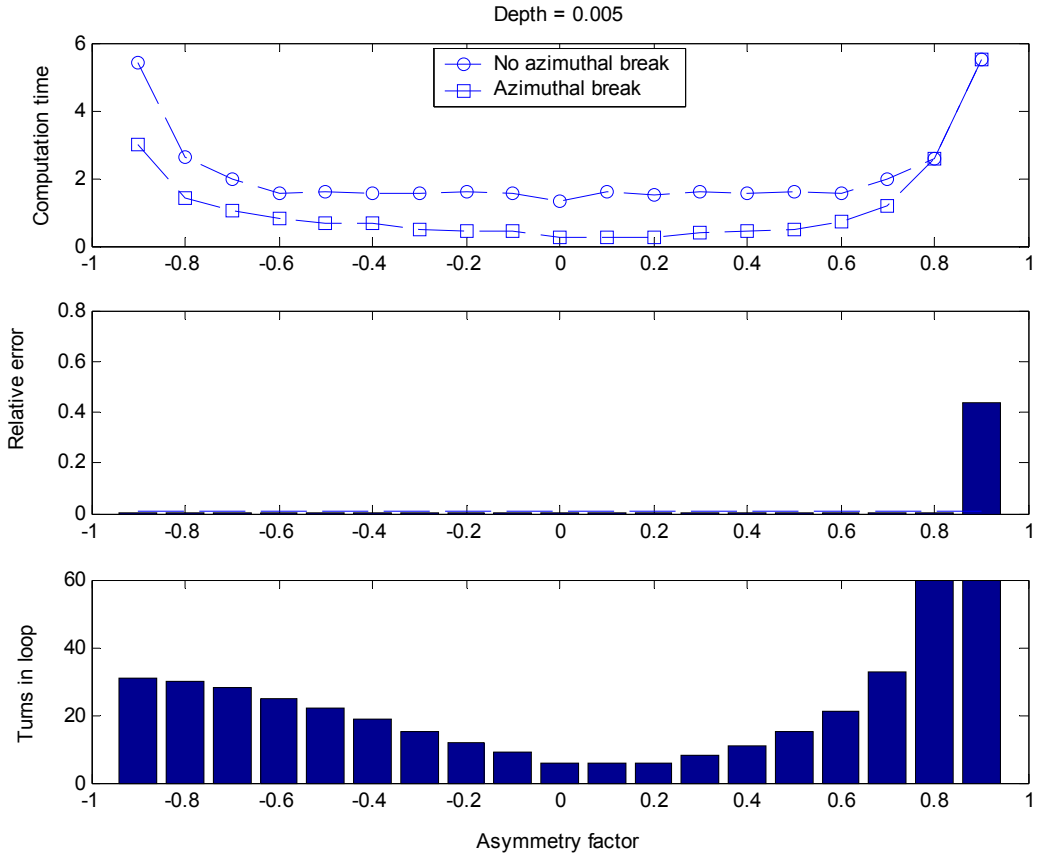

Depth = 0.0025
No azimuthal break
Azimuthal break
Computation time
Relative error
Turns in loop
Asymmetry factor



The fourth group of plots uses the basic parameter set, except for scattering coefficient = 0.01, and that there is no underlying surface. This is the same parameter set as in the third group of plots, except for scattering coefficient = 0.01. As can be seen below, the behavior is the same as in the third group of plots at the top and in the middle, while the performance is far better at the bottom.


Depth = 0
No azimuthal break
Azimuthal break
Computation time
Relative error
Turns in loop
Asymmetry factor


Depth = 0.0025
No azimuthal break
Azimuthal break
Computation time
Relative error
Turns in loop
Asymmetry factor



2.6.2. Multilayer Case

The performance of the loop-breaking algorithm for the multilayer case is studied in a wide variety of settings, divided in three major groups, in order to cover different interesting or extreme cases. The first group varies the single scattering albedo (through the scattering and absorption coefficients) with and without diffuse underlying surface. The second group varies the asymmetry factors for the different layers with and without diffuse illumination, all with a diffuse underlying surface. The third group varies the asymmetry factors for the different layers with and without diffuse illumination, all with no underlying surface.

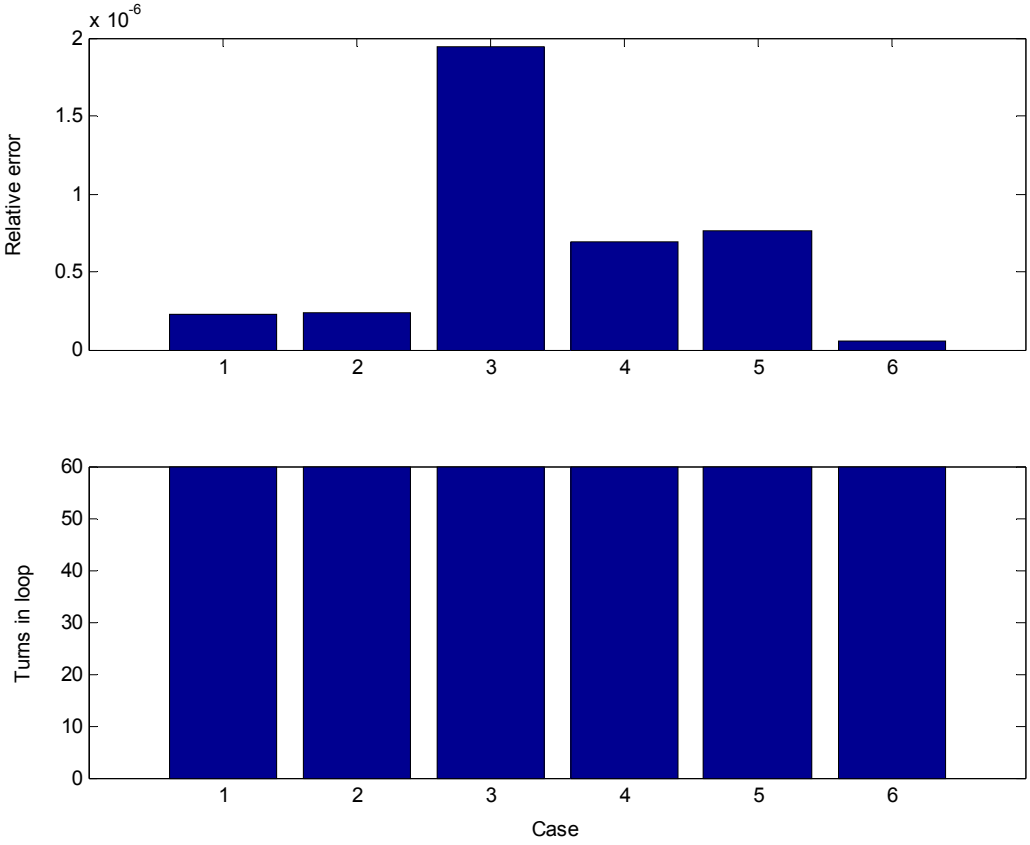
The plots show that the algorithm works well in most cases. Due to the more unpredictable behavior of the Fourier components of the intensity in multilayer media with different asymmetry factors in different layers, the algorithm may in some cases not break the loop although it would be possible. On the other hand there are (relatively rare) cases where the algorithm breaks the loop without meeting the convergence criterion. The reason for this is that the first Fourier components do not always reflect the behavior of the rest of the Fourier components in the multilayer case. Of the 18 cases studied below, 1 fails to meet the convergence criterion. All other 17 cases give the specified accuracy, and most of them with about 50% saving in computation time.

The first group varies the single scattering albedo (through the scattering and absorption coefficients) with and without diffuse underlying surface.

	Case 1-3	**Case 4-6**
Diffuse intensity	0	0
Beam intensity	1.0	1.0
Beam polar angle cosine	0.5	0.5
Beam azimuthal angle	$\pi/2$	$\pi/2$
Depth at upper boundary	0	0
Underlying surface	Diffuse	None
Underlying surface reflectance	0.5	-
Number of channels	60	60
Number of layers	3	3
Max relative error (convergence criterion)	0.01	0.01
Phase function	Henyey-Greenstein	Henyey-Greenstein

Case	**Layer**	**Scattering coefficient**	**Absorption coefficient**	**Asymmetry factor**	**Layer thickness**
1	1	100	10	0.7	0.002
	2	100	10	0.2	0.002
	3	100	10	-0.7	0.002
2	1	100	0.01	0.7	0.002
	2	100	0.01	0.2	0.002
	3	100	0.01	-0.7	0.002
3	1	0.01	10	0.7	0.002
	2	0.01	10	0.2	0.002
	3	0.01	10	-0.7	0.002
4	1	100	10	0.7	0.002
	2	100	10	0.2	0.002
	3	100	10	-0.7	0.002
5	1	100	0.01	0.7	0.002
	2	100	0.01	0.2	0.002
	3	100	0.01	-0.7	0.002
6	1	0.01	10	0.7	0.002
	2	0.01	10	0.2	0.002
	3	0.01	10	-0.7	0.002

The plots below show that the algorithm does not break the loop, for this particular combination of asymmetry factors, although it would be possible. The reason for this is that the first Fourier components do not reflect the behavior of the rest of the Fourier components in this case. As can be seen, the relative error is far below the specified limit. No computation time is saved.

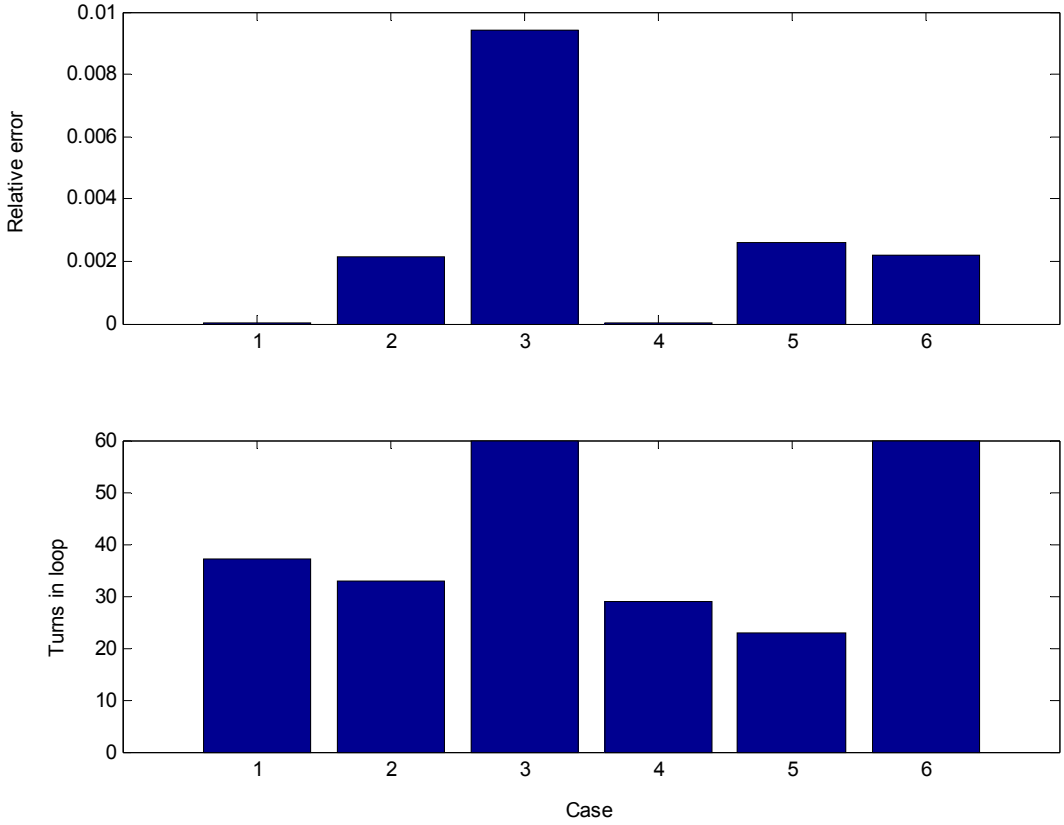

x 10^-6
Relative error
2
1.5
1
0.5
0
1
2
3
4
5
6
Turns in loop
60
50
40
30
20
10
0
1
2
3
4
5
6
Case

The second group varies the asymmetry factors for the different layers with and without diffuse illumination, all with a diffuse underlying surface.

	Case 1-3	**Case 4-6**
Diffuse intensity	0	0.3
Beam intensity	1.0	1.0
Beam polar angle cosine	0.5	0.5
Beam azimuthal angle	$\pi/2$	$\pi/2$
Depth at upper boundary	0	0
Underlying surface	Diffuse	Diffuse
Underlying surface reflectance	0.5	0.5
Number of channels	60	60
Number of layers	3	3
Max relative error (convergence criterion)	0.01	0.01
Phase function	Henyey-Greenstein	Henyey-Greenstein

Case	**Layer**	**Scattering coefficient**	**Absorption coefficient**	**Asymmetry factor**	**Layer thickness**
1	1	100	10	0.7	0.002
	2	100	10	0.2	0.002
	3	100	10	-0.3	0.002
2	1	100	10	0.7	0.002
	2	100	10	0.8	0.002
	3	100	10	0.9	0.002
3	1	100	10	-0.7	0.002
	2	100	10	0.9	0.002
	3	100	10	-0.3	0.002
4	1	100	10	0.7	0.002
	2	100	10	0.2	0.002
	3	100	10	-0.3	0.002
5	1	100	10	0.7	0.002
	2	100	10	0.8	0.002
	3	100	10	0.9	0.002
6	1	100	10	-0.7	0.002
	2	100	10	0.9	0.002
	3	100	10	-0.3	0.002

The plots below show that the algorithm breaks the loop when it would be possible in most but not all cases. As can be seen, the relative error is below the specified limit in all cases. About 50% computation time is saved.

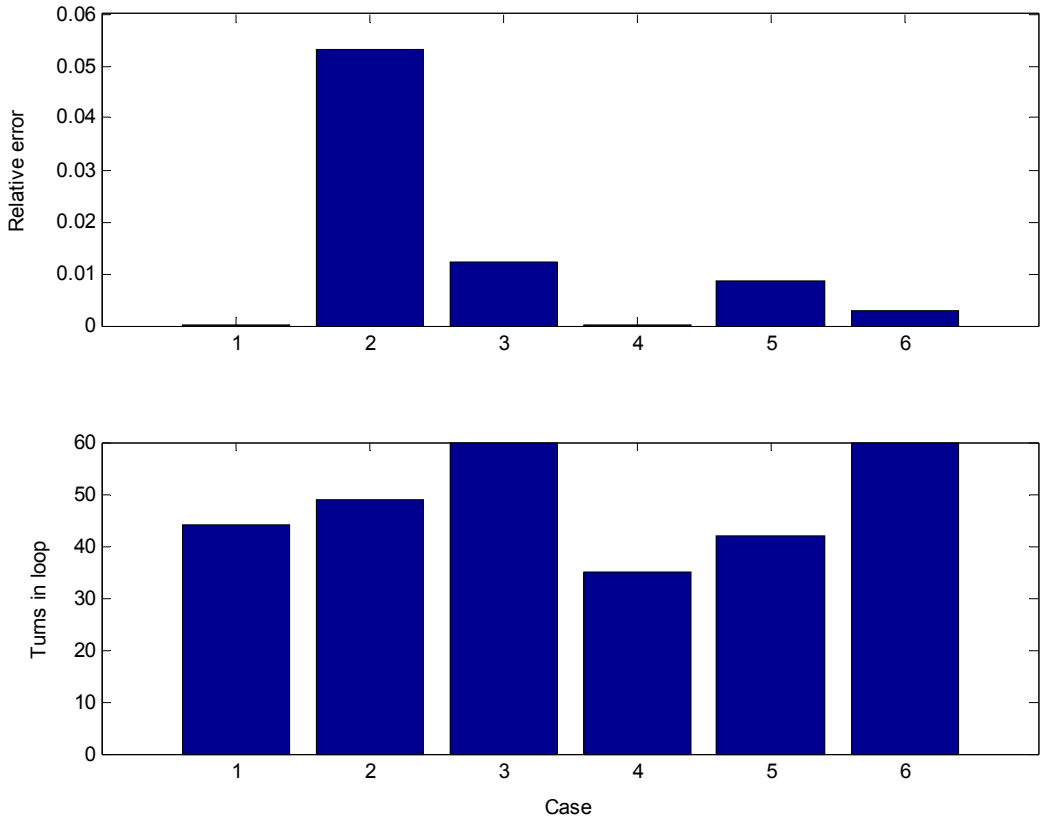

Relative error
0.01
0.008
0.006
0.004
0.002
0
1
2
3
4
5
6
Turns in loop
60
50
40
30
20
10
0
1
2
3
4
5
6
Case

The third group varies the asymmetry factors for the different layers with and without diffuse illumination, all without a diffuse underlying surface.

	Case 1-3	**Case 4-6**
Diffuse intensity	0	0.3
Beam intensity	1.0	1.0
Beam polar angle cosine	0.5	0.5
Beam azimuthal angle	$\pi/2$	$\pi/2$
Depth at upper boundary	0	0
Underlying surface	None	None
Underlying surface reflectance	-	-
Number of channels	60	60
Number of layers	3	3
Max relative error (convergence criterion)	0.01	0.01
Phase function	Henyey-Greenstein	Henyey-Greenstein

Case	**Layer**	**Scattering coefficient**	**Absorption coefficient**	**Asymmetry factor**	**Layer thickness**
1	1	100	10	0.7	0.002
	2	100	10	0.2	0.002
	3	100	10	-0.3	0.002
2	1	100	10	0.7	0.002
	2	100	10	0.8	0.002
	3	100	10	0.9	0.002
3	1	100	10	-0.7	0.002
	2	100	10	0.9	0.002
	3	100	10	-0.3	0.002
4	1	100	10	0.7	0.002
	2	100	10	0.2	0.002
	3	100	10	-0.3	0.002
5	1	100	10	0.7	0.002
	2	100	10	0.8	0.002
	3	100	10	0.9	0.002
6	1	100	10	-0.7	0.002
	2	100	10	0.9	0.002
	3	100	10	-0.3	0.002

The plots below show that the algorithm breaks the loop when it would be possible in most but not all cases. In one case the algorithm breaks the loop without meeting the convergence criterion. The reason for this is that the first Fourier components do not reflect the behavior of the rest of the Fourier components in that case. As can be seen, the relative error is below the specified limit in all cases but one. About 40% computation time is saved.



2.7. Computational Shortcut for Averaged Results

A computational shortcut has been implemented to allow for much faster calculation of variables that depend only on the azimuthally averaged intensity. These variables are total reflectance, total transmittance, total absorptance, and azimuthally averaged BSDF. If only variables from this group are required, DORT2002 breaks the azimuthal loop after the first time instead of fulfilling the prescribed $2N$ times, thus giving a large decrease in computation time. Typical savings in computation time are between 95% and 99%.

3. Application of DORT2002

This chapter covers comparison of accuracy, applicability and speed between DORT2002 and three other models when applied to different sets of relevant test problems.

3.1. DORT2002 vs Kubelka-Munk

Theoretically, Kubelka-Munk is the simple two channel special case for DORT2002 if illumination, phase function and underlying surface are perfectly diffuse. Therefore, DORT2002 should yield results identical to Kubelka-Munk under these conditions. This was tested for various scattering and absorption coefficients and different thickness by calculating total reflectance for a medium over a black background (R_0), over another medium with total reflectance R_g (R), and over an opaque pad of the medium itself (R_∞) in the single layer and multilayer cases. $R_g = 0.5$ was used in the single layer cases, and $R_g = 0$ was used in the multilayer cases. The Kubelka-Munk calculations were done according to the equations in Pauler [16]. The DORT2002 parameters that were used to get the Kubelka-Munk conditions are given in the table below.

Diffuse intensity	1
Beam intensity	0
Depth at upper boundary	0
Underlying surface	Diffuse
Underlying surface reflectance	Varying
Number of channels	2
Layer thickness	Varying
Scattering coefficient	Varying
Absorption coefficient	Varying
Phase function	Henyey-Greenstein
Asymmetry factor	0

The definitions for the scattering and absorption coefficients differ between Kubelka-Munk (*s* and *k*) and DORT2002 (σ_s and σ_a). Relations between the coefficients of the two models and when these relations are relevant to make has been studied by Edström [17]. A parameter file that gives Kubelka-Munk conditions and translates from Kubelka-Munk to DORT2002 parameters is supplied with DORT2002.

3.1.1. Single Layer Case

The following tables show the results of the different single layer cases studied. As can be seen, DORT2002 and Kubelka-Munk give identical results in all cases, as should be expected. This verifies that DORT2002 indeed becomes Kubelka-Munk when $N = 1$. This is theoretically already clear, but it is interesting to verify numerically, since the implementations have nothing in common.

Parameters	Quantity	Kubelka-Munk	DORT2002
s = 100.00000	R_∞	0.98596	0.98596
k = 0.01000	R_0	0.09091	0.09091
w = 0.00100	R	0.52380	0.52380

Parameters	Quantity	Kubelka-Munk	DORT2002
s = 0.10000	R_∞	0.00495	0.00495
k = 10.00000	R_0	0.00010	0.00010
w = 0.00100	R	0.49012	0.49012

Parameters	Quantity	Kubelka-Munk	DORT2002
s = 10.00000	R_∞	0.86823	0.86823
k = 0.10000	R_0	0.09082	0.09082
w = 0.01000	R	0.52283	0.52283

Parameters	Quantity	Kubelka-Munk	DORT2002
s = 0.01000	R_∞	0.00005	0.00005
k = 100.00000	R_0	0.00004	0.00004
w = 0.01000	R	0.06770	0.06770

Parameters	Quantity	Kubelka-Munk	DORT2002
s = 1.00000	R_∞	0.26795	0.26795
k = 1.00000	R_0	0.26015	0.26015
w = 1.00000	R	0.27572	0.27572

Parameters	Quantity	Kubelka-Munk	DORT2002
s = 100.00000	R_∞	0.98596	0.98596
k = 0.01000	R_0	0.98596	0.98596
w = 10.00000	R	0.98596	0.98596

3.1.2. Multilayer Case

The following tables show the results of the different multilayer cases studied. All are three-layer cases with various optical properties, and with $w = 0.01$ for all layers. As can be seen, DORT2002 and Kubelka-Munk give identical results in all cases, except one. That deviation (marked with an asterisk in the table below) is the only one ever encountered during the studies, and it is believed to be due to loss of accuracy in intermediate results in the Kubelka-Munk implementation because of finite arithmetic.

Parameters	Quantity	Kubelka-Munk	DORT2002
s =[100.0000 100.0000 100.0000]	R_∞	0.64174	0.64174
k = [10.0000 10.0000 10.0000]	R_0	0.61695	0.61695

Parameters	Quantity	Kubelka-Munk	DORT2002
s =[100.0000 10.0000 1.0000]	R_∞	0.53047	(*) 0.53081
k = [1.0000 10.0000 100.0000]	R_0	0.51768	0.51768

Parameters	Quantity	Kubelka-Munk	DORT2002
s = [0.1000 100.0000 1.0000]	R_∞	0.07065	0.07065
k = [100.0000 0.1000 1.0000]	R_0	0.06825	0.06825

3.1.3. Angle Resolved (Multichannel) Case

It is tempting to think that if illumination, phase function and underlying surface were perfectly diffuse, the resulting light distribution would be perfectly diffuse. There would therefore be no need for DORT2002 in this case at all, since it gives identical results as Kubelka-Munk for $N = 1$. This is however wrong. The light distribution deviates from the perfectly diffuse due to the finite thickness of the medium. With more channels, say $N = 10$, DORT2002 detects and quantifies this.

It is important to realize that scattering is a local phenomenon, and even if every scattering event is perfectly diffuse, the total scattered light distribution needs not be perfectly diffuse due to edge effects. Kubelka and Munk did not recognize this fact in the first paper on their model. They assume infinite horizontal extension to avoid edge effects, but do not consider edge effects due to finite thickness. Even the follow-up paper of Kubelka that aims to theoretically derive a range of validity for that model does not recognize this fact, but rather assumes the opposite in the line of reasoning: "Practically it will be so when the illumination is a perfectly diffuse one and when the material forming specimens of different thickness reflects and transmits always perfectly diffused light only". It is intuitively tempting to believe so, but this is not true when the medium has finite thickness since some light escapes through the lower boundary, and the light distribution becomes slightly changed.

This is illustrated by the tables below, which repeat some of the earlier single and multilayer simulations, but now also with $N = 10$. This can be considered to be a true value, since simple tests show that DORT2002 has already converged (it is shown in another chapter in this report that DORT2002 always converges to the true value when N increases). As can be seen, the difference between the erroneous total reflectance given by Kubelka-Munk and the true value given by DORT2002 can be up to 15% and more, depending on the properties of the medium, and this *even under the theoretically ideal conditions that Kubelka-Munk was created for*.

Parameters	Quantity	Kubelka-Munk	DORT2002, $N = 10$
s = 100.00000	R_∞	0.98596	0.98381
k = 0.01000	R_0	0.09091	0.08429
w = 0.00100	R	0.52380	0.52199

Parameters	Quantity	Kubelka-Munk	DORT2002, $N = 10$
s = 0.10000	R_∞	0.00495	0.00406
k = 10.00000	R_0	0.00010	0.00010
w = 0.00100	R	0.49012	0.49021

Parameters	Quantity	Kubelka-Munk	DORT2002, $N = 10$
s = 1.00000	R_∞	0.26795	0.23399
k = 1.00000	R_0	0.26015	0.22259
w = 1.00000	R	0.27572	0.25144

Parameters	Quantity	Kubelka-Munk	DORT2002, $N = 10$
s =[100.0000 10.0000 1.0000]	R_∞	0.53047	0.48434
k = [1.0000 10.0000 100.0000]	R_0	0.51768	0.46222

Parameters	Quantity	Kubelka-Munk	DORT2002, $N = 10$
s = [0.1000 100.0000 1.0000]	R_∞	0.07065	0.08342
k = [100.0000 0.1000 1.0000]	R_0	0.06825	0.07720

It is also worth to say something about absolute calculation times. DORT2002 is optimized for much larger and more complicated problems, and does much unnecessary work for the Kubelka-Munk case. Calculating total reflectance with Kubelka-Munk on a standard PC takes around 40μs, while DORT2002 with $N = 1$ takes around 4ms for the same result, i.e. Kubelka-Munk is about 100 times faster. However, using DORT2002 with $N = 10$ to get correct results takes around 5ms, while Kubelka-Munk cannot deliver correct results.

3.1.4. Comments

DORT2002 should not be used with $N = 1$ for perfectly diffuse conditions. It gives identical results as Kubelka-Munk, but is much slower. For perfectly diffuse conditions DORT2002 should be used with $N = 10$ for corrects answers, or Kubelka-Munk should be used if its accuracy is acceptable.

DORT2002 overpowers Kubelka-Munk in almost every other way, as indicated by the following list.

- DORT2002 is an angular resolved model, and can simulate the angular distribution of the reflected and transmitted light. Kubelka-Munk only handles the perfectly diffuse case, and even this more or less erroneously since the fact that the light distribution deviates from the perfectly diffuse due to the finite thickness of the medium is ignored.
- DORT2002 can handle different illumination and detection geometries, e.g. kombinations of collimated sources and diffuse light with asymmetric distribution. Kubelka-Munk only handles the perfectly diffuse case.
- DORT2002 can simulate instrument geometry, and take into consideration the deviation from perfectly diffuse conditions that are imposed by the instrument itself, i.e. from openings, gloss trap etc. While Kubelka-Munk assumes perfect conditions, DORT2002 can quantify the error and correct for it. It is also possible to calculate error corrections for different instrument geometries beforehand, to make the instruments correct their own errors.
- Different instrument geometries (e.g. d/0 and 45/0) rank the same samples differently, which can be explained by DORT2002 but not by Kubelka-Munk.
- DORT2002 is capable to identify and quantify other intrinsic errors of the Kubelka-Munk model, such as parameter dependencies for translucent or strongly absorbing media [18].
- DORT2002 has an open and modularized structure, which makes it possible to easily add functions for handling more phenomena. For example, DORT2002 is already prepared to handle refraction between layers with different index of refraction, to be combined with surface models to handle gloss, and to handle fluorescence, which is needed to simulate media with optical brightening agents and calculate whiteness and brightness. Kubelka-Munk has a close structure that does not allow for new phenomena, and handles none of the specified examples.

The conclusion is that Kubelka-Munk should be replaced by DORT2002 in all applications except where the accuracy of Kubelka-Munk is already acceptable.

3.2. DORT2002 vs Grace

The Grace model [19] is a Monte-Carlo model for light scattering in paper and print. As a Monte-Carlo model Grace uses statistics to handle a large number of light wave packets on their way through a medium. The interaction between the wave packets and the medium is governed by fundamental physical laws.

DORT2002 solves a system of integro-differential equations. This means that the implementations of Grace and DORT2002 have nothing in common. It is therefore interesting to compare them, since if they yield similar results they support each other.

Any differential equation model treats the medium as a continuum with certain properties. In order to be able to compare the results, Grace simulated a homogenous coating layer, so that no assumptions of fiber or pore size, orientation and distribution would affect the outcome. The same medium was used throughout all tests, and the direction (θ_i) of the incident beam and the asymmetry factor (g) of the medium were varied as indicated in the text in the plots below, where BRDF, BTDF, total reflectance

and total transmittance are compared. In the first 16 plots Grace uses 700 000 wave packets, and in the last 3 plots Grace uses 14 000 000 wave packets.

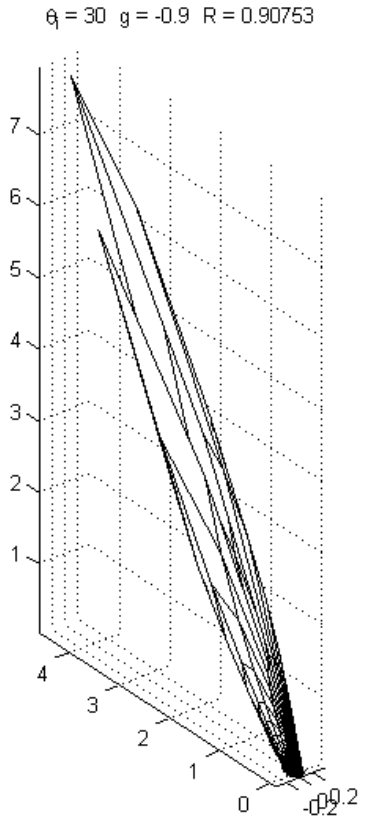
The DORT2002 parameters that were used to get the Kubelka-Munk conditions are given in the table below.

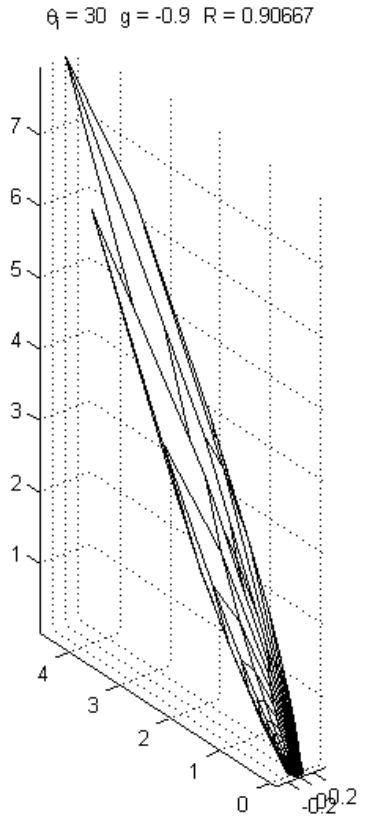
Diffuse intensity	0
Beam intensity	1
Beam polar angle cosine	Varying
Depth at upper boundary	0
Underlying surface	None
Number of channels	40
Number of layers	1
Layer thickness	1
Scattering coefficient	10
Absorption coefficient	0.01
Phase function	Henyey-Greenstein
Asymmetry factor	Varying

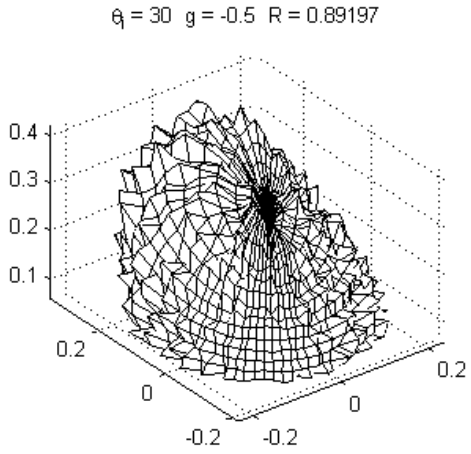
The BRDF and BTDF are not easy to compare numerically, since DORT2002 gives values in chosen distinct directions, while Grace must collect the statistically distributed wave packets over a finite solid angle. Furthermore, since the Grace results are indeed statistical, they show a somewhat spiky look. Therefore, the comparison is done by plotting the BRDF and BTDF next to each other, with Grace to the left and DORT2002 to the right in all plots. It is then up to the human eye to judge the similarity by imagining an averaging of the Grace results. The plots below show that the BRDF and BSDF have very similar size and shape. It is also convincing that with growing number of wave packets Grace converges to the DORT2002 results, which is shown in the last transmittance plots where the number of wave packets are 20-folded.

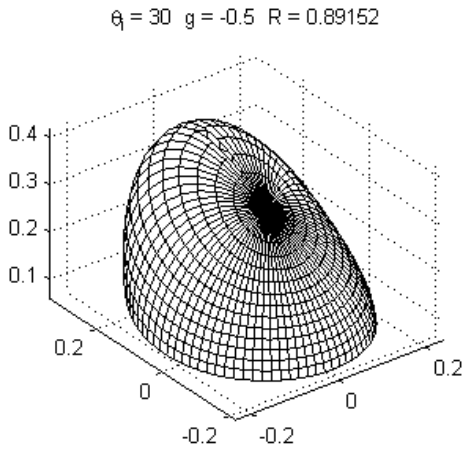
Total reflectance (R) and total transmittance (T) were compared numerically. As can be seen in the text in the plots below, the agreement was remarkably good considering the totally different implementations. The small differences are probably due to the statistical nature of the Grace results. The good agreement between the results of DORT2002 and Grace, in spite of their totally different implementations, supports both models.

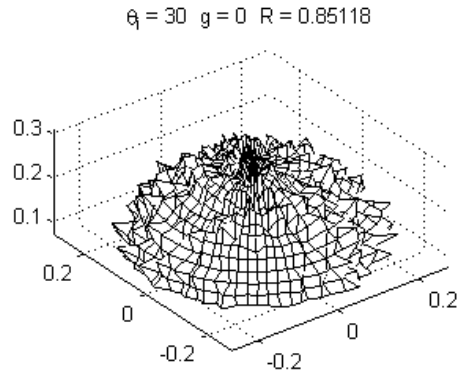
The following 16 plots are generated with 700 000 wave packets in Grace.

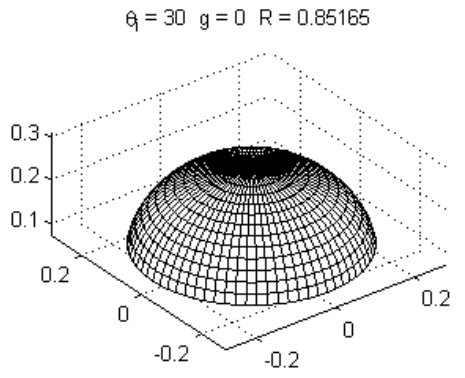


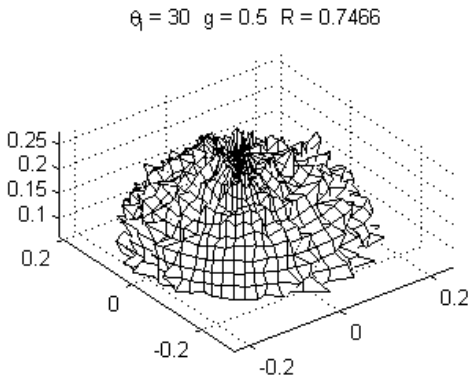


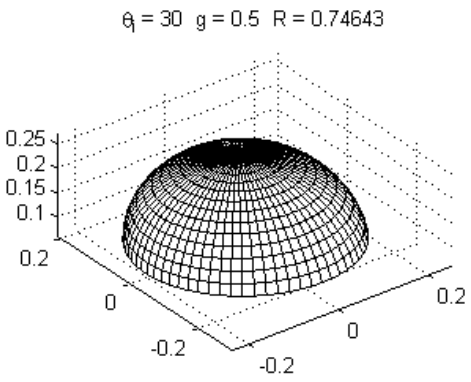


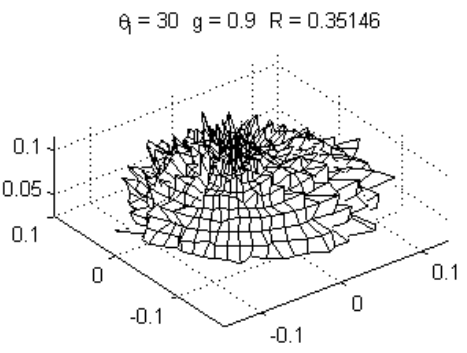


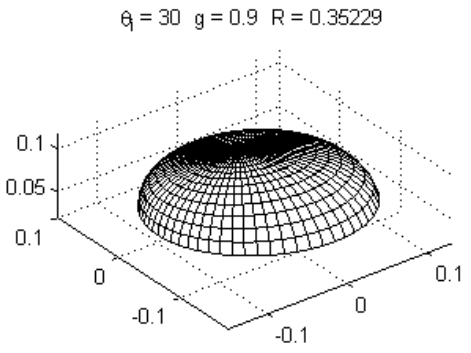

θi = 30 g = 0 R = 0.85118
0.3
0.2
0.1
0.2
0
-0.2
-0.2
0
0.2

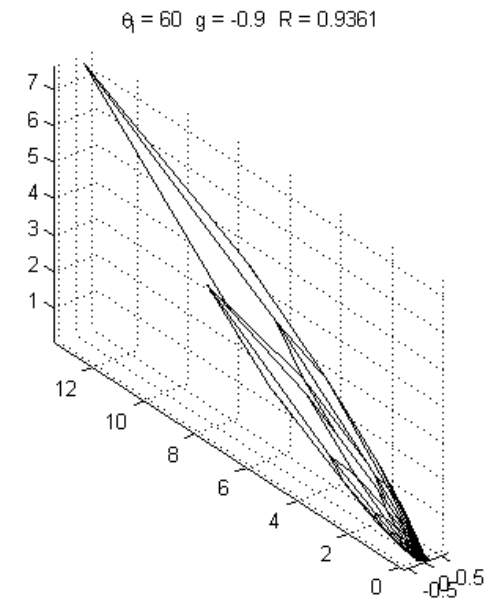

θi = 30 g = 0 R = 0.85165
0.3
0.2
0.1
0.2
0
-0.2
-0.2
0
0.2

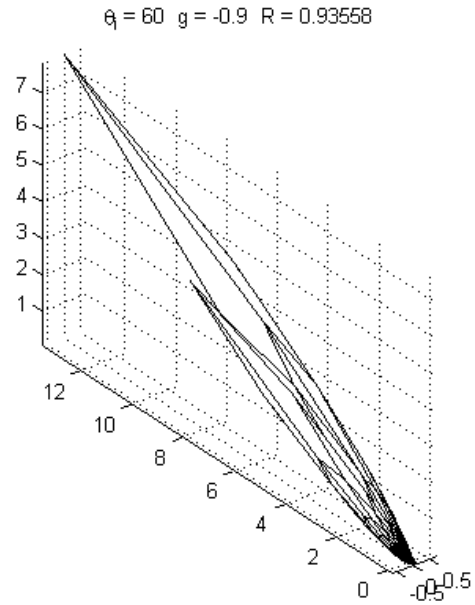

θi = 30 g = 0.5 R = 0.7466
0.25
0.2
0.15
0.1
0.2
0
-0.2
-0.2
0
0.2

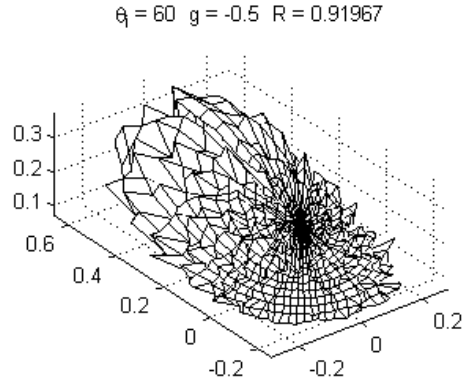

θi = 30 g = 0.5 R = 0.74643
0.25
0.2
0.15
0.1
0.2
0
-0.2
-0.2
0
0.2

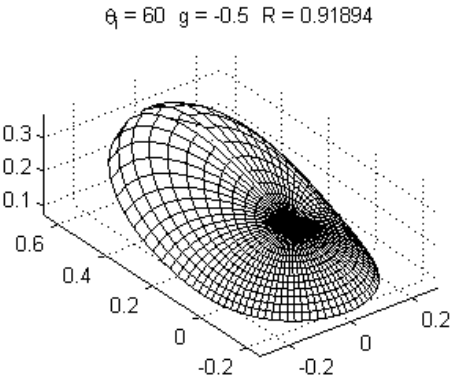

θi = 30 g = 0.9 R = 0.35146
0.1
0.05
0.1
0
-0.1
-0.1
0
0.1

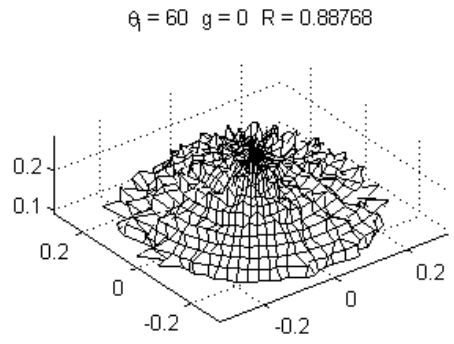

θi = 30 g = 0.9 R = 0.35229
0.1
0.05
0.1
0
-0.1
-0.1
0
0.1

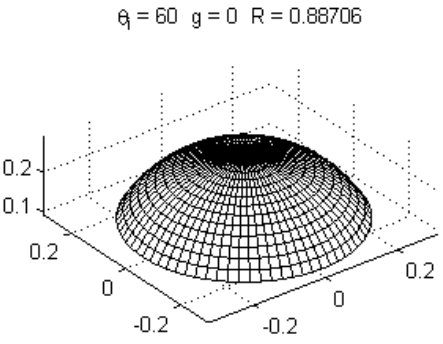

θi = 60 g = -0.9 R = 0.9361
7
6
5
4
3
2
1
12
10
8
6
4
2
0
-0.5
0
0.5

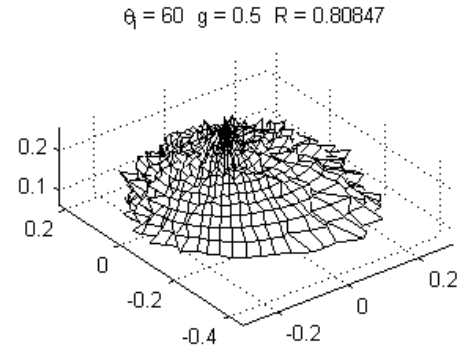

θi = 60 g = -0.9 R = 0.93558
7
6
5
4
3
2
1
12
10
8
6
4
2
0
-0.5
0
0.5

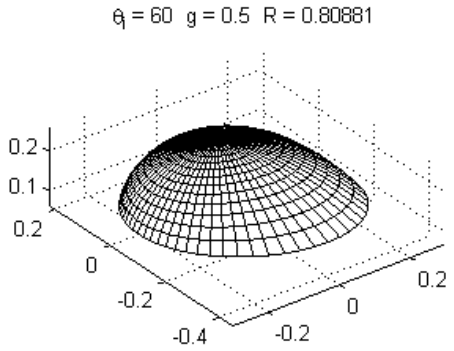

θi = 60 g = -0.5 R = 0.91967
0.3
0.2
0.1
0.6
0.4
0.2
0
-0.2
-0.2
0
0.2

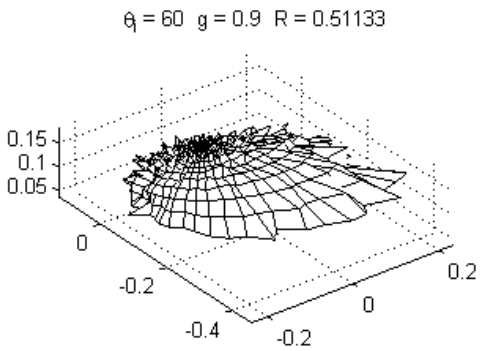

θi = 60 g = -0.5 R = 0.91894
0.3
0.2
0.1
0.6
0.4
0.2
0
-0.2
-0.2
0
0.2

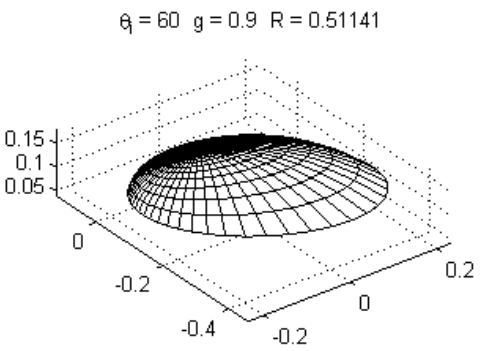

θi = 60 g = 0 R = 0.88768
0.2
0.1
0.2
0
-0.2
-0.2
0
0.2

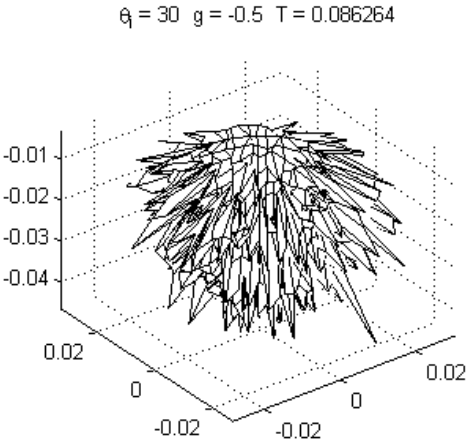

θi = 60 g = 0 R = 0.88706
0.2
0.1
0.2
0
-0.2
-0.2
0
0.2

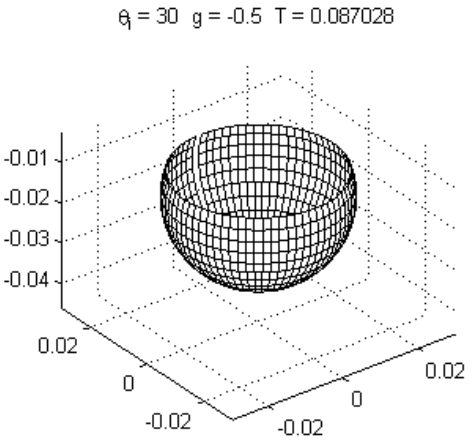

θi = 60 g = 0.5 R = 0.80847
0.2
0.1
0.2
0
-0.2
-0.4
-0.2
0
0.2

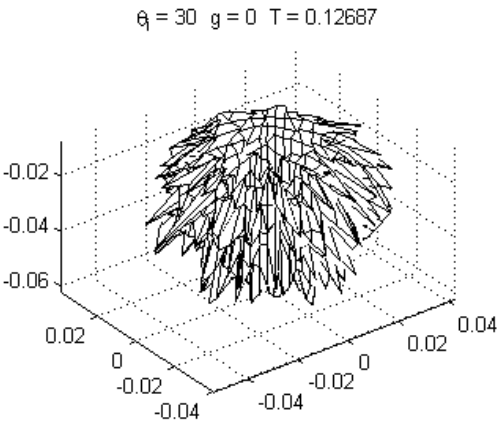

θi = 60 g = 0.5 R = 0.80881
0.2
0.1
0.2
0
-0.2
-0.4
-0.2
0
0.2

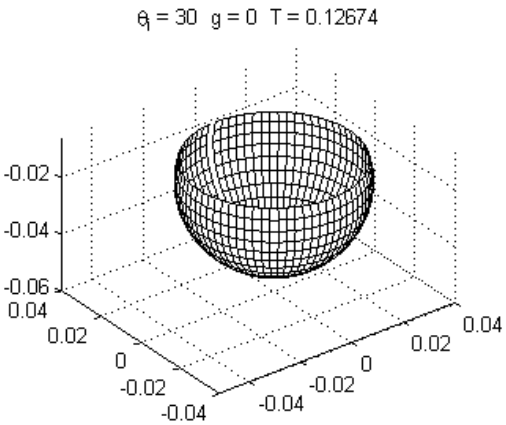

θi = 60 g = 0.9 R = 0.51133
0.15
0.1
0.05
0
-0.2
-0.4
-0.2
0
0.2

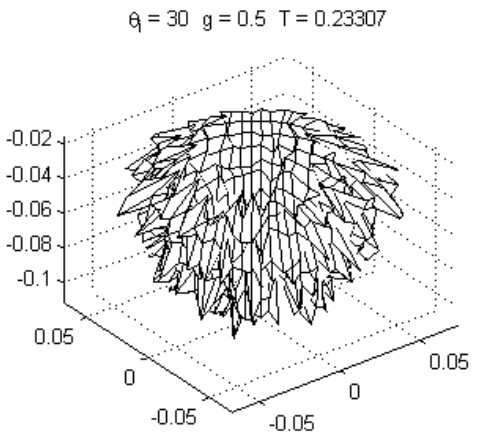

θi = 60 g = 0.9 R = 0.51141
0.15
0.1
0.05
0
-0.2
-0.4
-0.2
0
0.2

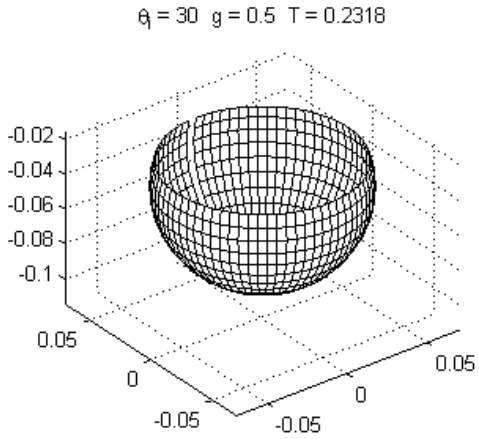

θi = 30 g = -0.5 T = 0.086264
-0.01
-0.02
-0.03
-0.04
0.02
0
-0.02
-0.02
0
0.02

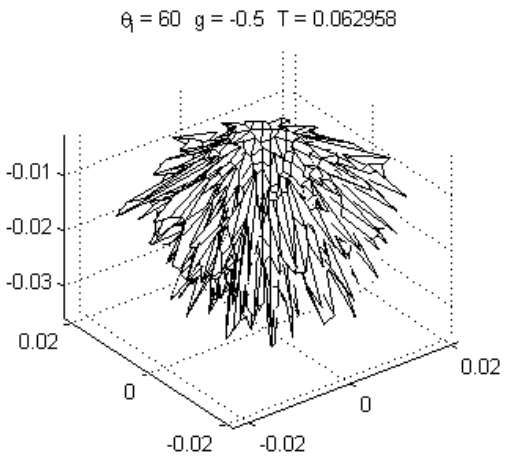

θi = 30 g = -0.5 T = 0.087028
-0.01
-0.02
-0.03
-0.04
0.02
0
-0.02
-0.02
0
0.02

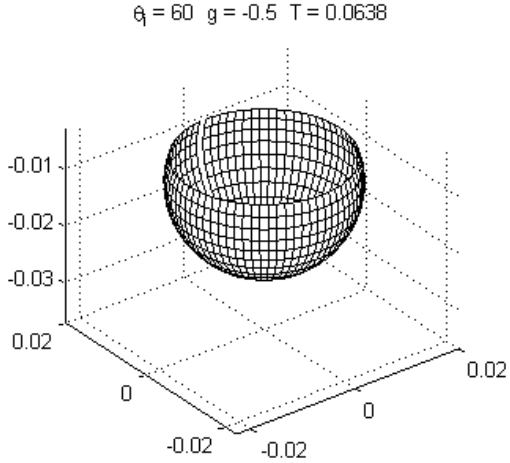

θi = 30 g = 0 T = 0.12687
-0.02
-0.04
-0.06
0.02
0
-0.02
-0.04
-0.04
0
0.02
0.04

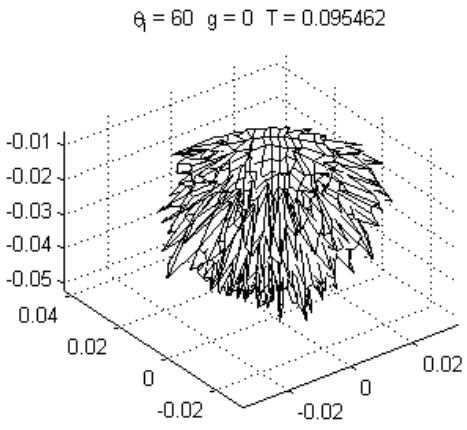

θi = 30 g = 0 T = 0.12674
-0.02
-0.04
-0.06
0.04
0.02
0
-0.02
-0.04
-0.02
0
0.02
0.04

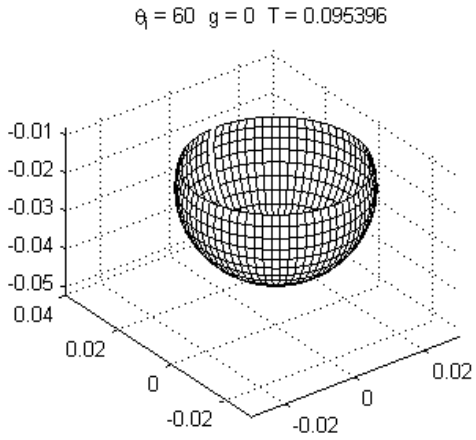

θi = 30 g = 0.5 T = 0.23307
-0.02
-0.04
-0.06
-0.08
-0.1
0.05
0
-0.05
0.05
0
-0.05

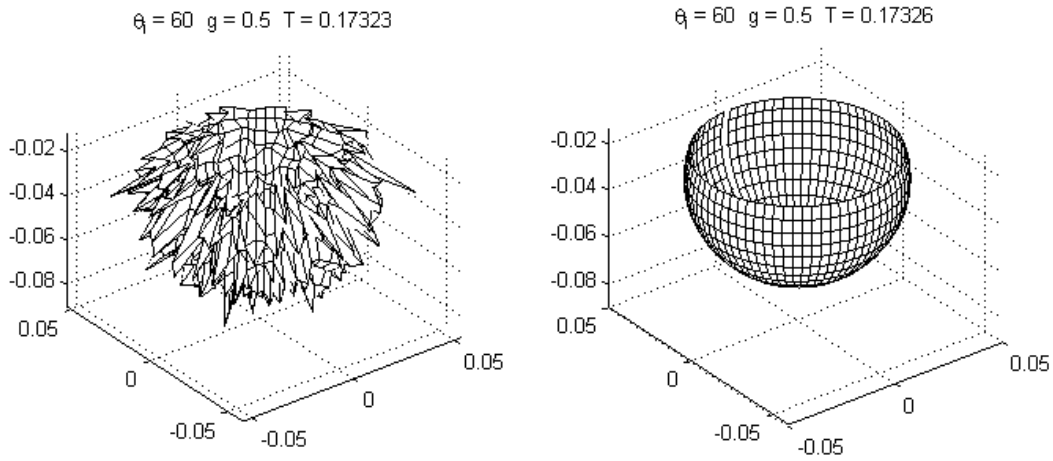

θi = 30 g = 0.5 T = 0.2318
-0.02
-0.04
-0.06
-0.08
-0.1
0.05
0
-0.05
0.05
0
-0.05


θi = 60 g = -0.5 T = 0.062958
-0.01
-0.02
-0.03
0.02
0
-0.02
0.02
0
-0.02

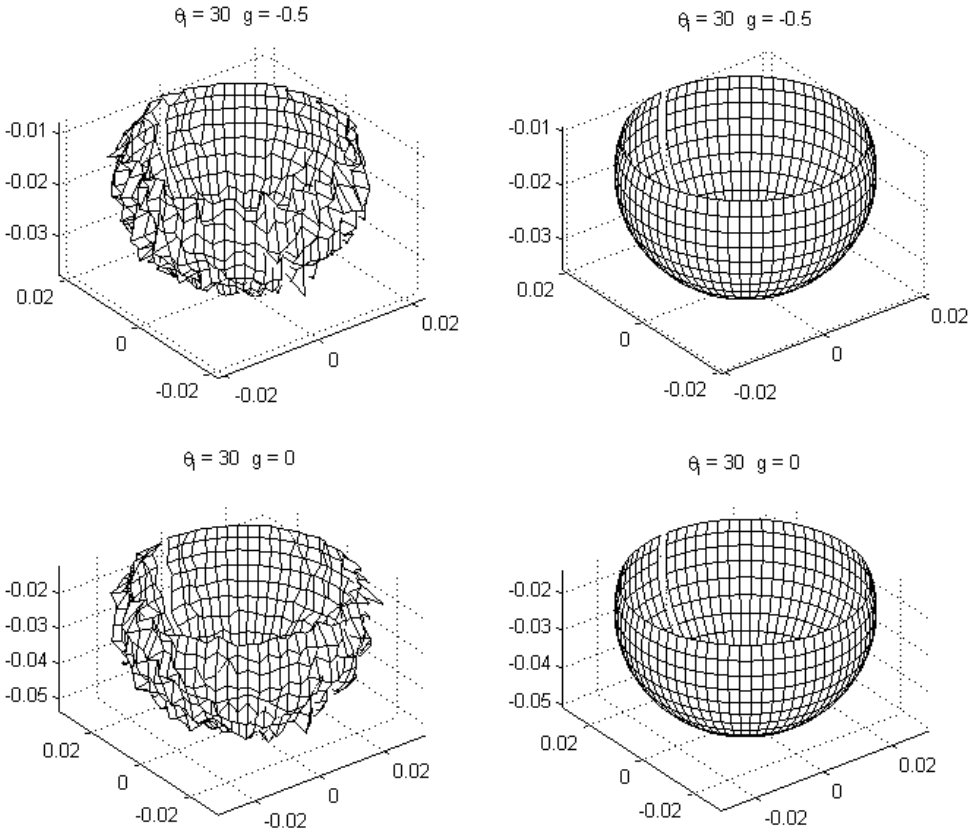

θi = 60 g = -0.5 T = 0.0638
-0.01
-0.02
-0.03
0.02
0
-0.02
0.02
0
-0.02

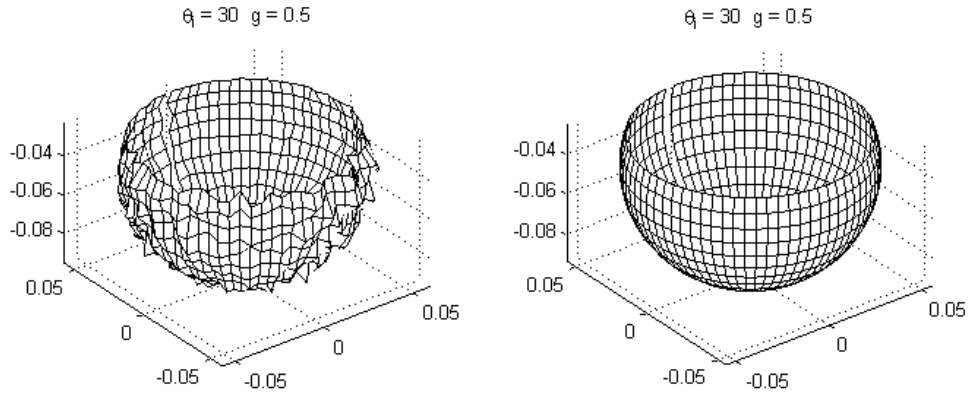

θi = 60 g = 0 T = 0.095462
-0.01
-0.02
-0.03
-0.04
-0.05
0.04
0.02
0
-0.02
0.02
0
-0.02


θi = 60 g = 0 T = 0.095396
-0.01
-0.02
-0.03
-0.04
-0.05
0.04
0.02
0
-0.02
0.02
0
-0.02



The following 3 plots are generated with 14 000 000 wave packets in Grace.





Of course, Grace is made for more complicated problems, and should not be used if the medium is indeed homogenous as in this study. Its strength instead lies in the ability to handle in principle any desired phenomena with fundamental physical laws, and thereby potentially contribute to the understanding of basic concepts of light scattering. On the negative side, which Grace shares with any Monte-Carlo model, is that quite a number of parameters are needed, several of which may be hard to give accurate values. Also, a very large number of wave packets is needed to get reasonable statistics, which makes the model computationally intensive with long calculation times.

It is therefore worth to say something about absolute calculation times. Calculating total reflectance, total transmittance, BRDF and BTDF with DORT2002 for any of the cases studied above on a standard PC takes around 9ms, while Grace takes around 3 hours for 700 000 wave packets and still 20 times longer for 14 000 000 wave packets. The total time for Grace consists of two parts, where the actual simulation is the smaller part, and forming the results from statistical simulation data is the larger. DORT2002 is thus about 1 million to 30 million times faster than Grace.

There are obviously some arguments for Grace instead of DORT2002 or the Kubelka-Munk model. Intuitively, paper and print are not homogeneous distributions of scattering and absorption sites, but rather specific distributions of pores, fibers, pigments and ink. However, DORT2002 is considerably faster and has fewer parameters than Grace, so the two models will probably both be used, with different demands on accuracy, complexity and speed.

The conclusion is that Kubelka-Munk should be replaced with DORT2002 for most applications, and that a combination of Grace and DORT2002 should be used for accurate modeling of paper and print.

3.3. DORT2002 vs DISORT

DISORT is a complete software for solving radiative transfer problems in a discrete ordinate model geometry, i.e. is has the same structure as DORT2002. In fact, DORT2002 has taken its overall structure from DISORT and uses the same textbook ideas in many cases. DORT2002 has however been implemented from scratch, many parts are entirely differently implemented, and the two models share no code. Numerical stability has been improved by reformulation of certain formulas. Also, a number of errors of Thomas and Stamnes have been corrected.

DISORT is claimed to be “a kind of standard against which to compare other modeling results” [12], and “no shortcuts were taken in which accuracy was sacrificed for speed”. DISORT has also been very thoroughly tested. DISORT is therefore ideally suited to verify the accuracy of DORT2002. The DISORT package includes a suite of published test problems [20] with parameter values and results. These test problems were used since they are publicly available, and since they cover most variations of normal cases as well as the most interesting extreme cases. Some of the problems in the suite were not used since they are not applicable. (DISORT is primarily designed for atmospheric research and includes for example internal thermal radiation, which is not present in the paper and print case.) The input parameters for the DISORT test cases were translated and used as input for DORT2002. The results from the two models were then compared, and absolute and relative differences were calculated.

The different test cases are briefly commented below. They include accuracy and consistency checks, variation of all parameters, and extreme cases with risk of breakdown.

- Test cases 1a-f, 3a-b and 4a-c involve a single layer with different thickness, varying proportions of scattering and absorption, and successively more difficult phase functions.
- Test cases 6a-c involve purely absorbing single layer cases with different boundary conditions.
- Test cases 8a-c involve two absorbing/isotropic-scattering layers with different thickness and varying proportions of scattering and absorption.
- Test cases 9a-b involve a general multilayer medium with every computational layer different, and all optical parameters varying.
- Test cases 11a-b checks consistency by ensuring that identical answers are obtained for a single layer and for several thinner layers, which together are identical to the single layer.

All parameters and results are found in tables in appendix. As can be seen in the tables, the agreement between the results from DORT2002 and DISORT is very good without exception. This is a strong support for the accuracy of DORT2002. It is interesting to verify this numerically, since the implementations have nothing in common although they share the overall structure, and DISORT is claimed to be “the finest radiative transfer software available”. The small deviations that are present in the result tables are of the same size as the round-off error in the DISORT test suite documentation, where results and parameters have a limited number of digits.

4. Suggestions for Future Work

There is room for improvements of DORT2002, and some developments have been suggested in previous reports. As a result of this study further investigations and developments can be suggested. This chapter shortly describes the most important issues.

Efficient generation of the sparse matrices from its diagonal blocks
One bottleneck that remains is the generation of the sparse matrix in the system of equations for the boundary and continuity conditions. Although the values and the indices of the nonzero elements are known, the assigning of these values to the sparse matrix is unsatisfactorily time consuming in MATLAB, to the extent that this purely administrative part of the code consumes around 75% of the total execution time. Since all computational parts of the code already are so optimized, this item is the first candidate to improve the speed of the code. This problem remains in spite of several different implementation strategies and consultation of experts at MathWorks (MATLAB is their product), and any suggestions to a solution would be most welcome.

Efficient evaluation of $\Lambda_l^m(u)$ for large *l* and *m*

The TMS and IMS methods take some relatively small time in themselves, but far more time takes the evaluation of the normalized associated Legendre functions, $\Lambda_l^m(u)$, for the larger *l* and *m* needed for these methods. Any studies that result in faster ways of evaluation of $\Lambda_l^m(u)$ for large *l* and *m* would be welcome.

Theoretical study of the terms in the Fourier expansion of the intensity
The method for breaking the azimuthal loop is based on an ad hoc assumption. It has been worked out with engineering skill and some trial and error. It works well in most tested cases, and it has a large positive impact on the computation time, but a closer study of the terms in the Fourier expansion of the intensity could give an even more efficient algorithm, and knowledge of when it is guaranteed to work. The needed number of turns in the azimuthal loop based on an upper limit for the relative error is sometimes unreliable for multilayer cases with different asymmetry factor. A theoretical study of the Fourier components could give a scientific basis to investigate this and propose an improvement.

5. Discussion

This report studies the performance and application of DORT2002, and is the documentation of an extensive test series. All major steps that are taken to improve stability and speed of DORT2002 are discussed, and the resulting improvements are illustrated. Comparison of accuracy, applicability and speed between DORT2002 and three other models when applied to different sets of relevant test problems is covered.

5.1. Performance Tests

It is shown that the system of equations for boundary and continuity conditions is very ill conditioned, with condition number near the largest positive floating-point number for the system ($10^{270} - 10^{300}$). After applying the preconditioner, the condition number is close to 1 in most cases, and around 30 in the worst case. The preconditioner also preserves the sparsity pattern, which is used to solve the system of equations efficiently. This indicates that the preconditioner works very well, and yields a system of equations well suited for numerical solution.

It is shown that two deliberate choices concerning the properties of the phase function and the numerical quadrature give the eigenvalue problem a symmetric structure that makes the eigenvalues come in positive/negative pairs. This allows reducing the size of the eigenvalue problem by a factor 2, and thus the eigenvalue calculation time is reduced by a factor 8. The reduced eigenvalue problem is very well conditioned, giving condition number close to 1 in all cases.

It is shown that DORT2002 always converges to the true value when N increases, and that a larger N is needed to maintain accuracy for larger g. The intensity correcting procedures make it possible to maintain accuracy for significantly lower N than otherwise needed in cases with strongly forward peaked scattering, and typically decreases the computation time with a factor ~1 000 – 10 000 in such cases.

It is shown that the algorithm for breaking the azimuthal loop based on a convergence criterion on the average gives 90% saving in computation time in single layer cases, and can thus be said to work very well in the single layer case. Due to the more unpredictable behavior of the Fourier components of the intensity in multilayer media with different asymmetry factors in different layers, the algorithm may in some cases not break the loop although it would be possible. On the other hand there are (relatively rare) cases where the algorithm breaks the loop without meeting the convergence criterion. Almost all cases give the specified accuracy, and most of them with about 50% saving in computation time.

It is shown that a computational shortcut, implemented to allow for much faster calculation of variables that depend only on the azimuthally averaged intensity, makes it possible to break the azimuthal loop after the first time instead of fulfilling the prescribed $2N$ times. Typical savings in computation time are between 95% and 99%.

It is clear from the performance tests that the steps that are taken to improve stability and speed of DORT2002 are very successful, together giving an unconditionally stable solution procedure to a problem previously considered numerically intractable, and

together decreasing computation time with a factor 1 000-10 000 in typical cases and with a factor up to and beyond 10 000 000 in extreme cases. Further investigations and developments are suggested, that can have a large positive impact on computation time.

5.2. Application Tests

It is shown that DORT2002 and Kubelka-Munk give identical results, which verifies that DORT2002 indeed becomes Kubelka-Munk when $N = 1$. This is theoretically already clear, but it is interesting to verify numerically, since the implementations have nothing in common. DORT2002 is optimized for much larger and more complicated problems, and does much unnecessary work for the Kubelka-Munk case. Kubelka-Munk is about 100 times faster in calculating total reflectance than DORT2002 with $N = 1$. However, using DORT2002 with $N = 10$ to get correct results takes no more time, while Kubelka-Munk generally cannot deliver correct results. A comprehensive list shows that DORT2002 overpowers Kubelka-Munk in almost every other way.

It is shown that DORT2002 and Grace have good agreement, and that Grace converges to DORT2002 with growing number of wave packets. Considering the totally different implementations, this supports both models. DORT2002 is about 1 million to 30 million times faster than Grace in calculating total reflectance, total transmittance, BRDF and BTDF. Of course, Grace is made for more complicated problems, and should not be used if the medium is indeed homogenous as in this study.

It is shown that the agreement between the results from DORT2002 and DISORT is very good without exception. This is a strong support for the accuracy of DORT2002. It is interesting to verify this numerically, since the implementations have nothing in common, and DISORT is claimed to be “the finest radiative transfer software available”.

The application tests show very good agreement with three other model types, and gives strong support for the accuracy of DORT2002. The accuracy, applicability and speed of the models are compared. The conclusion is that Kubelka-Munk should be replaced with DORT2002 for most applications, and that a combination of Grace and DORT2002 should be used for accurate modeling of paper and print.

6. Acknowledgements

This report was financially supported by T2F (“TryckTeknisk Forskning, a Swedish printing research program), which is gratefully acknowledged.

Appendix: DORT2002-DISORT Tests

Test 1a

Upper Boundary Conditions

```
Diffuse intensity =         0.0000
Beam intensity =           31.4200
Beam polar angle cosine =   0.1000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
No surface
Depth at lower boundary = 0.000313
```

Layer Properties

```
Number of computational layers = 1
        Layer      Scattering    Absorption   Asymmetry    Phase           No of phase
Layer:  thickness: coefficient:  coefficient: factor (g):  function type:  moments used:
-------------------------------------------------------------------------------------------
    1     0.00031     20.00000      80.00000      0.0000   Henyey-Greenstein          17
```

Calculation Settings

```
Half number of channels =       8
Number of calculation depths =  2
Output depths =     0.0000    0.0003
Output polar angle cosines = -1.0000  -0.5000  -0.1000   0.1000   0.5000   1.0000
Output azimuthal angles =     0.0000
```

DORT2002 Output Intensities

Depth:	Polar angle cosine:	Azimuthal angle(s): 0.0000
0.00000	1.0000	1.3406E-002
	0.5000	2.6420E-002
	0.1000	1.1779E-001
	-0.1000	0.0000E+000
	-0.5000	0.0000E+000
	-1.0000	0.0000E+000
0.00031	1.0000	0.0000E+000
	0.5000	0.0000E+000
	0.1000	0.0000E+000
	-0.1000	1.1591E-001
	-0.5000	2.6336E-002
	-1.0000	1.3384E-002

Calculation of Absolute and Relative diff. to DISORT Results

Depth:	Polar angle cosine:	Absolute diff.:	Relative diff.:
0.00000	1.0000	1.868218E-006	1.393776E-004
	0.5000	3.463353E-006	1.311032E-004
	0.1000	1.593265E-005	1.352862E-004
	-0.1000	0.000000E+000	0.000000E+000
	-0.5000	0.000000E+000	0.000000E+000
	-1.0000	0.000000E+000	0.000000E+000
0.00031	1.0000	0.000000E+000	0.000000E+000
	0.5000	0.000000E+000	0.000000E+000
	0.1000	0.000000E+000	0.000000E+000
	-0.1000	1.291318E-005	1.114166E-004
	-0.5000	3.765728E-006	1.430096E-004
	-1.0000	1.361939E-006	1.017663E-004

Test 1b

Upper Boundary Conditions

```
Diffuse intensity =         0.0000
Beam intensity =           31.4200
Beam polar angle cosine =   0.1000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
No surface
Depth at lower boundary = 0.000313
```

Layer Properties

```
Number of computational layers = 1
        Layer      Scattering    Absorption   Asymmetry    Phase           No of phase
Layer:  thickness: coefficient:  coefficient: factor (g):  function type:  moments used:
-------------------------------------------------------------------------------------------
    1     0.00031    100.00000       0.00000      0.0000   Henyey-Greenstein          17
```

Calculation Settings

```
DELTA-M method used in calculation.
Azimuthal break algorithm not used in calculation.
Half number of channels =       8
Number of calculation depths =  2
Output depths =     0.0000    0.0003
Output polar angle cosines = -1.0000  -0.5000  -0.1000   0.1000   0.5000   1.0000
Output azimuthal angles =     0.0000
```

DORT2002 Output Intensities

Depth:	Polar angle cosine:	Azimuthal angle(s):
		0.0000
0.00000	1.0000	7.0928E-002
	0.5000	1.3978E-001
	0.1000	6.2296E-001
	-0.1000	0.0000E+000
	-0.5000	0.0000E+000
	-1.0000	0.0000E+000
0.00031	1.0000	0.0000E+000
	0.5000	0.0000E+000
	0.1000	0.0000E+000
	-0.1000	6.1354E-001
	-0.5000	1.3935E-001
	-1.0000	7.0820E-002

Calculation of Absolute and Relative diff. to DISORT Results

Depth:	Polar angle cosine:	Absolute diff.:	Relative diff.:
0.00000	1.0000	1.235847E-005	1.742692E-004
	0.5000	2.105706E-005	1.506658E-004
	0.1000	8.445225E-005	1.355835E-004
	-0.1000	0.000000E+000	0.000000E+000
	-0.5000	0.000000E+000	0.000000E+000
	-1.0000	0.000000E+000	0.000000E+000
0.00031	1.0000	0.000000E+000	0.000000E+000
	0.5000	0.000000E+000	0.000000E+000
	0.1000	0.000000E+000	0.000000E+000
	-0.1000	7.749766E-005	1.263288E-004
	-0.5000	1.477130E-005	1.060091E-004
	-1.0000	5.116706E-006	7.225454E-005

Test 1c

Upper Boundary Conditions

```
Diffuse intensity =         1.0000
Beam intensity =            0.0000
Beam polar angle cosine =   0.1000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
No surface
Depth at lower boundary = 0.000313
```

Layer Properties

```
Number of computational layers = 1
        Layer      Scattering    Absorption   Asymmetry    Phase           No of phase
Layer:  thickness: coefficient:  coefficient: factor (g):  function type:  moments used:
-------------------------------------------------------------------------------------------
    1     0.00031     99.00000       1.00000      0.0000   Henyey-Greenstein          17
```

Calculation Settings

```
DELTA-M method used in calculation.
Azimuthal break algorithm not used in calculation.
Half number of channels =       8
Number of calculation depths =  2
Output depths =     0.0000    0.0003
Output polar angle cosines = -1.0000  -0.5000  -0.1000   0.1000   0.5000   1.0000
Output azimuthal angles =     0.0000
```

DORT2002 Output Intensities

Depth:	Polar angle cosine:	Azimuthal angle(s):
		0.0000
0.00000	1.0000	1.5223E-002
	0.5000	2.9988E-002
	0.1000	1.3318E-001
	-0.1000	1.0000E+000
	-0.5000	1.0000E+000
	-1.0000	1.0000E+000
0.00031	1.0000	0.0000E+000
	0.5000	0.0000E+000
	0.1000	0.0000E+000
	-0.1000	8.6395E-001
	-0.5000	9.6936E-001
	-1.0000	9.8445E-001

Calculation of Absolute and Relative diff. to DISORT Results

Depth:	Polar angle cosine:	Absolute diff.:	Relative diff.:
0.00000	1.0000	3.389593E-007	2.226626E-005
	0.5000	1.159858E-007	3.867739E-006
	0.1000	2.646649E-006	1.987272E-005
	-0.1000	0.000000E+000	0.000000E+000
	-0.5000	0.000000E+000	0.000000E+000
	-1.0000	0.000000E+000	0.000000E+000
0.00031	1.0000	0.000000E+000	0.000000E+000
	0.5000	0.000000E+000	0.000000E+000
	0.1000	0.000000E+000	0.000000E+000
	-0.1000	4.382969E-006	5.073174E-006
	-0.5000	2.629366E-006	2.712477E-006
	-1.0000	3.156068E-006	3.205920E-006

Test 1d

Upper Boundary Conditions

```
Diffuse intensity =         0.0000
Beam intensity =           31.4200
Beam polar angle cosine =   0.1000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
No surface
Depth at lower boundary = 0.320000
```

Layer Properties

```
Number of computational layers = 1
        Layer      Scattering    Absorption   Asymmetry    Phase           No of phase
Layer:  thickness: coefficient:  coefficient: factor (g):  function type:  moments used:
-------------------------------------------------------------------------------------------
    1     0.32000     20.00000      80.00000      0.0000   Henyey-Greenstein          17
```

Calculation Settings

```
DELTA-M method used in calculation.
Azimuthal break algorithm not used in calculation.
Half number of channels =       8
Number of calculation depths =  2
Output depths =     0.0000    0.3200
Output polar angle cosines = -1.0000  -0.5000  -0.1000   0.1000   0.5000   1.0000
Output azimuthal angles =     0.0000
```

DORT2002 Output Intensities

Depth:	Polar angle cosine:	Azimuthal angle(s):
		0.0000
0.00000	1.0000	5.0292E-002
	0.5000	9.0708E-002
	0.1000	2.6301E-001
	-0.1000	0.0000E+000
	-0.5000	0.0000E+000
	-1.0000	0.0000E+000
0.32000	1.0000	0.0000E+000
	0.5000	0.0000E+000
	0.1000	0.0000E+000
	-0.1000	6.8893E-018
	-0.5000	1.3072E-017
	-1.0000	1.2300E-015

Calculation of Absolute and Relative diff. to DISORT Results

Depth:	Polar angle cosine:	Absolute diff.:	Relative diff.:
0.00000	1.0000	6.802119E-006	1.352713E-004
	0.5000	1.145270E-005	1.262743E-004
	0.1000	3.640117E-005	1.384233E-004
	-0.1000	0.000000E+000	0.000000E+000
	-0.5000	0.000000E+000	0.000000E+000
	-1.0000	0.000000E+000	0.000000E+000
0.32000	1.0000	0.000000E+000	0.000000E+000
	0.5000	0.000000E+000	0.000000E+000
	0.1000	0.000000E+000	0.000000E+000
	-0.1000	8.930307E-022	1.296427E-004
	-0.5000	1.512512E-021	1.157240E-004
	-1.0000	1.570417E-019	1.276969E-004

Test 1e

Upper Boundary Conditions

```
Diffuse intensity =         0.0000
Beam intensity =           31.4200
Beam polar angle cosine =   0.1000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
No surface
Depth at lower boundary = 0.320000
```

Layer Properties

```
Number of computational layers = 1
        Layer      Scattering    Absorption   Asymmetry    Phase           No of phase
Layer:  thickness: coefficient:  coefficient: factor (g):  function type:  moments used:
-------------------------------------------------------------------------------------------
    1     0.32000    100.00000       0.00000      0.0000   Henyey-Greenstein          17
```

Calculation Settings

```
DELTA-M method used in calculation.
Azimuthal break algorithm not used in calculation.
Half number of channels =       8
Number of calculation depths =  2
Output depths =     0.0000    0.3200
Output polar angle cosines = -1.0000  -0.5000  -0.1000   0.1000   0.5000   1.0000
Output azimuthal angles =     0.0000
```

DORT2002 Output Intensities

Depth:	Polar angle cosine:	Azimuthal angle(s):
		0.0000
0.00000	1.0000	7.9730E-001
	0.5000	1.0275E+000
	0.1000	1.9335E+000
	-0.1000	0.0000E+000
	-0.5000	0.0000E+000
	-1.0000	0.0000E+000
0.32000	1.0000	0.0000E+000
	0.5000	0.0000E+000
	0.1000	0.0000E+000
	-0.1000	1.1639E-002
	-0.5000	1.8782E-002
	-1.0000	2.7134E-002

Calculation of Absolute and Relative diff. to DISORT Results

Depth:	Polar angle cosine:	Absolute diff.:	Relative diff.:
0.00000	1.0000	1.434690E-004	1.799751E-004
	0.5000	1.535310E-004	1.494510E-004
	0.1000	2.610456E-004	1.350329E-004
	-0.1000	0.000000E+000	0.000000E+000
	-0.5000	0.000000E+000	0.000000E+000
	-1.0000	0.000000E+000	0.000000E+000
0.32000	1.0000	0.000000E+000	0.000000E+000
	0.5000	0.000000E+000	0.000000E+000
	0.1000	0.000000E+000	0.000000E+000
	-0.1000	7.677329E-006	6.600747E-004
	-0.5000	1.467712E-005	7.820706E-004
	-1.0000	2.102202E-005	7.753484E-004

Test 1f

Upper Boundary Conditions

```
Diffuse intensity =         1.0000
Beam intensity =            0.0000
Beam polar angle cosine =   0.1000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
No surface
Depth at lower boundary = 0.320000
```

Layer Properties

```
Number of computational layers = 1
        Layer      Scattering    Absorption   Asymmetry    Phase           No of phase
Layer:  thickness: coefficient:  coefficient: factor (g):  function type:  moments used:
-------------------------------------------------------------------------------------------
    1     0.32000     99.00000       1.00000      0.0000   Henyey-Greenstein          17
```

Calculation Settings

```
DELTA-M method used in calculation.
Azimuthal break algorithm not used in calculation.
Half number of channels =       8
Number of calculation depths =  2
Output depths =     0.0000    0.3200
Output polar angle cosines = -1.0000  -0.5000  -0.1000   0.1000   0.5000   1.0000
Output azimuthal angles =     0.0000
```

DORT2002 Output Intensities

Depth:	Polar angle cosine:	Azimuthal angle(s):
		0.0000
0.00000	1.0000	7.5271E-001
	0.5000	8.1514E-001
	0.1000	8.7751E-001
	-0.1000	1.0000E+000
	-0.5000	1.0000E+000
	-1.0000	1.0000E+000
0.32000	1.0000	0.0000E+000
	0.5000	0.0000E+000
	0.1000	0.0000E+000
	-0.1000	7.7928E-004
	-0.5000	1.2649E-003
	-1.0000	1.8684E-003

Calculation of Absolute and Relative diff. to DISORT Results

Depth:	Polar angle cosine:	Absolute diff.:	Relative diff.:
0.00000	1.0000	5.247300E-006	6.971118E-006
	0.5000	4.242143E-006	5.204190E-006
	0.1000	2.404443E-007	2.740075E-007
	-0.1000	0.000000E+000	0.000000E+000
	-0.5000	0.000000E+000	0.000000E+000
	-1.0000	0.000000E+000	0.000000E+000
0.32000	1.0000	0.000000E+000	0.000000E+000
	0.5000	0.000000E+000	0.000000E+000
	0.1000	0.000000E+000	0.000000E+000
	-0.1000	9.608908E-009	1.233034E-005
	-0.5000	2.283469E-008	1.805257E-005
	-1.0000	4.583217E-009	2.453017E-006

Test 3a

Upper Boundary Conditions

```
Diffuse intensity =         0.0000
Beam intensity =            3.1420
Beam polar angle cosine =   1.0000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
No surface
Depth at lower boundary = 0.010000
```

Layer Properties

```
Number of computational layers = 1
        Layer      Scattering    Absorption   Asymmetry    Phase           No of phase
Layer:  thickness: coefficient:  coefficient: factor (g):  function type:  moments used:
-------------------------------------------------------------------------------------------
    1     0.01000    100.00000       0.00000      0.7500   Henyey-Greenstein          33
```

Calculation Settings

```
DELTA-M and TMS/IMS methods used in calculation.
Azimuthal break algorithm not used in calculation.
Half number of channels =       8
Number of calculation depths =  2
Output depths =     0.0000    0.0100
Output polar angle cosines = -1.0000  -0.5000  -0.1000   0.1000   0.5000   1.0000
Output azimuthal angles =     0.0000
```

DORT2002 Output Intensities

Depth:	Polar angle cosine:	Azimuthal angle(s): 0.0000
0.00000	1.0000	3.9551E-002
	0.5000	1.0113E-001
	0.1000	1.5124E-001
	-0.1000	0.0000E+000
	-0.5000	0.0000E+000
	-1.0000	0.0000E+000
0.01000	1.0000	0.0000E+000
	0.5000	0.0000E+000
	0.1000	0.0000E+000
	-0.1000	2.1371E-001
	-0.5000	2.6667E-001
	-1.0000	3.0589E+000

Calculation of Absolute and Relative diff. to DISORT Results

Depth:	Polar angle cosine:	Absolute diff.:	Relative diff.:
0.00000	1.0000	5.375883E-006	1.359400E-004
	0.5000	2.972233E-006	2.939022E-005
	0.1000	7.392449E-005	4.890156E-004
	-0.1000	0.000000E+000	0.000000E+000
	-0.5000	0.000000E+000	0.000000E+000
	-1.0000	0.000000E+000	0.000000E+000
0.01000	1.0000	0.000000E+000	0.000000E+000
	0.5000	0.000000E+000	0.000000E+000
	0.1000	0.000000E+000	0.000000E+000
	-0.1000	5.773237E-005	2.700677E-004
	-0.5000	6.007854E-006	2.253002E-005
	-1.0000	3.445062E-004	1.126353E-004

Test 3b

Upper Boundary Conditions

```
Diffuse intensity =         0.0000
Beam intensity =            3.1420
Beam polar angle cosine =   1.0000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
No surface
Depth at lower boundary = 0.080000
```

Layer Properties

```
Number of computational layers = 1
        Layer      Scattering    Absorption   Asymmetry    Phase           No of phase
Layer:  thickness: coefficient:  coefficient: factor (g):  function type:  moments used:
-------------------------------------------------------------------------------------------
    1     0.08000    100.00000       0.00000      0.7500   Henyey-Greenstein          33
```

Calculation Settings

```
DELTA-M and TMS/IMS methods used in calculation.
Azimuthal break algorithm not used in calculation.
Half number of channels =       8
Number of calculation depths =  2
Output depths =     0.0000    0.0800
Output polar angle cosines = -1.0000  -0.5000  -0.1000   0.1000   0.5000   1.0000
Output azimuthal angles =     0.0000
```

DORT2002 Output Intensities

Depth:	Polar angle cosine:	Azimuthal angle(s):
		0.0000
0.00000	1.0000	4.9336E-001
	0.5000	5.1968E-001
	0.1000	3.7983E-001
	-0.1000	0.0000E+000
	-0.5000	0.0000E+000
	-1.0000	0.0000E+000
0.08000	1.0000	0.0000E+000
	0.5000	0.0000E+000
	0.1000	0.0000E+000
	-0.1000	2.3635E-001
	-0.5000	4.2239E-001
	-1.0000	6.6967E-001

Calculation of Absolute and Relative diff. to DISORT Results

Depth:	Polar angle cosine:	Absolute diff.:	Relative diff.:
0.00000	1.0000	5.471552E-005	1.109151E-004
	0.5000	6.989517E-005	1.345147E-004
	0.1000	7.819067E-005	2.059004E-004
	-0.1000	0.000000E+000	0.000000E+000
	-0.5000	0.000000E+000	0.000000E+000
	-1.0000	0.000000E+000	0.000000E+000
0.08000	1.0000	0.000000E+000	0.000000E+000
	0.5000	0.000000E+000	0.000000E+000
	0.1000	0.000000E+000	0.000000E+000
	-0.1000	1.620900E-005	6.857468E-005
	-0.5000	3.972736E-005	9.406266E-005
	-1.0000	8.927585E-005	1.333311E-004

Test 4a

Upper Boundary Conditions

```
Diffuse intensity =         0.0000
Beam intensity =            3.1420
Beam polar angle cosine =   1.0000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
No surface
Depth at lower boundary = 0.010000
```

Layer Properties

```
Number of computational layers = 1
          Layer        Scattering     Absorption    Asymmetry    Phase           No of phase
Layer:    thickness:   coefficient:   coefficient:  factor (g):  function type:  moments used:
-------------------------------------------------------------------------------------------
    1       0.01000      100.00000        0.00000       -        User defined               33
Layer:  Phase Function Moments:
    1   1.000000   0.804200   0.646094   0.481851   0.359056   0.263045   0.191995   0.140907
        0.102831   0.076154   0.056054   0.042019   0.031295   0.023746   0.017919   0.013730
        0.010512   0.008100   0.006302   0.004862   0.003851   0.002963   0.002392   0.001829
        0.001506   0.001143   0.000960   0.000722   0.000618   0.000461   0.000402   0.000297
        0.000263
```

Calculation Settings

```
DELTA-M and TMS/IMS methods used in calculation.
Azimuthal break algorithm not used in calculation.
Half number of channels =      16
Number of calculation depths =  3
Output depths =     0.0000    0.0050    0.0100
Output polar angle cosines = -1.0000  -0.5000  -0.1000   0.1000   0.5000   1.0000
Output azimuthal angles =     0.0000
```

DORT2002 Output Intensities

Depth:	Polar angle cosine:	Azimuthal angle(s):
		0.0000
0.00000	1.0000	3.6496E-002
	0.5000	6.6115E-002
	0.1000	9.3043E-002
	-0.1000	0.0000E+000
	-0.5000	0.0000E+000
	-1.0000	0.0000E+000
0.00500	1.0000	1.7778E-002
	0.5000	4.0244E-002
	0.1000	1.2396E-001
	-0.1000	1.3493E-001
	-0.5000	1.1927E-001
	-1.0000	2.5164E+000
0.01000	1.0000	0.0000E+000
	0.5000	0.0000E+000
	0.1000	0.0000E+000
	-0.1000	1.5657E-001
	-0.5000	2.1971E-001
	-1.0000	3.3734E+000

Calculation of Absolute and Relative error to DISORT Results

Depth:	Polar angle cosine:	Absolute error:	Relative error:
0.00000	1.0000	3.400E-005	9.000E-004
	0.5000	1.499E-004	2.300E-003
	0.1000	3.489E-004	3.800E-003
	-0.1000	0.000E+000	0.000E+000
	-0.5000	0.000E+000	0.000E+000
	-1.0000	0.000E+000	0.000E+000
0.00500	1.0000	5.700E-006	3.000E-004
	0.5000	4.400E-005	1.100E-003
	0.1000	8.040E-005	6.000E-004
	-0.1000	5.300E-005	4.000E-004
	-0.5000	3.180E-005	3.000E-004
	-1.0000	3.099E-004	1.000E-004
0.01000	1.0000	0.000E+000	0.000E+000
	0.5000	0.000E+000	0.000E+000
	0.1000	0.000E+000	0.000E+000
	-0.1000	3.391E-004	2.200E-003
	-0.5000	1.067E-004	5.000E-004
	-1.0000	4.475E-004	1.000E-004

Test 4b

Upper Boundary Conditions

```
Diffuse intensity =         0.0000
Beam intensity =            3.1420
Beam polar angle cosine =   1.0000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
No surface
Depth at lower boundary = 0.010000
```

Layer Properties

```
Number of computational layers = 1
          Layer        Scattering     Absorption    Asymmetry    Phase           No of phase
Layer:    thickness:   coefficient:   coefficient:  factor (g):  function type:  moments used:
-------------------------------------------------------------------------------------------
    1       0.01000       90.00000       10.00000       -        User defined              33
Layer:  Phase Function Moments:
    1   1.000000   0.804200   0.646094   0.481851   0.359056   0.263045   0.191995   0.140907
        0.102831   0.076154   0.056054   0.042019   0.031295   0.023746   0.017919   0.013730
        0.010512   0.008100   0.006302   0.004862   0.003851   0.002963   0.002392   0.001829
        0.001506   0.001143   0.000960   0.000722   0.000618   0.000461   0.000402   0.000297
        0.000263
```

Calculation Settings

```
DELTA-M and TMS/IMS methods used in calculation.
Azimuthal break algorithm not used in calculation.
Half number of channels =      16
Number of calculation depths =  3
Output depths =     0.0000    0.0050    0.0100
Output polar angle cosines = -1.0000  -0.5000  -0.1000   0.1000   0.5000   1.0000
Output azimuthal angles =     0.0000
```

DORT2002 Output Intensities

Depth:	Polar angle cosine:	Azimuthal angle(s):
		0.0000
0.00000	1.0000	2.8276E-002
	0.5000	4.5521E-002
	0.1000	6.5316E-002
	-0.1000	0.0000E+000
	-0.5000	0.0000E+000
	-1.0000	0.0000E+000
0.00500	1.0000	1.3887E-002
	0.5000	2.7951E-002
	0.1000	8.4340E-002
	-0.1000	9.6149E-002
	-0.5000	9.6616E-002
	-1.0000	2.2429E+000
0.01000	1.0000	0.0000E+000
	0.5000	0.0000E+000
	0.1000	0.0000E+000
	-0.1000	1.0813E-001
	-0.5000	1.6772E-001
	-1.0000	2.9710E+000

Calculation of Absolute and Relative error to DISORT Results

Depth:	Polar angle cosine:	Absolute error:	Relative error:
0.00000	1.0000	7.100E-006	2.516E-004
	0.5000	6.900E-006	1.525E-004
	0.1000	1.080E-005	1.656E-004
	-0.1000	0.000E+000	0.000E+000
	-0.5000	0.000E+000	0.000E+000
	-1.0000	0.000E+000	0.000E+000
0.00500	1.0000	3.700E-006	2.656E-004
	0.5000	4.200E-006	1.506E-004
	0.1000	1.200E-005	1.427E-004
	-0.1000	1.620E-005	1.681E-004
	-0.5000	1.110E-005	1.151E-004
	-1.0000	2.664E-004	1.188E-004
0.01000	1.0000	0.000E+000	0.000E+000
	0.5000	0.000E+000	0.000E+000
	0.1000	0.000E+000	0.000E+000
	-0.1000	2.080E-005	1.924E-004
	-0.5000	1.800E-005	1.075E-004
	-1.0000	3.538E-004	1.191E-004

Test 4c

Upper Boundary Conditions

```
Diffuse intensity =         0.0000
Beam intensity =            3.1420
Beam polar angle cosine =   0.5000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
No surface
Depth at lower boundary = 0.010000
```

Layer Properties

```
Number of computational layers = 1
          Layer        Scattering     Absorption    Asymmetry    Phase           No of phase
Layer:    thickness:   coefficient:   coefficient:  factor (g):  function type:  moments used:
-------------------------------------------------------------------------------------------
    1       0.01000       90.00000       10.00000       -        User defined              33
Layer:  Phase Function Moments:
    1   1.000000   0.804200   0.646094   0.481851   0.359056   0.263045   0.191995   0.140907
        0.102831   0.076154   0.056054   0.042019   0.031295   0.023746   0.017919   0.013730
        0.010512   0.008100   0.006302   0.004862   0.003851   0.002963   0.002392   0.001829
        0.001506   0.001143   0.000960   0.000722   0.000618   0.000461   0.000402   0.000297
        0.000263
```

Calculation Settings

```
DELTA-M and TMS/IMS methods used in calculation.
Azimuthal break algorithm not used in calculation.
Half number of channels =      16
Number of calculation depths =  3
Output depths =     0.0000    0.0050    0.0100
Output polar angle cosines = -1.0000  -0.5000  -0.1000   0.1000   0.5000   1.0000
Output azimuthal angles =     0.0000   1.5708   3.1416
```

DORT2002 Output Intensities

Depth:	Polar angle cosine:	Azimuthal angle(s): 0.0000	1.5708	3.1416
0.00000	1.0000	2.2760E-002	2.2760E-002	2.2760E-002
	0.5000	2.2499E-001	5.7747E-002	5.0298E-002
	0.1000	8.7092E-001	8.8818E-002	6.9833E-002
	-0.1000	0.0000E+000	0.0000E+000	0.0000E+000
	-0.5000	0.0000E+000	0.0000E+000	0.0000E+000
	-1.0000	0.0000E+000	0.0000E+000	0.0000E+000
0.00500	1.0000	9.3299E-003	9.3299E-003	9.3299E-003
	0.5000	1.0914E-001	2.9588E-002	2.4775E-002
	0.1000	6.9778E-001	9.1617E-002	5.9135E-002
	-0.1000	1.4121E+000	1.0451E-001	6.2601E-002
	-0.5000	3.0267E+000	5.8104E-002	2.5858E-002
	-1.0000	4.7707E-002	4.7707E-002	4.7707E-002
0.01000	1.0000	0.0000E+000	0.0000E+000	0.0000E+000
	0.5000	0.0000E+000	0.0000E+000	0.0000E+000
	0.1000	0.0000E+000	0.0000E+000	0.0000E+000
	-0.1000	8.7663E-001	8.9557E-002	4.6720E-002
	-0.5000	2.7057E+000	9.4230E-002	3.9943E-002
	-1.0000	8.3859E-002	8.3859E-002	8.3859E-002

Calculation of Absolute and Relative error to DISORT Results

Depth:	Polar angle cosine:	Absolute error:	Relative error:
0.00000	1.0000	3.500E-006	1.525E-004
	0.5000	2.800E-005	1.244E-004
	0.1000	1.111E-004	1.276E-004
	-0.1000	0.000E+000	0.000E+000
	-0.5000	0.000E+000	0.000E+000
	-1.0000	0.000E+000	0.000E+000
0.00500	1.0000	1.300E-006	1.387E-004
	0.5000	1.360E-005	1.248E-004
	0.1000	9.150E-005	1.312E-004
	-0.1000	1.284E-004	9.100E-005
	-0.5000	3.977E-004	1.314E-004
	-1.0000	5.100E-006	1.079E-004
0.01000	1.0000	0.000E+000	0.000E+000
	0.5000	0.000E+000	0.000E+000
	0.1000	0.000E+000	0.000E+000
	-0.1000	1.150E-004	1.312E-004
	-0.5000	3.262E-004	1.206E-004
	-1.0000	1.000E-005	1.194E-004

Test 6a

Upper Boundary Conditions

```
Diffuse intensity =         0.0000
Beam intensity =          200.0000
Beam polar angle cosine =   0.5000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
No surface
Depth at lower boundary = 0.000000
```

Layer Properties

```
Number of computational layers = 1
        Layer      Scattering    Absorption   Asymmetry    Phase           No of phase
Layer:  thickness: coefficient:  coefficient: factor (g):  function type:  moments used:
-------------------------------------------------------------------------------------------
    1     0.00000      0.00000     100.00000      0.8480   Henyey-Greenstein          17
```

Calculation Settings

```
DELTA-M method used in calculation.
Azimuthal break algorithm not used in calculation.
Half number of channels =       8
Number of calculation depths =  2
Output depths =     0.0000    0.0000
Output polar angle cosines = -1.0000  -0.1000   0.1000   1.0000
Output azimuthal angles =     1.5708
```

DORT2002 Output Intensities

```
Depth:   Polar angle cosine:    Azimuthal angle(s):
-----------------------------------------------------------
                                1.5708

0.00000          1.0000         0.0000E+000
                 0.1000         0.0000E+000
                -0.1000         0.0000E+000
                -1.0000         0.0000E+000
0.00000          1.0000         0.0000E+000
                 0.1000         0.0000E+000
                -0.1000         0.0000E+000
                -1.0000         0.0000E+000
```

Calculation of Absolute and Relative diff. to DISORT Results

```
Depth:   Polar angle cosine:  Absolute diff.:  Relative diff.:
-------------------------------------------------------------
0.00000          1.0000         0.000000E+000    0.000000E+000
                 0.1000         0.000000E+000    0.000000E+000
                -0.1000         0.000000E+000    0.000000E+000
                -1.0000         0.000000E+000    0.000000E+000
0.00000          1.0000         0.000000E+000    0.000000E+000
                 0.1000         0.000000E+000    0.000000E+000
                -0.1000         0.000000E+000    0.000000E+000
                -1.0000         0.000000E+000    0.000000E+000
```

Test 6b

Upper Boundary Conditions

```
Diffuse intensity =         0.0000
Beam intensity =          200.0000
Beam polar angle cosine =   0.5000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
No surface
Depth at lower boundary = 0.010000
```

Layer Properties

```
Number of computational layers = 1
        Layer      Scattering    Absorption   Asymmetry    Phase           No of phase
Layer:  thickness: coefficient:  coefficient: factor (g):  function type:  moments used:
-------------------------------------------------------------------------------------------
    1     0.01000      0.00000     100.00000      0.8480   Henyey-Greenstein          17
```

Calculation Settings

```
DELTA-M method used in calculation.
Azimuthal break algorithm not used in calculation.
Half number of channels =       8
Number of calculation depths =  3
Output depths =     0.0000    0.0050    0.0100
Output polar angle cosines = -1.0000  -0.1000   0.1000   1.0000
Output azimuthal angles =     1.5708
```

DORT2002 Output Intensities

Depth:	Polar angle cosine:	Azimuthal angle(s):
		1.5708
0.00000	1.0000	0.0000E+000
	0.1000	0.0000E+000
	-0.1000	0.0000E+000
	-1.0000	0.0000E+000
0.00500	1.0000	0.0000E+000
	0.1000	0.0000E+000
	-0.1000	0.0000E+000
	-1.0000	0.0000E+000
0.01000	1.0000	0.0000E+000
	0.1000	0.0000E+000
	-0.1000	0.0000E+000
	-1.0000	0.0000E+000

Calculation of Absolute and Relative diff. to DISORT Results

Depth:	Polar angle cosine:	Absolute diff.:	Relative diff.:
0.00000	1.0000	0.000000E+000	0.000000E+000
	0.1000	0.000000E+000	0.000000E+000
	-0.1000	0.000000E+000	0.000000E+000
	-1.0000	0.000000E+000	0.000000E+000
0.00500	1.0000	0.000000E+000	0.000000E+000
	0.1000	0.000000E+000	0.000000E+000
	-0.1000	0.000000E+000	0.000000E+000
	-1.0000	0.000000E+000	0.000000E+000
0.01000	1.0000	0.000000E+000	0.000000E+000
	0.1000	0.000000E+000	0.000000E+000
	-0.1000	0.000000E+000	0.000000E+000
	-1.0000	0.000000E+000	0.000000E+000

Test 6c

Upper Boundary Conditions

```
Diffuse intensity =         0.0000
Beam intensity =          200.0000
Beam polar angle cosine =   0.5000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
Diffuse lower surface (Henyey-Greenstein)
Surface reflectance =     0.500000
Depth at lower boundary = 0.010000
```

Layer Properties

```
Number of computational layers = 1
        Layer      Scattering    Absorption   Asymmetry    Phase           No of phase
Layer:  thickness: coefficient:  coefficient: factor (g):  function type:  moments used:
-------------------------------------------------------------------------------------------
    1     0.01000      0.00000     100.00000      0.8480   Henyey-Greenstein          17
```

Calculation Settings

```
DELTA-M method used in calculation.
Azimuthal break algorithm not used in calculation.
Half number of channels =       8
Number of calculation depths =  3
Output depths =     0.0000    0.0050    0.0100
Output polar angle cosines = -1.0000  -0.1000   0.1000   1.0000
Output azimuthal angles =     1.5708
```

DORT2002 Output Intensities

Depth:	Polar angle cosine:	Azimuthal angle(s):
		1.5708
0.00000	1.0000	7.9239E-001
	0.1000	9.7788E-005
	-0.1000	0.0000E+000
	-1.0000	0.0000E+000
0.00500	1.0000	1.3064E+000
	0.1000	1.4513E-002
	-0.1000	0.0000E+000
	-1.0000	0.0000E+000
0.01000	1.0000	2.1539E+000
	0.1000	2.1539E+000
	-0.1000	2.1539E+000
	-1.0000	2.1539E+000

Calculation of Absolute and Relative diff. to DISORT Results

Depth:	Polar angle cosine:	Absolute diff.:	Relative diff.:
0.00000	1.0000	4.196720E-006	5.296281E-006
	0.1000	1.767438E-010	1.807419E-006
	-0.1000	0.000000E+000	0.000000E+000
	-1.0000	0.000000E+000	0.000000E+000
0.00500	1.0000	2.332847E-005	1.785706E-005
	0.1000	5.223344E-008	3.599079E-006
	-0.1000	0.000000E+000	0.000000E+000
	-1.0000	0.000000E+000	0.000000E+000
0.01000	1.0000	2.793018E-005	1.296726E-005
	0.1000	2.793018E-005	1.296726E-005
	-0.1000	Not comparable	
	-1.0000	Not comparable	

Test 8a

Upper Boundary Conditions

```
Diffuse intensity =         0.3183
Beam intensity =            0.0000
Beam polar angle cosine =   0.5000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
No surface
Depth at lower boundary = 0.005000
```

Layer Properties

```
Number of computational layers = 2
        Layer      Scattering    Absorption   Asymmetry    Phase           No of phase
Layer:  thickness: coefficient:  coefficient: factor (g):  function type:  moments used:
-------------------------------------------------------------------------------------------
    1     0.00250     50.00000      50.00000      0.0000   Henyey-Greenstein           9
    2     0.00250     30.00000      70.00000      0.0000   Henyey-Greenstein           9
```

Calculation Settings

```
DELTA-M method used in calculation.
Azimuthal break algorithm not used in calculation.
Half number of channels =       4
Number of calculation depths =  3
Output depths =     0.0000    0.0025    0.0050
Output polar angle cosines = -1.0000  -0.2000   0.2000   1.0000
Output azimuthal angles =     1.0472
```

DORT2002 Output Intensities

Depth:	Polar angle cosine:	Azimuthal angle(s):
		1.0472
0.00000	1.0000	1.9442E-002
	0.2000	5.6255E-002
	-0.2000	3.1830E-001
	-1.0000	3.1830E-001
0.00250	1.0000	5.5217E-003
	0.2000	1.8490E-002
	-0.2000	1.3695E-001
	-1.0000	2.6270E-001
0.00500	1.0000	0.0000E+000
	0.2000	0.0000E+000
	-0.2000	5.6036E-002
	-1.0000	2.1001E-001

Calculation of Absolute and Relative diff. to DISORT Results

Depth:	Polar angle cosine:	Absolute diff.:	Relative diff.:
0.00000	1.0000	3.048359E-007	1.567925E-005
	0.2000	2.159868E-006	3.839288E-005
	-0.2000	1.000000E-005	3.141592E-005
	-1.0000	1.000000E-005	3.141592E-005
0.00250	1.0000	1.889971E-007	3.422683E-005
	0.2000	6.408454E-007	3.465715E-005
	-0.2000	2.505449E-006	1.829462E-005
	-1.0000	6.819903E-006	2.595981E-005
0.00500	1.0000	0.000000E+000	0.000000E+000
	0.2000	0.000000E+000	0.000000E+000
	-0.2000	2.121882E-006	3.786506E-005
	-1.0000	2.236128E-006	1.064772E-005

Test 8b

Upper Boundary Conditions

```
Diffuse intensity =         0.3183
Beam intensity =            0.0000
Beam polar angle cosine =   0.5000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
No surface
Depth at lower boundary = 0.005000
```

Layer Properties

```
Number of computational layers = 2
        Layer      Scattering    Absorption   Asymmetry    Phase           No of phase
Layer:  thickness: coefficient:  coefficient: factor (g):  function type:  moments used:
-------------------------------------------------------------------------------------------
    1     0.00250     80.00000      20.00000      0.0000   Henyey-Greenstein           9
    2     0.00250     95.00000       5.00000      0.0000   Henyey-Greenstein           9
```

Calculation Settings

```
DELTA-M method used in calculation.
Azimuthal break algorithm not used in calculation.
Half number of channels =       4
Number of calculation depths =  3
Output depths =     0.0000    0.0025    0.0050
Output polar angle cosines = -1.0000  -0.2000   0.2000   1.0000
Output azimuthal angles =     1.0472
```

DORT2002 Output Intensities

Depth:	Polar angle cosine:	Azimuthal angle(s):
		1.0472
0.00000	1.0000	4.9557E-002
	0.2000	1.2368E-001
	-0.2000	3.1830E-001
	-1.0000	3.1830E-001
0.00250	1.0000	2.5057E-002
	0.2000	8.3567E-002
	-0.2000	1.8394E-001
	-1.0000	2.7749E-001
0.00500	1.0000	0.0000E+000
	0.2000	0.0000E+000
	-0.2000	1.2929E-001
	-1.0000	2.4072E-001

Calculation of Absolute and Relative diff. to DISORT Results

Depth:	Polar angle cosine:	Absolute diff.:	Relative diff.:
0.00000	1.0000	1.421284E-006	2.867921E-005
	0.2000	6.727705E-006	5.439166E-005
	-0.2000	1.000000E-005	3.141592E-005
	-1.0000	1.000000E-005	3.141592E-005
0.00250	1.0000	2.817534E-007	1.124450E-005
	0.2000	3.068033E-006	3.671213E-005
	-0.2000	5.577022E-006	3.031814E-005
	-1.0000	9.364565E-006	3.374618E-005
0.00500	1.0000	0.000000E+000	0.000000E+000
	0.2000	0.000000E+000	0.000000E+000
	-0.2000	3.261937E-006	2.522962E-005
	-1.0000	6.836837E-006	2.840044E-005

Test 8c

Upper Boundary Conditions

```
Diffuse intensity =         0.3183
Beam intensity =            0.0000
Beam polar angle cosine =   0.5000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
No surface
Depth at lower boundary = 0.030000
```

Layer Properties

```
Number of computational layers = 2
        Layer      Scattering    Absorption   Asymmetry    Phase           No of phase
Layer:  thickness: coefficient:  coefficient: factor (g):  function type:  moments used:
-------------------------------------------------------------------------------------------
    1     0.01000     80.00000      20.00000      0.0000   Henyey-Greenstein           9
    2     0.02000     95.00000       5.00000      0.0000   Henyey-Greenstein           9
```

Calculation Settings

```
DELTA-M method used in calculation.
Azimuthal break algorithm not used in calculation.
Half number of channels =       4
Number of calculation depths =  3
Output depths =     0.0000    0.0100    0.0300
Output polar angle cosines = -1.0000  -0.2000   0.2000   1.0000
Output azimuthal angles =     1.0472
```

DORT2002 Output Intensities

Depth:	Polar angle cosine:	Azimuthal angle(s):
		1.0472
0.00000	1.0000	1.0476E-001
	0.2000	1.4933E-001
	-0.2000	3.1830E-001
	-1.0000	3.1830E-001
0.01000	1.0000	6.5442E-002
	0.2000	9.6516E-002
	-0.2000	9.8813E-002
	-1.0000	1.8901E-001
0.03000	1.0000	0.0000E+000
	0.2000	0.0000E+000
	-0.2000	2.9669E-002
	-1.0000	6.8474E-002

Calculation of Absolute and Relative diff. to DISORT Results

Depth:	Polar angle cosine:	Absolute diff.:	Relative diff.:
0.00000	1.0000	7.422761E-006	7.084816E-005
	0.2000	4.527185E-008	3.031665E-007
	-0.2000	1.000000E-005	3.141592E-005
	-1.0000	1.000000E-005	3.141592E-005
0.01000	1.0000	2.505354E-006	3.828182E-005
	0.2000	2.846120E-006	2.948766E-005
	-0.2000	3.263639E-006	3.302744E-005
	-1.0000	5.539658E-006	2.930726E-005
0.03000	1.0000	0.000000E+000	0.000000E+000
	0.2000	0.000000E+000	0.000000E+000
	-0.2000	1.095935E-006	3.693747E-005
	-1.0000	1.888120E-006	2.757345E-005

Test 9a

Upper Boundary Conditions

```
Diffuse intensity =         0.3183
Beam intensity =            0.0000
Beam polar angle cosine =   0.5000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
No surface
Depth at lower boundary = 0.210000
```

Layer Properties

```
Number of computational layers = 6
          Layer        Scattering     Absorption    Asymmetry    Phase           No of phase
Layer:    thickness:   coefficient:   coefficient:  factor (g):  function type:  moments used:
-------------------------------------------------------------------------------------------
    1       0.01000       65.00000       35.00000       0.0000   Henyey-Greenstein           9
    2       0.02000       70.00000       30.00000       0.0000   Henyey-Greenstein           9
    3       0.03000       75.00000       25.00000       0.0000   Henyey-Greenstein           9
    4       0.04000       80.00000       20.00000       0.0000   Henyey-Greenstein           9
    5       0.05000       85.00000       15.00000       0.0000   Henyey-Greenstein           9
    6       0.06000       90.00000       10.00000       0.0000   Henyey-Greenstein           9
```

Calculation Settings

```
DELTA-M method used in calculation.
Azimuthal break algorithm not used in calculation.
Half number of channels =       4
Number of calculation depths =  5
Output depths =     0.0000    0.0105    0.0210    0.0600    0.2100
Output polar angle cosines = -1.0000  -0.2000   0.2000   1.0000
Output azimuthal angles =     1.0472
```

DORT2002 Output Intensities

Depth:	Polar angle cosine:	Azimuthal angle(s):
		1.0472
0.00000	1.0000	5.9133E-002
	0.2000	9.9888E-002
	-0.2000	3.1830E-001
	-1.0000	3.1830E-001
0.01050	1.0000	2.3190E-002
	0.2000	3.6699E-002
	-0.2000	5.0952E-002
	-1.0000	1.5350E-001
0.02100	1.0000	9.7228E-003
	0.2000	1.4854E-002
	-0.2000	2.0911E-002
	-1.0000	7.0659E-002
0.06000	1.0000	5.9472E-004
	0.2000	8.8329E-004
	-0.2000	1.0881E-003
	-1.0000	3.7277E-003
0.21000	1.0000	0.0000E+000
	0.2000	0.0000E+000
	-0.2000	1.0592E-007
	-1.0000	2.8765E-007

Calculation of Absolute and Relative diff. to DISORT Results

Depth:	Polar angle cosine:	Absolute diff.:	Relative diff.:
0.00000	1.0000	1.363106E-006	2.305113E-005
	0.2000	3.565609E-006	3.569464E-005
	-0.2000	1.000000E-005	3.141592E-005
	-1.0000	1.000000E-005	3.141592E-005
0.01050	1.0000	4.430523E-007	1.910532E-005
	0.2000	1.587491E-006	4.325470E-005
	-0.2000	1.494898E-006	2.933876E-005
	-1.0000	8.211237E-006	5.348992E-005
0.02100	1.0000	3.311865E-007	3.406183E-005
	0.2000	2.759774E-008	1.857933E-006
	-0.2000	7.163803E-007	3.425690E-005
	-1.0000	1.818672E-006	2.573799E-005
0.06000	1.0000	1.582365E-008	2.660599E-005
	0.2000	3.099865E-008	3.509334E-005
	-0.2000	8.301546E-008	7.628695E-005
	-1.0000	7.878868E-008	2.113544E-005
0.21000	1.0000	0.000000E+000	0.000000E+000
	0.2000	0.000000E+000	0.000000E+000
	-0.2000	2.503478E-012	2.363555E-005
	-1.0000	1.272267E-011	4.422814E-005

Test 9b

Upper Boundary Conditions

```
Diffuse intensity =         0.3183
Beam intensity =            0.0000
Beam polar angle cosine =   0.5000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
No surface
Depth at lower boundary = 0.210000
```

Layer Properties

```
Number of computational layers = 6
```

Layer:	Layer thickness:	Scattering coefficient:	Absorption coefficient:	Asymmetry factor (g):	Phase function type:	No of phase moments used:
1	0.01000	65.00000	35.00000	-	User defined	9
2	0.02000	70.00000	30.00000	-	User defined	9
3	0.03000	75.00000	25.00000	-	User defined	9
4	0.04000	80.00000	20.00000	-	User defined	9
5	0.05000	85.00000	15.00000	-	User defined	9
6	0.06000	90.00000	10.00000	-	User defined	9

```
Layer:  Phase Function Moments:
    1   1.000000  0.669720  0.312678  0.096296  0.024683  0.004295  0.000516  0.000045  0.000003
    2   1.000000  0.669720  0.312678  0.096296  0.024683  0.004295  0.000516  0.000045  0.000003
    3   1.000000  0.669720  0.312678  0.096296  0.024683  0.004295  0.000516  0.000045  0.000003
    4   1.000000  0.669720  0.312678  0.096296  0.024683  0.004295  0.000516  0.000045  0.000003
    5   1.000000  0.669720  0.312678  0.096296  0.024683  0.004295  0.000516  0.000045  0.000003
    6   1.000000  0.669720  0.312678  0.096296  0.024683  0.004295  0.000516  0.000045  0.000003
```

Calculation Settings

```
DELTA-M method used in calculation.
Azimuthal break algorithm not used in calculation.
Half number of channels =       4
Number of calculation depths =  5
Output depths =     0.0000    0.0105    0.0210    0.0600    0.2100
Output polar angle cosines = -1.0000  -0.2000   0.2000   1.0000
Output azimuthal angles =     1.0472
```

DORT2002 Output Intensities

Depth:	Polar angle cosine:	Azimuthal angle(s):
		1.0472
0.00000	1.0000	1.3276E-002
	0.2000	7.3917E-002
	-0.2000	3.1830E-001
	-1.0000	3.1830E-001
0.01050	1.0000	7.0554E-003
	0.2000	3.0022E-002
	-0.2000	5.9235E-002
	-1.0000	1.9660E-001
0.02100	1.0000	4.0692E-003
	0.2000	1.5267E-002
	-0.2000	3.0180E-002
	-1.0000	1.1547E-001
0.06000	1.0000	7.7787E-004
	0.2000	2.3829E-003
	-0.2000	3.8558E-003
	-1.0000	1.4617E-002
0.21000	1.0000	0.0000E+000
	0.2000	0.0000E+000
	-0.2000	1.2085E-005
	-1.0000	3.3773E-005

Calculation of Absolute and Relative diff. to DISORT Results

Depth:	Polar angle cosine:	Absolute diff.:	Relative diff.:
0.00000	1.0000	6.203533E-007	4.672390E-005
	0.2000	2.531483E-006	3.424626E-005
	-0.2000	1.000000E-005	3.141592E-005
	-1.0000	1.000000E-005	3.141592E-005
0.01050	1.0000	2.613779E-007	3.704493E-005
	0.2000	9.759753E-007	3.250759E-005
	-0.2000	1.928866E-006	3.256185E-005
	-1.0000	7.104144E-006	3.613318E-005
0.02100	1.0000	1.065420E-007	2.618189E-005
	0.2000	2.966042E-007	1.942780E-005
	-0.2000	1.045523E-006	3.464178E-005
	-1.0000	5.496984E-006	4.760117E-005
0.06000	1.0000	2.400598E-008	3.086038E-005
	0.2000	6.286154E-008	2.637916E-005
	-0.2000	1.245503E-007	3.230123E-005
	-1.0000	7.683790E-007	5.256389E-005
0.21000	1.0000	0.000000E+000	0.000000E+000
	0.2000	0.000000E+000	0.000000E+000
	-0.2000	5.437450E-010	4.498965E-005
	-1.0000	8.207794E-010	2.430211E-005

Test 11a

Upper Boundary Conditions

```
Diffuse intensity =         0.1592
Beam intensity =            1.0000
Beam polar angle cosine =   0.5000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
Diffuse lower surface (Henyey-Greenstein)
Surface reflectance =     0.500000
Depth at lower boundary = 0.010000
```

Layer Properties

```
Number of computational layers = 1
        Layer      Scattering    Absorption   Asymmetry    Phase           No of phase
Layer:  thickness: coefficient:  coefficient: factor (g):  function type:  moments used:
-------------------------------------------------------------------------------------------
    1     0.01000     90.00000      10.00000      0.0000   Henyey-Greenstein          17
```

Calculation Settings

```
DELTA-M method used in calculation.
Azimuthal break algorithm not used in calculation.
Half number of channels =       8
Number of calculation depths =  4
Output depths =     0.0000    0.0005    0.0050    0.0100
Output polar angle cosines = -1.0000  -0.1000   0.1000   1.0000
Output azimuthal angles =     0.0000   1.5708
```

DORT2002 Output Intensities

Depth:	Polar angle cosine:	Azimuthal angle(s): 0.0000	1.5708
0.00000	1.0000	1.4088E-001	1.4088E-001
	0.1000	2.1007E-001	2.1007E-001
	-0.1000	1.5920E-001	1.5920E-001
	-1.0000	1.5920E-001	1.5920E-001
0.00050	1.0000	1.3686E-001	1.3686E-001
	0.1000	2.0401E-001	2.0401E-001
	-0.1000	1.8274E-001	1.8274E-001
	-1.0000	1.6213E-001	1.6213E-001
0.00500	1.0000	1.0652E-001	1.0652E-001
	0.1000	1.5051E-001	1.5051E-001
	-0.1000	1.7216E-001	1.7216E-001
	-1.0000	1.7079E-001	1.7079E-001
0.01000	1.0000	8.5962E-002	8.5962E-002
	0.1000	8.5962E-002	8.5962E-002
	-0.1000	8.5962E-002	8.5962E-002
	-1.0000	8.5962E-002	8.5962E-002

Calculation of Absolute and Relative diff. to DISORT Results

Depth:	Polar angle cosine:	Absolute diff.:	Relative diff.:
0.00000	1.0000	1.901116E-005	1.349266E-004
	0.1000	7.009193E-005	3.337711E-004
	-0.1000	0.000000E+000	0.000000E+000
	-1.0000	0.000000E+000	0.000000E+000
0.00050	1.0000	6.356220E-005	4.646360E-004
	0.1000	1.162111E-005	5.696623E-005
	-0.1000	3.925702E-005	2.148715E-004
	-1.0000	2.618363E-005	1.615276E-004
0.00500	1.0000	1.606425E-005	1.508380E-004
	0.1000	1.299966E-005	8.637646E-005
	-0.1000	6.434792E-005	3.738985E-004
	-1.0000	5.936111E-006	3.475475E-005
0.01000	1.0000	1.155343E-005	1.344204E-004
	0.1000	1.155343E-005	1.344204E-004
	-0.1000	Not comparable	
	-1.0000	Not comparable	

Test 11b

Upper Boundary Conditions

```
Diffuse intensity =         0.1592
Beam intensity =            1.0000
Beam polar angle cosine =   0.5000
Beam azimuthal angle =      0.0000
Depth at upper boundary =   0.0000
```

Lower Boundary Conditions

```
Diffuse lower surface (Henyey-Greenstein)
Surface reflectance =     0.500000
Depth at lower boundary = 0.010000
```

Layer Properties

```
Number of computational layers = 3
          Layer        Scattering     Absorption    Asymmetry    Phase           No of phase
Layer:    thickness:   coefficient:   coefficient:  factor (g):  function type:  moments used:
-------------------------------------------------------------------------------------------------
    1       0.00050       90.00000       10.00000       0.0000   Henyey-Greenstein          17
    2       0.00450       90.00000       10.00000       0.0000   Henyey-Greenstein          17
    3       0.00500       90.00000       10.00000       0.0000   Henyey-Greenstein          17
```

Calculation Settings

```
DELTA-M method used in calculation.
Azimuthal break algorithm not used in calculation.
Half number of channels =       8
Number of calculation depths =  4
Output depths =     0.0000    0.0005    0.0050    0.0100
Output polar angle cosines = -1.0000  -0.1000   0.1000   1.0000
Output azimuthal angles =     0.0000   1.5708
```

DORT2002 Output Intensities

Depth:	Polar angle cosine:	Azimuthal angle(s): 0.0000	1.5708
0.00000	1.0000	1.4088E-001	1.4088E-001
	0.1000	2.1007E-001	2.1007E-001
	-0.1000	1.5920E-001	1.5920E-001
	-1.0000	1.5920E-001	1.5920E-001
0.00050	1.0000	1.3686E-001	1.3686E-001
	0.1000	2.0401E-001	2.0401E-001
	-0.1000	1.8274E-001	1.8274E-001
	-1.0000	1.6213E-001	1.6213E-001
0.00500	1.0000	1.0652E-001	1.0652E-001
	0.1000	1.5051E-001	1.5051E-001
	-0.1000	1.7216E-001	1.7216E-001
	-1.0000	1.7079E-001	1.7079E-001
0.01000	1.0000	8.5962E-002	8.5962E-002
	0.1000	8.5962E-002	8.5962E-002
	-0.1000	8.5962E-002	8.5962E-002
	-1.0000	8.5962E-002	8.5962E-002

Calculation of Absolute and Relative diff. to DISORT Results

Depth:	Polar angle cosine:	Absolute diff.:	Relative diff.:
0.00000	1.0000	2.098884E-005	1.490050E-004
	0.1000	3.009193E-005	1.432676E-004
	-0.1000	5.000000E-005	3.141690E-004
	-1.0000	5.000000E-005	3.141690E-004
0.00050	1.0000	2.356220E-005	1.721880E-004
	0.1000	3.162111E-005	1.550206E-004
	-0.1000	3.925702E-005	2.148715E-004
	-1.0000	4.618363E-005	2.849434E-004
0.00500	1.0000	1.606425E-005	1.508380E-004
	0.1000	2.299966E-005	1.528318E-004
	-0.1000	2.434792E-005	1.414426E-004
	-1.0000	3.406389E-005	1.994840E-004
0.01000	1.0000	1.255343E-005	1.460567E-004
	0.1000	1.255343E-005	1.460567E-004
	-0.1000	Not comparable	
	-1.0000	Not comparable	

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